Abstract

This paper analyzes optimal regulation to address the issue of “Too Big To Fail” (TBTF) in a simple model with government bailouts. Due to the commitment issue of government bailouts, a combination of capital requirement and size regulation can only achieve a second-best allocation, where the government has to bail out the banking sector in order to save on bankruptcy costs. A capital requirement can alleviate the cost of bailouts while a size regulation addresses the issue of TBTF. Meanwhile, a combination of capital requirement and issuance of Contingent Convertible Bonds (CoCos) can fully address the inefficiency and help implement the first-best allocation. In the optimal regulatory mechanism, CoCos improve the risk-sharing between the banker and depositors while the capital requirement addresses the moral hazard issue incurred by government bailouts.

Keywords: Too Big To Fail, Bailout, Optimal Regulation, CoCos

JEL Classification: G21, G28
1 Introduction

During the 2008-09 global financial crisis, government interventions in failing financial institutions, such as American International Group (AIG), fueled a resurgence of interest in the notion of “Too Big To Fail” (TBTF). The notion refers to the idea that some financial institutions are so large and complex that their failures are costlier to society than the required expenses to save them. As a result, policymakers will have to bail out those big financial institutions when they are in trouble, which could potentially lead to ex-ante moral hazard problems. Hence, there are many policy proposals aiming to address TBTF, such as capital requirements, a direct cap/tax on size, leverage ratio, etc. How effective are these policies? What is the optimal regulation?

In this paper, we first provide a simple framework to explain the emergence of TBTF. It happens because there is a costly bankruptcy process and government has an incentive to reduce the potentially detrimental associate deadweight loss. The inability for the government to credibly commit no-bailout in the bad state generates a typical moral hazard problem ex-ante—the banking sector thus takes advantage of the bailout and becomes Too-big-to-fail. In order to address the inefficiency associated with the TBTF, we consider a range of possible policies, including a minimum Tier-1 or Tier-2 capital requirement, a direct cap/tax on size, and issuance of CoCos. We find that neither the capital requirement nor the size regulation alone is sufficient to address the TBTF. The size regulation, price or quantity based, can reduce excessive size taken by the banking sector. However, there will still be the government bailout in the bad state due to the commitment issue. To minimize such a cost, a capital requirement is needed to force the banker to put more stake into the investment. In equilibrium, it is necessary to combine both the size regulation and capital requirement to reach the second-best allocation.

1The idea began to attract public attention in the 1970s when the Federal Deposit Insurance Corporation (FDIC) repeatedly bailed out institutions that it deemed “essential” to the community. In 1991, TBTF received an official status in the financial industry when Congress passed the FDIC Improvement Act, authorizing the FDIC to grant special treatments to a number of large banks. Although Congress attempted to abolish the notion of TBTF with the passage of the Dodd-Frank Act in 2010, the Act created a new category—Systemically Important Financial Institutions (SIFIs)—which not only included all previously TBTF institutions but also many new ones.
Our second result features an implementation of the first-best allocation. The key market friction is a limited risk-sharing between the banking sector and depositors. As a result, a more equity-like security such as the Contingent Convertible bonds (CoCos) can help to improve the risk-sharing. However, CoCos alone cannot implement the first best allocation since the banker always finds it optimal to take advantage of the bailout and issue defaultable debts, a typical moral hazard issue. Therefore, the capital requirement is necessary to rule out such a possibility. Intuitively, the existence of capital requirement helps to address the moral hazard issue incurred by the government bailout. As a result, when it is used together with the issuance of CoCos, the first-best allocation can be achieved.

Our model captures several layers of the issues surrounding TBTF. First, quite literally, banks can become too big to fail. This is because bank failure is typically costly to creditors and depositors, as well as disruptive to the local and even national economies (see Bernanke (1983); Chabot (2011); Bernanke (2013)). The larger the bank, the more costly and disruptive its failure will be (see White and Yorulmazer (2014); McAndrews et al. (2014)). When a large bank finds itself on the brink of collapse, the government is inclined to intervene in the form of recapitalization by using public funds (i.e., a bailout). Second, knowing that the government will intervene, banks have a strong incentive to become TBTF. Naturally, a bank that has received either the implicit or explicit status of TBTF will face less scrutiny from the market and will be able to raise more and cheaper debts (see Jacewitz and Pogach (2018); Strahan (2013); Santos (2014) for empirical evidence). Furthermore, TBTF banks will be more willing to gamble with their investments (see Davila and Walther (forthcoming); Afonso et al. (2014); Gropp et al. (2014)). Third, on anticipating such intervention and banks’ behavior, authorities have tried to regulate those banks that are (or may become) TBTF. For instance, under the authority of the FDIC Improvement Act, banks that received the TBTF status (implicitly or explicitly) were subject to a broader scope of regulation and supervision. However, TBTF banks continue to get larger in good times and require ever more public assistance in bad times (see Strahan (2013)).

As is known to all, regulating TBTF is not a simple task. According to Stern and Feldman (2004), it is difficult to identify and measure the TBTF problem be-
cause financial markets have grown not only in size but also in complexity. Furthermore, the benefit that TBTF institutions receive is mostly at the margin, which can vary greatly across firms of different sizes with different portfolio compositions and performance histories (see Ennis and Malek (2005)). Even considering the negative impacts of TBTF as given, optimal regulation remains debatable.\footnote{Furthermore, another fundamental concern regarding TBTF is the use of public funds to assist open banks. Strahan (2013) provides an excellent survey of the issue, arguing that TBTF is partly due to—and always reinforced by—the government’s commitment to assist large financial institutions in distress. The justification for an ex-post intervention can be traced back to Bagehot (1873), who explained the need and presented the principles for lending of last resort, and to Diamond and Dybvig (1983), whose model provides the rationale for policy actions that prevent widespread contagion of liquidity shocks. A formal argument for bailouts can also be found in the representation hypothesis by Dewatripont and Tirole (1994), who argue that depositors are too small, and thus need protection. Based on these reasons, governments have repeatedly provided bailouts to failing institutions throughout history, and they seemingly use more of the taxpayers’ money each time. To address this issue, a growing body of literature has advocated for “bail-in” regulation, thereby shifting the burden of saving failing banks from taxpayers to holders of high-yielding bonds. In particular, Sommer (2014) and Flannery (2014) support the use of convertible debts at the largest financial institutions as a counter-measure to the moral hazard of TBTF.} For instance, Johnson and Kwak (2011) argued for a straightforward cap on size and called for division of the largest financial institutions in the United States. Others, however, strongly resisted the idea for fear of inhibiting innovation and economies of scales.\footnote{Krugman, Paul. Financial Reform 101. April 1, 2010. http://www.nytimes.com/2010/04/02/opinion/02krugman.html. Retrieved April 20, 2015.} \footnote{Indeed, a number of studies have found evidence of economies of scale in banking (see Hughes and Mester (1998); Feng and Serletis (2010); Wheelock and Wilson (2012); Kovner et al. (2014)).}

In a simple model like ours, one can understand why regulating TBTF is a difficult task. According to the model, one need the capital requirement to address the moral hazard issue generated by the government bailout. Size regulation, however, is insufficient to address TBTF if it is used alone. Even though the size regulation is used optimally with the capital requirement, only the second-best allocation can be achieved due to the government’s inability to commit no bailout. To implement the first-best allocation, one will need to introduce CoCos and use capital requirement to address the moral hazard issue generated by government bailouts.

It is noteworthy that, in this paper, we aim to highlight the discussion about TBTF in terms of bank size, and thus choose to simplify our main model with
respect to systemic risk and its associated regulations. As Afonso et al. (2014), Cetorelli et al. (2014), and Laeven et al. (2014) argue, banks tend to become larger, riskier, and more complex simultaneously. Their complexity can generate systemic risk; in other words, the failure of one institution can lead to a wave of asset fire sales and credit flow disruptions in the financial system, such as the case of Lehman Brothers. For this reason, researchers and policymakers have spent a great deal of effort understanding, measuring, and mitigating systemic risk, especially over the last few years. However, bank size regulation remains a crucial aspect of TBTF. With the failure of Lehman Brothers, for instance, the resolution, which happened well after the financial crisis had passed, recovered less than 30 cents on the dollar for creditors, at a cost of more than $9 billion in administrative and other expenses (see Fleming and Sarkar (2014)). Meanwhile, Brewer III and Jagtiani (2013) examined the data on bank mergers in the United States between 1991 and 2004, finding that banks were willing to pay additional premiums in acquisitions that expanded their size into the TBTF regime. As size continues to play an important role in bankers’ business decisions and policymakers’ responses, the pros and cons of bank size regulation require more attention.

The paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 describes the model and examines the market outcome in a laissez-faire environment. Section 4 provides evaluate a range of policy responses to address the issue of TBTF. Section 5 concludes.

2 Literature Review

There are two bodies of literature that are closely related to this paper. The first one, which examines the interactions between bank size and the decisions of bankers and central bankers, provides rationales for TBTF and offers a range of optimal bailout strategies to offset it. The second one examines alternative ex-ante prudential policies to prevent banks from becoming TBTF. However, this latter strand of literature, while placing attention on systemic risk regulations, has not made conclusive statements regarding bank size regulation. The two bodies of literature are discussed

\footnote{For an exposition, see Acharya et al. (2010a).}
The literature regarding the rationales behind TBTF and bailout policies is expansive. Freixas (1999) provides a game-theoretic model of bank failure under the crucial assumption that the cost of bank liquidation increases with bank size. If the regulator cannot commit to an ex-ante policy, the model results in the regulator’s pure strategy of bailing out sufficiently large banks. In this case, banks anticipate the policy and structure their liabilities to maximize the value of a bailout. If the regulator can commit to an ex-ante policy, however, it is optimal for the regulator to follow a mixed strategy. In this case, banks are qualified for—but not guaranteed—a bailout when their uninsured funding is sufficiently small. Similarly, Goodhart and Huang (1999) analyze TBTF from the perspective of the lender of last resort (LOLR). In their model, the LOLR faces a request for liquidity injection from an illiquid bank, which may or may not be insolvent. If the LOLR complies and the bank is actually insolvent, there is a direct loss of capital. If the illiquid bank’s request is declined, however, the resulting failure triggers a loss of confidence by the bank’s depositors that is socially costly. Given that the cost of bank failure rises (with respect to bank size) faster than the cost of bank rescue, the authors find that a sufficiently large bank will always receive liquidity injections.

The link between bank size and ex-post bailouts has also been a subject of contention in this literature. Acharya and Yorulmazer (2007) provide a model that demonstrates a herding behavior by small banks. In their analysis, when the number of failures is small, the regulator will let other banks acquire the failed banks. However, when the number is large, the social cost is sufficiently high that an ex-post bailout is optimal. Anticipating the regulator’s ex-post decision, small banks tend to correlate their risk of failure to increase their bailout subsidy. The authors then argue that, in this case, a TBTF bank will differentiate itself from the small banks because its bailout subsidy does not increase with the herd. Meanwhile, a recent paper by Davila and Walther (forthcoming) contends that the presence of large banks exacerbates the risk-taking behavior of small banks and can lead to higher bailout costs. Using a model with a continuum of small banks and a definitive number of large banks, the author shows that by internalizing their size, large banks take on more risk to increase their chances of receiving bailouts. Even though
small banks cannot directly influence the equilibrium bailout probability via their individual leverage decisions, they handle more risk in the presence of large banks, and hence, in aggregate, increase the probability of bailouts. Size, as Davila and Walther (forthcoming) concludes, does matter.

With respect to this literature, our paper focuses on ex-ante regulations rather than bailout policies, while employing similar assumptions on the costs of bank failure and rescue. In particular, we focus on bank size regulation, and demonstrate that TBTF can be resolved with a combination of policies, including a cap on size and the use of CoCos.

While a general discussion of banking regulation can be found in Dewatripont and Tirole (1994) and Freixas and Rochet (1997), a number of recent papers have considered specific ex-ante prudential regulations of TBTF. Employing similar modeling techniques as Acharya and Yorulmazer (2007), Acharya (2009) finds that banks tend to invest in the same industry, increase the correlation of their returns and failures, and thereby extract greater bailout subsidies. The paper then proposes a capital requirement that considers the banks’ joint risk, and shows that such a policy can alleviate systemic risk. Similarly, Farhi and Tirole (2012) examine a model in which banks tend to hold little liquidity and take on too much correlated risk. Therefore, in their model, it is optimal for the regulator to impose an ex-ante liquidity requirement to eliminate ex-post bailout equilibriums. Furthermore, Calomiris and Herring (2013), Chen et al. (2013), and Sommer (2014) propose the use of convertible debts to ensure bank solvency. Directly tackling the banks’ incentive to take risk, Acharya et al. (2010b) build on their earlier framework in Acharya et al. (2017) and argue that the optimal policy to regulate systemic risk is a “Pigouvian tax.” This tax, which is based on the banks’ expected losses in a systemic crisis, varies with the banks’ size, leverage, risk, and correlation with the rest of the financial sector.

Our paper differs from this body of literature in considering a range of ex-ante regulations. In particular, we focus on the combinations of those regulations that can reach the first-best and second-best allocations.
3 The Model

Let us consider a two-period economy, $t = 1, 2$, with a continuum of depositors of mass 1 and a representative entrepreneurial banker, all of whom are risk-neutral.

**Depositors** are assumed to be deep pocket and they (collectively) receive an arbitrarily large endowment, $w$, at $t = 1$. Their utilities are given as follows:

$$U^d = E_1 \left[ c^d_1 + c^d_2 \right],$$

where $c^d_t$ is their time-$t$ consumption.

**Banker** receives an endowment, $e_0$, at $t = 1$, and has utility function as follows:

$$U^b = E_1 \left[ c^b_1 + c^b_2 \right],$$

where $c^b_t$ is her time-$t$ consumption. Moreover, $c^b_t \geq 0$, which captures the idea that equity issuance is costly and the risk-sharing between depositors and the banker is limited (see Brunnermeier and Sannikov (2014)).

**Technology** Only the banker has access to an investment technology. At date 1, the banker chooses investment scale $I \geq 0$. At date 2 the investment yields either $\rho(I)I$ in good state with probability $\alpha$ or $\rho I$ in bad state with probability $1 - \alpha$. Here $\rho < 1$ captures a loss in bad state.\(^6\)

**Possibility of Bankruptcy** The banker can use either internal equity $E$ or issue debt $D$ with a promised return $R = 1 + r$ to finance investment. Bankruptcy occurs whenever the promised repayment is not satisfied. In the case of bankruptcy, limited liability requires a liquidation of bank’s asset. However, there is a deadweight loss in this process and we assume that only a fraction $\lambda \in [0, 1)$ of investment can be recovered.

**Issuance of Risky Debt** There are three options for the banker to conduct investment: (1) use internal equity, (2) issue risk-free debt, or (3) issue risky debt. In the case of option (1) and (2), there is no bankruptcy cost and thus no role for government intervention. In the real world, neither of them is realistic. Hence, we

\(^6\)The usual concavity assumption on technology applies here, i.e. $\rho(I)I$ is strictly increasing and concave.
impose the following assumption such that the banker finds it optimal to issue risky
debt and the bankruptcy occurs in the bad state.

**Assumption 1. (Issuance of Risky Debt)**

Let $I_0 = \frac{e_0}{1-\rho}$, where $I_0$ is the maximum amount of investment the banker can have when issuing only risk-free deposits. The following relations hold,

$$\alpha \left[ \rho(I_0)I_0 \right]' + (1 - \alpha)\rho > 1$$

and

$$I_0 \geq \alpha \rho(I_0)I_0 + (1 - \alpha)\rho I_0.$$

Assumption 1 implies that the banker finds it optimal to expand its investment scale beyond the maximum amount of investment funded only by internal equity and risk-free debt. The internal equity and risk-free debt are perfect substitute to the banker since their costs are the same.

### 3.1 Laissez Faire

We only consider a Laissez Faire with risky debt due to Assumption 1. The maximization problem is given as follows, where the banker chooses internal equity $E$, investment size $I$ and promised return $r$ to maximize her utility function.\(^7\)

$$\Pi^{LF} = \max_{E,I,r} E \left[ c_1^b + c_2^b \right] = e_0 - E + \alpha \left( \rho(I)I - (1+r)(I-E) \right)$$

s.t. $e_0 \geq E$ \hspace{1cm} (1)

$$\rho(I)I \geq (1+r)(I-E)$$ \hspace{1cm} (2)

$$\alpha(1+r)(I-E) + (1-\alpha)\rho I = I-E.$$ \hspace{1cm} (3)

where $E,I,r$ are the internal equity, investment scale and promised return of risky debt $D = I-E$ for the banker. Condition (1) and (2) are imposed by the limited risk-sharing between the banker and depositors, i.e. $c_t^b \geq 0$ for $t = 1,2$.

\(^7\)Assumption 1 implies that banks would like to issue risky debt since she finds it optimal to increase the size $I$ above the maximum amount of investment that she can have by issuing only the risk-free debt.
Condition (3) is the break-even condition for depositors. Substituting condition (3) into the utility function, we get the following problem.

$$\Pi^{LF} = \max_I e_0 + \alpha \rho(I)I + (1 - \alpha)\rho\lambda I - I$$

The first order condition implies that

$$\alpha [\rho (I^{LF}) I^{LF}]' + (1 - \alpha)\rho\lambda = 1$$

Notice the marginal benefit of the project at the bad state is \(\rho\lambda\) rather than \(\rho\) due to the bankruptcy cost.\(^8\) The cost is beared by the banker since the depositors only care about the expected return.

**Note** There is no difference between the internal equity and risky debt since their marginal costs are the same. The introduction of risky debt only reduces the marginal benefit of the project in the bad state due to the bankruptcy cost. To illustrate this point, we introduce the first best benchmark.

### 3.2 The First Best

We define the first best investment \(I^*\) that solves the following problem.

$$\Pi^{FB} = \max_I e_0 + w + \alpha \rho(I)I + (1 - \alpha)\rho I - I$$

The optimality condition implies that

$$\alpha [\rho (I^*) I^*]' + (1 - \alpha)\rho = 1$$

The difference between the Laissez Faire and the First Best lies in the marginal benefit of investment. Due to the bankruptcy cost in the bad state, the marginal benefit is lower in the Laissez Faire, corresponding to the inefficiently lower investment.

\(^8\)One might realize that the marginal benefit under Laissez Faire is smaller than the marginal benefit of the project since \(\lambda < 1\). Hence we need to impose an additional assumption such that the banker wants to take the risky debt as opposed to the risk-free debt. The assumption is that \(\Pi^{LF} > e_0 + \alpha \rho(l_0)l_0 + (1 - \alpha)\rho l_0 - l_0\), which is the utility of issuing the maximum amount of risk-free debt.
than the first best. A government, in this case, might have an incentive to bail out the banker so as to save the cost of bankruptcy. As noted later, we argue that this motive creates a moral hazard problem and leads to the issue of TBTF.

3.3 Government Bailout and TBTF

In the bad state, the banker declares bankruptcy. The government has an incentive to recapitalize (bail out) the bank since it saves the bankruptcy cost. However, there will be a cost to bail out banks. The government needs to tax depositors in order to finance the bailout funds. In reality, the taxation is distortionary. For simplicity, we assume that there is a fixed marginal cost in raising public funds, $\eta > 1$. For the government, it is optimal to bail out the bank if the total cost of liquidation exceeds the total cost of raising the necessary public funds. Formally, the condition for a bailout is:

$$\eta\left(\frac{(1+r)D - \rho I}{D} - \rho I\right) \leq \frac{(1-\lambda)\rho I}{\rho I},$$

where $D = I - E$ is the amount of risky debt and $r$ is the interest rate on the debt.

The bailout is an important feature in financial crisis. Arguably, the government finds it difficult to commit no bailout ex ante. To capture this phenomenon, we impose the following assumption such that bailout occurs whenever the bad state is realized.

**Assumption 2.**

$$\frac{(1-\lambda)\rho}{1-\rho} > \eta.$$

Assumption 2 says that the cost of bailout is sufficiently small than the bankruptcy cost. Hence, the government cannot commit not to bail out in the event of bankruptcy.

Due to the existence of bailout, TBTF emerges. The maximization problem for

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$^9$In reality, the justification can also come from the protection of small depositors as in Dewatripont and Tirole (1994).

$^{10}$This is a simplified assumption as in Jeanne and Korinek (2013). Introducing more structure on the distortion cost of taxation will not change the insight of this paper.
the banker is given as
\[
\Pi_{TBTF} = \max_{E,I} e_0 - E + \alpha \left[ \rho(I) I - (I - E) \right]
\]
s.t. \[ e_0 \geq E \]
\[ \rho(I) I \geq I - E \]

The optimality conditions imply that \( E = 0 \) and the optimal level of investment satisfies
\[
\left[ \rho \left( I_{TBTF} \right) I_{TBTF} \right]' = 1
\]

Note that bailout increases the marginal benefit of investment for the banker since she/he only cares about the good state. One can easily show that the size of investment \( I_{TBTF} \) exceeds the level of investment in the First Best level, \( I^* \) and Laissez Faire level, \( I_{LF} \), which is the notion of TBTF. Furthermore, the banker finds it optimal not to use the internal equity since the banker uses risky debt to gamble the profit in the good state. In other words, the existence of bailout creates a moral hazard problem in our model.

**Proposition 1. Government Bailout and TBTF**

*TBTF emerges due to the government’s bailout. In equilibrium, the banker takes zero internal equity and borrows to undertake an investment, whose size exceeds the First Best and Laissez Faire level.*

**Proof.** See the discussion above. \(\square\)

## 4 TBTF and Policy Responses

In this section, we evaluate a range of policy responses to address the issue of TBTF. In particular, we focus on capital requirement, size regulation and CoCos. To facilitate analysis, we define a second-best benchmark, where the government cannot commit no bailout.
4.1 Optimal Regulation for the Second Best

The second best allocation is defined as the case where the bailout occurs and the cost of bailout is internalized by the agents. To this end, the maximization problem for the society is given as follows

$$U_{SB} = \max_{I, E} e_0 + w - E + \alpha \left[ \rho(I) I - I + E \right] - (1 - \alpha) \eta \left( I - E - \rho I \right)$$

s.t. $E \leq e_0$.

where $(1 - \alpha) \eta \left( I - E - \rho I \right)$ captures the bailout cost in the bad state.

The optimality conditions require that $E = e_0$

and

$$\alpha \left[ \rho \left( I_{SB} \right) I_{SB} \right]' + (1 - \alpha) \left[ 1 - \eta \left( 1 - \rho \right) \right] = 1$$

Proposition 2. Implementation of the Second Best

To implement the second-best allocation, we need a combination of capital requirement and size regulation. In terms of size regulation, it is equivalent to adopt a price- or quantity- based regulation.

Proof. See Appendix A.

Even if there is only one source of inefficiency in the model, i.e. the commitment issue of bailout, one need two instruments to correct it. The reason is that the only inefficiency affects two decision margins of the banker, the issuance of internal equity and the risky debt. The government wants the size regulation to address the issue of TBTF so as to reduce the size of investment. Conditional on the optimal size of investment, the government also wants to reduce the cost of bailout, i.e. forcing the banker to put more internal equity in the project. As a result, a combination of capital requirement and size regulation is needed. In terms of the size regulation, there is an equivalent result between price- and quantity- based tools in the spirit of Weitzman (1974) since there is no asymmetric information between the government and the banker.
Corollary 1. Capital requirement or size regulation alone is insufficient to implement the second-best allocation. The capital requirement forces the banker to put more stake into the project but fails to address the issue of TBTF. The size regulation addresses the issue of TBTF but does not minimize the cost of bailout.

Proof. See Appendix B. □

Corollary 1 says that neither capital requirement nor size regulation can implement the second best allocation. Consistent with the conventional wisdom, the size regulation is efficient to address the issue of TBTF since it can restrict to size of the project. However, the banker still has an incentive to put zero capital into the project. The cost of bailout is not minimized. Differently, capital requirement forces the banker to put more stake into the project and minimizes the cost of bailout in the bad state. However, it is ineffective to address the issue of TIBTF. The banker finds it optimal to increase the size of investment so as to gamble in the good state.

4.2 Optimal Regulation for the First Best

The main inefficiency in the second best world is the issue of bankruptcy. In the world of the first best, there is no need for bankruptcy and thus such cost. One way to avoid the process of bankruptcy is to increase the risk-sharing mechanism between the banker and depositors. The issuance of CoCos is one option since the security converts to equity in the bad state. One might expect that the banker might choose to issue CoCos so as to achieve the first best level of investment. However, the following proposition shows that the capital requirement is needed in order to achieve the first best allocation.

Proposition 3. Implementation of the First Best
To implement the first-best allocation, we need a combination of capital requirement and issuance of Contingent Convertible bonds.

Proof. See Appendix C. □

Proposition 3 says that the implementation of the first best needs a combination of capital requirement and issuance of CoCo. The role of CoCo provides a better
risk-sharing mechanism between the banker and depositors. However, CoCo alone cannot implement the first-best since the banker always finds it optimal to issue risky debt and thus incur the bailout. Imposing the capital requirement is one way to reduce the moral hazard problem caused by the bailout. In a way, the capital requirement complements the risk-sharing role of CoCos. The following corollary illustrates this point.

**Corollary 2.** Contingent Convertible bonds alone cannot implement the first-best allocation. Instead, the banker finds it optimal to issue risky debt and trigger bailout.

*Proof.* See Appendix D.

5 Conclusion

This paper analyzes the optimal regulation in a simple model with government bailout. We find that TBTF emerges due to the existence of government bailout. To address the issue of TBTF, we find that a combination of capital requirement and issuance of CoCos is needed to implement the first-best allocation. The issuance of CoCos alone fails to increase the risk-sharing in the economy since the banker finds it optimal to issue risky debt and trigger bailout. The existence of capital requirement reduces the moral hazard problem incurred by the bailout and complements the risk-sharing role of CoCos. Furthermore, the use of capital requirement cannot address the issue of TBTF since the banker still issues risky debt to take advantage of government bailout. Therefore, the capital requirement is used to reduce the cost of bailout. To address the issue of TBTF in a world where the issuance of CoCos is not available and the government bailout is inevitable, a size regulation is needed. In other words, a combination of capital requirement and size regulation is required to achieve the second best allocation, where the capital requirement minimizes the cost of bailout and the size regulation addresses the issue of TBTF.

Our paper has important policy implications. First, our paper provides a simple framework to evaluate different policy proposals. In particular, our model provides a welfare benchmark framework to design policy packages. Second, our paper points out two important market frictions that policymakers need to address—limited
risk-sharing and moral hazard problem. To the extent that policy tools resolve such frictions, TBTF problem can be alleviated or completely removed.

There are several issues that we left for future research. First, our model is silent on the excessive risk-taking behavior by the banking sector. Introducing such a feature will enrich the analysis. It is interesting to explore such a case. Second, it is interesting to introduce TBTF in a full-fledged DSGE framework to understand its inefficiency quantitatively. In such an environment, understanding the tradeoff of regulatory policy instrument is worth investigating.
References


A Proof of Proposition 2

Proof. Suppose that the government imposes a capital requirement $E \geq \bar{E}$ and a quantity-based size regulation $I \leq \bar{I}$. The maximization problem for the banker is given as follows.

\[
\Pi(\bar{E}, \bar{I}) = \max_{E,I} e_0 - E + \alpha \left( \rho(I)I - (I - E) \right) \\
\text{s.t. } e_0 \geq E, E \geq \bar{E}, I \leq \bar{I}.
\]

The optimality conditions implies that

\[
E = \bar{E}, \ I = \min \left\{ I^{TBF}, \bar{I} \right\}.
\]

By choosing $\bar{E} = e_0$ and $\bar{I} = I^{SB}$, the second best allocation is implemented.

Similar procedure can be applied to a combination of capital requirement and price-based size regulation. The maximization problem for the banker is given as follows.

\[
\Pi(\bar{E}, \tau) = \max_{E,I} e_0 - E - \tau I + T + \alpha \left( \rho(I)I - (I - E) \right) \\
\text{s.t. } e_0 \geq E, E \geq \bar{E}.
\]

where in equilibrium $T = \tau I$.

The optimality conditions implies that

\[
E = \bar{E}, \ \alpha \left[ \rho(I)I \right]' = \tau + \alpha
\]

By choosing $\bar{E} = e_0$ and $\tau = (1 - \alpha) \left( 1 - \rho \right) \eta$, the second best allocation is implemented.

\[
\Box
\]
B Proof of Corollary 1

Proof. Assume that the government only has the policy tool of capital requirement. The maximization problem is given by
\[
\Pi(\bar{E}) = \max_{E,I} e_0 - E + \alpha \left( \rho(I)I - (I - E) \right)
\]
s.t. \(e_0 \geq E, E \geq \bar{E}\).

The optimality conditions for the banker are given by
\[ E = \bar{E}, \quad [\rho(I)I]' = 1. \]

The Envelope theorem implies that \(\Pi'(\bar{E}) = -1 + \alpha\). For the government, her maximization problem is to choose \(\bar{E}\) to maximize the social welfare
\[
\max_{\bar{E}} \Pi(\bar{E}) + w - (1 - \alpha)\eta \left( I(\bar{E}) - \bar{E} - \rho I(\bar{E}) \right)
\]

The optimality condition implies that
\[-1 + \alpha - (1 - \alpha)\eta(-1) > 0\]

And the government chooses \(\bar{E} = e_0\).

Suppose the government only has the policy tool of size regulation. For the sake of argument, we only consider the quantity-based regulation. One can easily show that a price-based regulation can achieve the same allocation due to Weitzman (1974). The maximization problem is given by
\[
\Pi(\bar{I}) = \max_{E,I} e_0 - E + \alpha \left( \rho(I)I - (I - E) \right)
\]
s.t. \(e_0 \geq E, I \leq \bar{I}\).

The optimality conditions implies that
\[ E = 0, \quad I = \min \left\{ I^{TBF}, \bar{I} \right\}. \]
The Envelope theorem implies that \( \Pi'(\bar{I}) = \alpha \left( \rho(\bar{I}) \bar{I}' - \rho \bar{I} \right) \). For the government, her maximization problem is to choose \( \bar{I} \) to maximize the social welfare

\[
\max_{\bar{I}} \pi(\bar{I}) + w - (1 - \alpha) \eta \left( \bar{I} - \rho \bar{I} \right)
\]

The optimality condition implies that

\[
\alpha \left( \rho(\bar{I}) \bar{I}' \right) + (1 - \alpha) \left( 1 - \eta \left( 1 - \rho \right) \right) = 1
\]

And the government chooses \( \bar{I} = I_{SB} \). Since \( E = 0 \), the government cannot implement the second best. \( \square \)

C Proof of Proposition 3

Proof. Suppose that the government has the policy of capital requirement and issuance of CoCos. In the bad state, CoCos become external equity, which saves the cost of bankruptcy. The maximization problem is given as

\[
\Pi(\bar{E}) = \max_{E, B, I} e_0 - E - \alpha \left( \rho(I) I - (I - E - B) - (1 + r_B)B \right)
\]

s.t. 
\[
e_0 \geq E, E + B \geq \bar{E}, \]
\[
\alpha(1 + r_B)B + (1 - \alpha) \left[ \rho I - (I - E - B) \right] = B, \]
\[
\rho I - (I - E - B) \geq 0.
\]

where the last inequality ensures that the banker can repay debt and no bankruptcy is declared.

Substituting the break-even condition into the problem changes the problem into the following one.

\[
\Pi(\bar{E}) = \max_{E, B, I} e_0 + \alpha \rho(I) I + (1 - \alpha) \rho I - I
\]

s.t. 
\[
e_0 \geq E, E + B \geq \bar{E}, \]
\[
\rho I - (I - E - B) \geq 0.
\]
The optimality condition implies that

\[ \alpha \left[ \rho (I^*) I^* \right]' + (1 - \alpha) \bar{\rho} = 1 \]

Furthermore, \(\{E, B\}\) are indetermined. One can choose \(\bar{E} \in (1 - \bar{\rho}) I^*, I^*\) so as to satisfy the last inequality constraint. Importantly, the existence of \(\bar{E}\) is to rule out the possibility that the banker finds it optimal to issue only risky debt and triggers TBTF. See the proof in Appendix D where there is no capital requirement \(\bar{E}\).

\[ \square \]

D Proof of Corollary 2

Proof. Without regulatory constraint, the banker’s maximization problem is as follows

\[
\Pi^{CoCo} = \max_{E, B, I} e_0 - E + \alpha \left( \rho (I) I - (I - E - B) - (1 + r_B)B \right)
\]

s.t.
\[
e_0 \geq E,
\]
\[
\alpha (1 + r_B)B + (1 - \alpha) \left[ \rho I - (I - E - B) \right] = B,
\]
\[
\rho I - (I - E - B) \geq 0.
\]

Substituting the break-even condition into the problem changes the problem into the following one.

\[
\Pi^{CoCo} = \max_{E, B, I} e_0 + \alpha \rho (I) I + (1 - \alpha) \rho I - I
\]

s.t.
\[
e_0 \geq E,
\]
\[
\rho I - (I - E - B) \geq 0.
\]

The optimality condition is given as follows

\[ \alpha \left[ \rho (I^*) I^* \right]' + (1 - \alpha) \bar{\rho} = 1 \]

The last constraint can be easily satisfied by setting \(E + B = I^*\). Without loss of
generality, choose $E = 0, B = I^*$.

However, the banker finds it optimal to issue risky debt rather than CoCo since
\[
\Pi^{\text{CoCo}} = e_0 + \alpha \rho(I^*)I^* + (1 - \alpha) \rho I^* - I^* < e_0 + \alpha \rho(I^*)I^* + (1 - \alpha) I^* - I^* = e_0 + \alpha \left( \rho(I^*)I^* - I^* \right) < e_0 + \alpha \left( \rho(I^{\text{TBF}})I^{\text{TBF}} - I^{\text{TBF}} \right) = \Pi^{\text{TBF}}.
\]
\qed