Financial Stability, Growth and Macroprudential Policy

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Abstract

This paper studies the effect of optimal macroprudential policy in a small open economy model where growth is endogenous. By introducing endogenous growth, this model is able to capture the persistent effect of financial crises on output, which is different from previous literature but consistent with the data. Furthermore, there is a new policy trade-off between the cyclical and trend consumption growth. By constraining external borrowing to reduce systemic risk, the macroprudential policy hurts trend growth in good times but reduces the permanent output loss from a crisis. In a calibrated version of my model, I find that the optimal macroprudential policy significantly enhances financial stability (reducing the probability of crisis by two-thirds) at the cost of lowering average growth by a small amount. The welfare gains from policy intervention do not increase with endogenous growth because crises are rare events.

Keywords: Macroprudential Policy, Financial Crises, Endogenous Growth

JEL Classification: F38, F41, G18

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1 Introduction

In the wake of the Global Financial Crisis in 2008-2009, the use of macroprudential policy to manage boom-bust cycles came to the forefront of macroeconomic research (see Lorenzoni (2008), Benigno et al. (2013), and Dávila and Korinek (2017)). By limiting excessive capital inflows, the goal of macroprudential policy is to mitigate the risk of financial crises and the resulting highly persistent output losses.1 However, financial crises in current models of macroprudential policy have a temporary effect on output.2 This raises the question of how the optimal macroprudential policy changes in these models when financial crises have a permanent effect on the output level.

The main contributions of this paper are twofold. First, I provide a new framework such that financial crises have a persistent effect on output level. To achieve this goal, I introduce endogenous growth into a small open economy (SOE) model with occasionally binding collateral constraints that has been widely used in the literature (see Jeanne and Korinek (2010b) and Benigno et al. (2016)). In a quantitative exercise, I show that my model is able to match the output dynamics during the crises episodes. Second, I analyze the impact of macroprudential policy on financial stability and growth in the new framework. Unlike the existing literature, there is a new policy trade-off between the cyclical and trend consumption growth. By constraining external borrowing to reduce financial instability, the optimal macroprudential policy hurts trend growth in good times but reduces the permanent output losses from crises. A quantitative exercise suggests that the optimal macroprudential policy significantly enhances financial stability (reducing the probability of crisis by two thirds) at the cost of lowering average growth by a small amount.

The key feature of my model is an endogenous productivity process, which can be affected by the occasionally binding collateral constraints. In each period, private agents can use resources to invest in a technology that increases productivity. In a crisis, when the collateral constraint binds, they are forced to cut spending and thus investment in the technology. As a result, crises are associated with lower productivity growth. Importantly, growth rates only converge to the long-run average level after crises, which captures the persistent effect of financial crises. Unlike existing models in the literature, output in my model follows a trajectory that is parallel to its pre-crisis trend after financial crises, consistent with the data (see Figure 1).

This new framework is appropriate to analyze the impact of macroprudential policy on

1There is strong evidence that financial crises have very persistent effects on output. See Cerra and Saxena (2008), Reinhart and Reinhart (2009), Rogoff and Reinhart (2009), and Ball (2014).

2In the existing literature, productivity growth is by assumption exogenous. See Jeanne and Korinek (2010b) and Benigno et al. (2013).
growth. Unsurprisingly, there is room in my model for policy intervention to address over-
borrowing. Like other papers (e.g., Jeanne and Korinek (2010b)), I analyze the role of macro-
prudential policy by considering a social planner with an instrument to manage capital flows, i.e. macroprudential capital controls.\footnote{This policy is prudential capital control. See Korinek (2011), Jeanne (2012), Jeanne et al. (2012), and IMF (2012) for a detailed overview.} Unlike the existing literature, however, I do so in an environment that allows me to evaluate the policy’s impact on average growth. There is a new policy trade-off between the cyclical and trend consumption growth. Specifically, macroprudential policy reduces the permanent output losses from crises at the cost of lowering trend growth in good times.

In general, the impact of macroprudential policy on average growth is ambiguous. On one hand, macroprudential policy increases growth during crises because it reduces financial vulner-
abilities. On the other hand, it also lowers growth during normal periods because it reduces external borrowing and thus the expenditures to increase productivity. The calibrated version of my model reveals that optimal macroprudential policy reduces the probability of crises from 6.2 percent to 1.9 percent (about two-thirds), at the cost of lowering average growth by 0.01
Furthermore, I find that the welfare gains from optimal macroprudential policy are equivalent to a 0.06 percent permanent increase in annual consumption. Like existing literature, macroprudential policy increases welfare by limiting the likelihood of financial crises, therefore helping agents to smooth consumption. In fact, in the model, that effect is stronger with endogenous growth. However, macroprudential policy successfully restricts over-borrowing in the upswing, thus reducing growth in normal periods. The cost of lowering trend growth has significant welfare consequences, which explains why the optimal policy only lowers average growth by a small amount. Furthermore, even if the welfare gains from smoothing the cyclical consumption growth have been enhanced by endogenous growth, the probability of crisis has been driven down considerably by this optimal policy. Overall, macroprudential policy still improves welfare. The gains are similar to models with exogenous productivity (see Jeanne and Korinek (2010b)).

**Relation to Literature**

This paper is related to the literature on the relationship between growth and stability, in which empirical evidence often leads to mixed results. There are papers on the cross-country relationship between average growth and volatility of growth. For example, Ramey and Ramey (1995) find a negative relationship between average growth and volatility of growth, while Rancière et al. (2008) argue that countries experiencing more crises (more volatile growth) have higher average growth (see Levine (2005) for a summary). Moreover, there are also papers on the impact of policy on growth and financial stability. For example, Sánchez and Gori (2016) find that certain growth-promoting policies can have negative side-effects on financial stability, while Boar et al. (2017) find that macroprudential policy can increase both financial stability and long-run economic growth. This paper finds a negative relationship between average growth and financial stability for macroprudential policy, consistent with Rancière et al. (2008) and Sánchez and Gori (2016). However, this relationship depends on calibrations and might become positive in some cases, which is consistent with the findings in Ramey and Ramey (1995) and Boar et al. (2017).

This paper is also related to the literature on short-run fluctuations and growth. There are two existing approaches in the literature to introduce endogenous growth into a standard DSGE framework: One approach models growth following Romer (1990), such as Comin and Gertler (2006), Queraltó (2015), and Guerron-Quintana and Jinnai (2014). The other approach models growth following Aghion and Howitt (1992), such as Ates and Saffie (2016) and Benigno and Fornaro (2017). My way of modeling growth is similar to the first approach, which preserves
the representative-agent framework. However, unlike the existing literature, which focuses on a positive analysis, my paper is interested in the characterization of optimal policy and the policy’s impact on growth and welfare.

Finally, this paper belongs to the literature on optimal macroprudential policy and capital flow management. The theoretical rationale for macroprudential policy includes pecuniary externalities (see Lorenzoni (2008), Jeanne and Korinek (2010a), and Dávila and Korinek (2017)) and aggregate demand externalities (see Farhi and Werning (2016) and Korinek and Simsek (2016)). The general takeaway from the theories is that ex-ante policy intervention can be welfare-improving, since it addresses over-borrowing in the credit market and thus reduces financial instability. However, the literature has been silent on the effect of ex-ante intervention on economic growth, which is the central focus of this paper. Specifically, this paper introduces endogenous growth into a standard SOE-DSGE model with occasional binding constraints (see Jeanne and Korinek (2010b), Bianchi (2011) and Bianchi and Mendoza (2018)). Unlike in other literature, crises have persistent output-level effects in this model, consistent with the empirical evidence.

The organization of this paper is as follows: Section 2 presents a benchmark model; Section 3 presents the calibration procedure and model performance; Section 4 presents a normative analysis for macroprudential policy; Section 5 presents quantitative analysis of the policy; and Section 6 concludes.

2 Model Economy

This section introduces an analytical framework that incorporates endogenous growth into an SOE model as in Jeanne and Korinek (2010b). One feature of the model is an occasionally binding collateral constraint, which can capture financial crises and justifies the policy intervention (see Benigno et al. (2013) and Dávila and Korinek (2017)). In the model, normal periods are when the constraint is slack, and crisis periods are when the constraint binds. In order to capture the persistent effect of financial crises, I make two departures from the standard literature. First, I introduce a technology that allows agents to change the productivity level. By doing so, crises can have an impact on growth. Second, I make a modification to utility functions such that growth rates fall at a level that is consistent with the data. As I will explain later, this modification can be interpreted as one form of internal habit. Its role is to increase the local concavity of the utility functions (see Campbell and Cochrane (1999)).
2.1 Analytical Framework

In my model, the economy is populated by a continuum of identical households that have access to an international capital market and a technology that increases productivity. Due to friction in the financial market, there exist collateral borrowing constraints, and the maximum amount of external borrowing cannot exceed the value of collateral. In normal periods, when the constraints are slack, households can finance their desired levels of expenditure through external borrowing. The economy thus grows at a normal rate. In crises, when the collateral constraints bind, households cannot finance enough expenditures for the technology. As a result, the growth rate drops.

Preferences: Households have the following Constant Relative Risk Aversion (CRRA) preferences with one modification:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t - H_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t - H_t)^{1-\gamma}}{1-\gamma} \tag{1} \]

where \( \beta \in (0,1) \) is the discount factor, \( \gamma \) is the coefficient of risk aversion, \( c_t \) is consumption, and \( H_t \) is the modification. Given that the economy is growing, I assume that \( H_t \) depends on the level of endogenous productivity (trend) \( z_t \) and takes the functional form as follows (see Christiano (1989)).

\[ H_t = h z_t \tag{2} \]

Interpretation of \( H_t \): One interpretation of \( H_t \) is a form of internal habit. The stock of habit depends on a pre-determined economic trend \( z_t \). As I will explain later, households can spend on a technology to change the trend from \( z_t \) to \( z_{t+1} \) at period \( t \), which will affect the term \( H_{t+1} \). Importantly, the private agent internalizes this effect. Therefore, this is a form of internal habit. Rather than modelling \( H_t \) as a function of past consumption, I assume that it depends on past trend, which reduces the number of endogenous state variables and thus the computational burden. The other interpretation of \( H_t \) is a form of subsistence level of consumption as in the Stone-Geary functional form (see Geary (1950) and Stone (1954)). A subsistence level of consumption has been introduced before in the literature on growth in open economies (see Rebelo (1992) and Steger (2000)). I assume that the subsistence level of consumption increases with the economy. As argued by Ravn et al. (2008), “Luxuries in a poor society, such as tap water, inside plumbing, and health care, are considered necessities in developed countries.”

\(^4 h \) is a constant.
Role of $H_t$: The main role of $H_t$ is to increase the local concavity of utility functions as in the habit formation literature (see Campbell and Cochrane (1999)). Without $H_t$, private agents find it costly to cut $z_{t+1}$, since that implies a permanent future loss in output.\(^5\) Instead, private agents cut consumption spending. As a result, the endogenous growth rate, $\frac{z_{t+1}}{z_t}$, barely falls when there is a negative shock. Therefore, crises only have a temporary impact on output level in the model even after introducing endogenous growth. To have a large decrease in growth, one need to raise the cost of cutting consumption for private agents, which is achieved here by increasing the local concavity of the utility functions as in the habit formation literature.\(^6\)

Production Function: Production only requires a productive asset $n_t$ as an input and takes the following form:

$$y_t = A_t n_t^\alpha$$

where $A_t$ represents the productivity level in the economy and $\alpha \in (0, 1)$. Productive asset $n_t$ is an endowment to households and is normalized to 1. It corresponds to an asset in fixed supply, such as land. In each period, households trade the productive asset $n_t$ at a market-determined price $q_t$.

Endogenous Productivity: The level of productivity $A_t$ takes the following form:

$$A_t = \theta_t z_t$$

where $\theta_t$ is a stationary exogenous productivity shock, and $z_t$ is non-stationary endogenous productivity chosen by private agents.

Source of Growth: Growth in the economy comes from the endogenous productivity $z_t$ that households can choose. Specifically, there is a technology that costs $\Psi(z_{t+1}, z_t)$ units of consumption to elevate endogenous productivity from $z_t$ to $z_{t+1}$. I call $\Psi(z_{t+1}, z_t)$ “growth-enhancing expenditures,” which include all the expenditures that facilitate long-term economic growth. Here I do not take a stand on any particular form of endogenous growth, but use a generic form that includes many models in the growth literature.\(^7\) For example, $\Psi(z_{t+1}, z_t)$ includes physical

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\(^5\) As I will explain below, future output $y_{t+1}$ depends on productivity $z_{t+1}$.

\(^6\) One might also want to increase the risk-aversion coefficient of utility functions or introduce Epstein-Zin preference. However, neither of these modifications leads to a large decrease in growth following a crisis.

\(^7\) Admittedly, it is important to understand the source of growth. However, the main focus of this paper is to understand the policy’s impact on growth. Therefore, I adopt a reduced-form function of endogenous growth so as
capital investment in the AK growth framework as in Romer (1986), human capital investment as in Lucas (1988), R&D expenditure as in Romer (1990) and Aghion and Howitt (1992), etc. The only restriction is that there are no externalities in the process of choosing $z_{t+1}$. When private agents choose $z_{t+1}$, they internalize its impact on not only the future term $H_{t+1}$ in the utility function but also the future cost function, $\Psi(z_{t+2}, z_{t+1})$. This restriction thus shuts down any externalities in endogenous growth.\(^8\) This departs from the literature on short-run fluctuations and growth, where economic growth is typically suboptimal (see Comin and Gertler (2006) and Kung and Schmid (2015)).

Financial Friction: I introduce a collateral constraint on external borrowing following Jeanne and Korinek (2010b). Specifically, households can purchase $b_{t+1}$ units of a one-period bond from the international market in each period, and these bonds promise a gross interest rate $1 + r$ in the next period. The domestic economy is atomistic in the international world and takes the interest rate as given. Furthermore, bonds are supplied with infinite elasticity. However, there is a source of financial friction in the market: Private agents need to post their productive assets as collateral for external borrowing, and the maximum amount of external borrowing cannot exceed a fraction $\phi \in (0, 1)$ of the collateral value $q_t$.\(^9\) Therefore, the collateral constraint can be written as\(^10\)

$$-b_{t+1} \leq \phi q_t$$

**Budget Constraint:** In each period, households make expenditure plans for consumption $c_t$, growth-enhancing expenditures $\Psi(z_{t+1}, z_t)$, productive assets $q_t n_{t+1}$, and bond holdings $b_{t+1}$. Their incomes come from the output $y_t$, sale of productive assets $q_t n_t$, and existing bond hold-

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\(^8\)As I will explain in the next section, there are pecuniary externalities in the economy that justify an optimal policy. However, both externalities in growth and pecuniary externalities typically call for policy intervention to increase national saving. If both of them are present in the economy, it is hard to disentangle their effects. Furthermore, externalities in endogenous growth tend to dominate pecuniary externalities.

\(^9\)One rationale for the collateral constraint is as follows: There is a moral hazard problem between domestic households and international investors (see Jeanne and Korinek (2010b)). Households have the option to invest in a scam that prevents international investors from seizing future productive assets. This implies that households can default on their debts without any punishment. The investors, however, cannot coordinate to punish the households by excluding them from the market. The only recourse is to take legal action before the scam is completed. By doing so, they can only seize a fraction $\phi$ of productive assets and sell them to other households at the prevailing market price $q_t$. As a result, rational international investors will restrict the amount of external borrowing up to $\phi q_t$.

\(^10\)One can also specify the collateral constraint in the form of $-b_{t+1} \leq \phi q_t n_t$. I check this alternative formulation and find that its quantitative results are similar to the current setting. Following Jeanne and Korinek (2010b), I adopt the form as in (5) since it makes the math simpler.
nings \((1 + r)b_t\). As a result, the budget constraint can be written as follows:

\[
c_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + b_{t+1} = y_t + q_t n_t + (1 + r)b_t,
\]

(6)

**Market Clearing:** There are two markets in the economy: the final goods market and the productive asset market. Given that the productive asset is in fixed supply and owned by the households, the equilibrium condition implies that

\[
n_t = 1, \forall t
\]

(7)

The final goods market can be pinned down by aggregating the budget constraint for each household and applying the equilibrium condition (7) in the productive asset market.

\[
c_t + \Psi(z_{t+1}, z_t) + b_{t+1} = y_t + (1 + r)b_t,
\]

(8)

### 2.2 Competitive Equilibrium (CE)

**Competitive Equilibrium:** In this economy, a competitive equilibrium consists of a stochastic process \(\{c_t, z_{t+1}, n_{t+1}, b_{t+1}\}_{t=0}^{\infty}\) chosen by the households and an asset price \(\{q_t\}_{t=0}^{\infty}\), given initial values \(\{b_0, z_0\}\) and the exogenous shock \(\{\theta_t\}_{t=0}^{\infty}\) such that utility (1) is maximized, constraints (5) and (6) are satisfied, and the productive assets and goods market clear, i.e., conditions (7) and (8) are satisfied.

**Recursive Formulation:** It is convenient to define net consumption by \(c^h_t = c_t - H_t\) and write the problem in a recursive formulation. State variables at time \(t\) include the endogenous variables \(\{z_t, n_t, b_t\}\) and the exogenous variable \(\theta_t\). I can write the optimization problem as follows:

\[
V^CE_t(z_t, n_t, b_t, \theta_t) = \max_{c^h_t, z_{t+1}, n_{t+1}, b_{t+1}} u \left( c^h_t \right) + \beta E \left[ V^CE_{t+1}(z_{t+1}, n_{t+1}, b_{t+1}, \theta_{t+1}) \right]
\]

s.t.

\[
c^h_t + h z_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + (1 + r)b_t, \quad -b_{t+1} \leq \phi q_t
\]
The maximization problem yields the following optimality conditions for each period:

\[ \lambda_{t}^{CE} = u'(c_t^h) \]  
(9)

\[ \lambda_{t}^{CE} \Psi_{1,t} = \beta E_t \left[ \lambda_{t+1}^{CE} (\theta_{t+1} - h - \Psi_{2,t+1}) \right] \]  
(10)

\[ \lambda_{t}^{CE} q_t = \beta E_t \left[ \lambda_{t+1}^{CE} (\alpha \theta_{t+1} z_{t+1} + q_{t+1}) \right] \]  
(11)

\[ \lambda_{t}^{CE} = \mu_{t}^{CE} + \beta (1 + r) E_t \left[ \lambda_{t+1}^{CE} \right] \]  
(12)

where \( \Psi_{1,t} = \frac{\partial \Psi(z_{t+1}, \bar{z}_t)}{\partial z_{t+1}} \) and \( \Psi_{2,t+1} = \frac{\partial \Psi(z_{t+2}, \bar{z}_{t+1})}{\partial z_{t+1}} \). \( \lambda_{t}^{CE} \) and \( \mu_{t}^{CE} \) are Lagrangian multipliers associated with the budget constraint and collateral constraint, respectively.

Condition (9) is the marginal valuation of household wealth. Condition (10) is the key equation for growth in this model, where private agents equate the marginal cost of choosing \( z_{t+1} \) with the marginal benefit. The cost is reflected in the partial derivative of the technology function \( \Psi_{1,t} \), while the benefit includes a future output \( \theta_{t+1} \), excluding the normalized future habit term (or subsistence level of consumption term), \( h \) and the partial derivative of future technology function, \( \Psi_{2,t+1} \). The marginal cost and marginal benefit are evaluated at the marginal valuation of wealth in periods \( t \) and \( t+1 \) respectively. The third condition (11) is a standard asset pricing function, where holding productive asset \( n_{t+1} \) yields a dividend income \( \alpha \theta_{t+1} z_{t+1} \) and capital gains \( q_{t+1} \). The last condition (12) is the Euler equation for holding bonds. The additional term \( \mu_{t}^{CE} \) captures the effect of collateral constraint on the external borrowing. When the collateral constraint (5) binds, the marginal benefit of borrowing to increase consumption exceeds the expected marginal cost by an amount equal to the shadow price of relaxing collateral constraint \( \mu_{t}^{CE} \).

**Normalized Economy:** To solve for a stationary equilibrium, I normalize all the endogenous variables by \( z_t \) and denote this by variables with hats. Specifically, I denote \( \hat{x}_t = \frac{x_t}{z_t} \), where \( x_t = \{c_t^h, b_t, q_t, V_t^{CE}, \cdots \} \), and endogenous growth rate \( g_{t+1} = \frac{z_{t+1}}{z_t} \). The normalized equilibrium conditions are given in Appendix C.

### 3 Calibration

This section first describes empirical evidence on the persistent effect of crises, i.e. an 11-year event window that the model targets. It then shows parameter values and the model’s ability to fit the data.
3.1 Targeted Event Window

One key feature of the model is its generation of such persistent output-level effects of financial crises as found in the data (see Cerra and Saxena (2008), Rogoff and Reinhart (2009), and Ball (2014)). To quantify the magnitude of output cost for later calibration, I construct an 11-year event window of output growth rates centering on one specific type of financial crisis in emerging markets, i.e., sudden stop episodes.¹¹ These episodes occur when there is a sudden slowdown in private capital inflows to emerging market economies and a corresponding sharp reversal in current account balances. For the identification of sudden stops I use the episodes in Calvo et al. (2006) (“Calvo episodes”), whose criterion is based on a sharp reversal in current account balances and a spike in spreads. For robustness, I also use episodes identified in Korinek and Mendoza (2014) (“KM episodes”) and report the results in Appendix B.

The left panel of Figure 2 shows that the growth rate of real GDP per capita is a stationary process and falls to $-5.65$ percent at the time of crises. I also construct an event window for “Total Factor Productivity (TFP)” in the right panel of Figure 2 and find that productivity displays a similar pattern to output, consistent with the predictions of my model.

![Figure 2: Growth Rates in Sudden Stop Episodes (%)](image)

*Note:* The series are constructed using an 11-year window centering on the sudden stop episodes.

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¹¹The source of real GDP per capita is explained in Appendix A.
3.2 Parameter Values

I calibrate the model to annual frequency using 55 countries’ data from between 1961 and 2015 (see Appendix A for details). The model can be solved using a variant of the endogenous gridpoint method, as in Carroll (2006) (see Appendix F for details). There is only one shock in the economy: the exogenous technology shock $\theta_t$, which follows the process below. I discretize the process using Rouwenhorst method as in Kopecky and Suen (2010).

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma^2)$$

where $\rho$ and $\sigma$ are persistence and volatility of the shock, and $\varepsilon_t$ is a random variable following a normal distribution.

It is important to have the shock $\theta_t$ in the model to capture the fall of output growth during crises, as seen in Figure 2. Without a fall in $\theta_t$, one cannot explain the negative output growth rate in crises, since output $y_t$ depends on the predetermined productivity $z_t$ and the exogenous productivity $\theta_t$. Furthermore, the endogenous response of productivity $z_{t+1}$ prevents the output growth rate after crises from being higher than its long-run average, consistent with the event window.

Assumption 1. Cost function $\Psi(z_{t+1}, z_t)$ is quadratic and takes the following form:

$$\Psi(z_{t+1}, z_t) = \left[ \left( \frac{z_{t+1}}{z_t} - \psi \right) + \kappa \left( \frac{z_{t+1}}{z_t} - \psi \right)^2 \right] z_t,$$

where $\psi > 0$ and $\frac{z_{t+1}}{z_t} \geq \psi$.

I impose a simple quadratic form on $\Psi(z_{t+1}, z_t)$ so as to calibrate my model. Given that this way of modeling growth is generic, I calibrate the functions parameter values using references to moments in the data. For example, $\kappa$ is a scale parameter and is used to match the average share of consumption in GDP. The parameter $\psi$ is the minimum level of endogenous growth $g_{t+1}$ in the model and is used to match the output growth rate after crises in the targeted event window.

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12 Admittedly, other shocks, such as financial shocks and interest rate shocks, are important for understanding financial crises. However, these shocks alone cannot lead to a drop of output growth in crises in the model, since the productivity $z_t$ is predetermined.

13 One could also have an exogenous trend shock, as in Aguiar and Gopinath (2007). Introducing an exogenous trend shock, however, does not allow me to analyze the policy’s impact on growth.
I need to assign values to 10 parameters in the model: \{\beta, r, \gamma, h, \psi, \kappa, \alpha, \rho, \sigma, \phi\}. The calibration proceeds in two steps. First, some parameter values are standard in the literature. For example, I choose the interest rate \( r \) to be 6 percent and the coefficient of risk aversion parameter \( \gamma \) to be 2. The parameter \( \alpha \) equals productive asset income’s share of total income, and I choose 0.2 following Jeanne and Korinek (2010b). Second, given these parameter values, I jointly choose the remaining parameters to match relevant moments in the data and the targeted event window in Figure 2.

Specifically, I use the following parameters to match data moments. Parameter \( \beta \) determines the incentive to borrow and is chosen to match the long-run Net Foreign Asset (NFA) to GDP ratio \((-30\%\))

Parameter \( \rho \) is chosen to match the correlation between the current account and output at \(-0.25\), since I focus on the relationship between capital flows and output growth.\(^{14}\) Parameter \( \phi \) determines the maximum value of borrowing in the economy and thus the probability of crises.\(^{15}\) In the model, I define crisis episodes as periods when constraints bind and the magnitude of current account reversal exceeds 1 standard deviation of its long-run average (see Bianchi (2011)). The parameter \( \phi \) is chosen to match the probability of crises at 5.5 percent, a standard value in the literature (see Bianchi (2011) and Eichengreen et al. (2008)). Furthermore, parameters \( h \) and \( \kappa \) are jointly chosen to match the average growth rate, 2.3 percent and the share of consumption in GDP, 77.6 percent. Specifically, \( h \) and \( \kappa \) must satisfy the normalized resource constraint (8) and the Euler equation of \( z_{t+1} \) (10) as follows:

\[
\frac{\hat{c}_{ss}}{77.6\%} + \Psi\left(\frac{g_{ss}}{1+2.3\%}\right) = 1 + \frac{1 + r - g_{ss} \hat{b}_{ss} g_{ss}}{g_{ss}} \Psi\left(\frac{g_{ss}}{-30\%}\right) \\
\Psi_1(g_{ss}) = \beta g_{ss}^{-\gamma}(1 - h - \Psi_2(g_{ss}))
\]

where the average value of \( \theta_t \) is normalized at 1, and the value of \( h \) and \( \kappa \) depend on the value of \( \beta \) and \( \psi \).\(^{16}\)

\(^{14}\)Aguiar and Gopinath (2007) find that the persistence of shocks governs the correlation between the current account and output. The correlation is constructed by first de-trending the output series with a HP filter and then calculating the correlation between the current account to GDP ratio and the cyclical component of output.

\(^{15}\)I calibrate the model such that the collateral constraint marginally binds in the long run and the following relationship holds in the steady states:

\[
\frac{-\hat{b}_{gss}}{30\%} = \phi \hat{q} \\
\hat{q} = \frac{\beta g_{ss}^{1-\gamma}}{1 - \beta g_{ss}^{1-\gamma}} \alpha
\]

\(^{16}\)Here, I calibrate the economy so that in the long run it is unconstrained and the collateral constraint marginally
As explained before, I also want to match the event window in Figure 2. The volatility $\sigma$ governs the minimum level of the exogenous shock $\theta_t$ and thus the decline in the output growth rate during crises. Parameter $\psi$ determines the minimum level of the endogenous growth rate $g_{t+1}$ and thus the decline in the output growth rate one year after crises. Therefore, I choose $\sigma$ and $\psi$ to jointly match the output growth rate during crises ($-5.65$ percent) and one period after crises ($3.28$ percent) in the event window.

In sum, given the values of $\{r, \gamma, \alpha, \eta\}$, I pick values of $\{\beta, \psi, \rho, \sigma\}$, which determine values of $\{\phi, \kappa, h\}$. I then simulate the model, calculate moments of the simulated data, construct an event window as in Figure 2, and then compare the simulation results with the actual data moments and the targeted event window. The values of all parameters are reported in Table 1.

Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter in production function</th>
<th>Value</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.2</td>
<td>Jeanne and Korinek (2010b)</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r = 6%$</td>
<td>Benigno et al. (2013)</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma = 2$</td>
<td>Standard in the literature</td>
</tr>
<tr>
<td>Volatility of technology shock</td>
<td>$\sigma = 0.04$</td>
<td>Output growth rate at time of crises = $-5.65 %$</td>
</tr>
<tr>
<td>Parameter in $\Psi$ functions</td>
<td>$\psi = 0.95$</td>
<td>Output growth rate one year after crises = $3.28%$</td>
</tr>
<tr>
<td>Parameter in $\Psi$ functions</td>
<td>$\kappa = 26.29$</td>
<td>Consumption-GDP ratio = $77.6%$</td>
</tr>
<tr>
<td>Parameter in the utility function</td>
<td>$h = 0.51$</td>
<td>Average GDP growth = $2.3%$</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta = 0.968$</td>
<td>Probability of crisis = $5.5%$</td>
</tr>
<tr>
<td>Persistence of technology shock</td>
<td>$\rho = 0.83$</td>
<td>Correlation between current account and output = $-0.25$</td>
</tr>
<tr>
<td>Collateral constraint parameter</td>
<td>$\phi = 0.0852$</td>
<td>NFA-GDP ratio = $-30%$</td>
</tr>
</tbody>
</table>

3.3 Model Performance

Table 2 reports model and data moments. One can see that the model matches targeted moments in the data. As with other models with occasionally binding collateral constraints, crisis episodes are rare events in my model and occur with a probability of 6.2 percent in the simulation.

Unlike existing models in the literature, my model can generate the growth rate dynamics in Figure 2. To see this, I simulate the model, identify crisis episodes and construct an 11-period event window for different variables in Figure 3. Not surprisingly, crises occur when there is a large drop in the exogenous shock $\theta_t$. The current account experiences a large reversal because the borrowing constraints bind and private agents have to cut their external borrowing, i.e., an binds.

Specifically, I simulate the model for 11,000 periods and throw away the first 1000 periods. Data moments are calculated based on the remaining 10,000 periods of simulated data. Furthermore, I identify crisis episodes in the simulated data and calculate the output growth rate during crises and one period after crises.
increase in $\hat{b}_{t+1}$. Furthermore, these events are accompanied by a decline in spending such as consumption $\hat{c}_t$ and growth-enhancing expenditures (reflected in a decline in the endogenous growth rate $g_{t+1}$). The asset price $\hat{q}_t$ also drops, which leads to an amplification effect through collateral constraints. Fortunately, my model captures the empirical regularity of crises. Importantly, it can capture the persistent output-level effects of crises as in the data: Output growth rates fall during crises with a decline in $\theta_t$ and only go back to the long-run average level after crises. This occurs because the endogenous growth rate $g_{t+1}$ decreases during crises.

Figure 3: Event Window: Model and Data
4 Optimal Macroprudential Policy

Consistent with the literature, there is a role for macroprudential policy in the economy due to the presence of pecuniary externalities (see Lorenzoni (2008) and Dávila and Korinek (2017)). These pecuniary externalities are related to a vicious cycle associated with the collateral borrowing constraints. Intuitively, private agents need to cut spending when a negative shock hits and the constraints bind. However, asset prices fall with a decline in spending and private agents need to cut spending further due to lower collateral values and tighter borrowing constraints. Therefore, the initial shock is endogenously amplified through the constraints. Importantly, private agents, taking the asset price as given, fail to internalize their contributions to this vicious cycle, which represents pecuniary externalities in the economy. As a result, they over-borrow in normal periods. The optimal macroprudential policy is designed to correct this over-borrowing in the credit market.

Following the literature, I first define the social planner’s problem and then choose macroprudential policy to implement the allocation (see Jeanne and Korinek (2010b), Bianchi (2011), and Bianchi and Mendoza (2018)). This is similar to the “primal approach” in optimal policy analysis (originally from Stiglitz (1982)), in which the social planner can choose allocations subject to resource, implementability, and collateral constraints. This formulation allows me to see the wedge between the social planner and private agents in choosing allocations and understand the inefficiencies in the economy. To implement the social planner’s allocation, I consider what tax or subsidy with lump-sum transfers is needed to close the wedge. In this case, a tax on capital flows is needed.

Specifically, I consider the social planner who chooses allocations on behalf of the representative household to be subject to the same constraints as private agents, but who lacks the ability to commit to future policies. Importantly, I assume that the asset price $q_t$ remains market determined and that the Euler equation of asset price (11) enters the social planner’s problem as an implementability constraint. The implicit rationale is that the social planner cannot directly intervene with respect to the asset price but internalizes how the allocations affect it and thus the collateral constraint.

Furthermore, I assume that endogenous productivity $z_{t+1}$ is chosen by private agents and that the Euler equation of productivity (10) also enters the social planner’s problem as an additional

---

18 Pecuniary externalities refer to externalities associated with prices. In an economy with incomplete markets, allocations with pecuniary externalities are generically sub-optimal. For a detailed proof, see early contributions by Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986).

19 I do not allow the social planner to trade assets on behalf of private agents. One rationale is that private agents are better than the planner at observing fundamental payoffs of financial assets (see Jeanne and Korinek (2010b)).
implementability constraint. This is because I use macroprudential policy to decentralize this social planner’s allocation and the policy is designed to correct the wedge only in the bond holdings.

I call the social planner with macroprudential policy a macroprudential social planner and denote her allocation with a superscript “MP”. As described before, the maximization problem can be written as

\[
V_t^{MP}(z_t, b_t, \Theta_t) = \max_{c_t^h, z_{t+1}, b_{t+1}, q_t} u(c_t^h) + \beta E \left[ V_{t+1}^{MP}(z_{t+1}, b_{t+1}, \Theta_{t+1}) \right]
\]

s.t.

\[
c_t^h + h_{z_t} + \Psi(z_{t+1}, z_t) + b_{t+1} = \theta_t z_t + (1 + r) b_t,
- b_{t+1} \leq \phi q_t,
\]

\[
u_t' \left[ u_t'(c_t^h) (\alpha \theta_{t+1} z_{t+1} + q_{t+1}) \right] = \beta E_t \left[ \left. \frac{\partial}{\partial c_t^h} \right|_{c_t^h} \right] G(z_{t+1}, b_{t+1})
\]

where equations (13) and (14) are two implementation constraints, i.e., the Euler equations of choosing productive assets and productivity. I write implementation constraints as functions of future endogenous state variables \( z_{t+1} \) and \( b_{t+1} \), since I want to solve for time-consistent policy functions as in Jeanne and Korinek (2010b) and Bianchi and Mendoza (2018).

Given the definition of the macroprudential social planner, it is straightforward to define constrained inefficiency as follows:

**Definition 1. Constrained Inefficiency**

The competitive equilibrium displays constrained inefficiency if it differs from the allocation chosen by the macroprudential social planner.

To understand the difference between private agents and the macroprudential social planner, I derive the optimality conditions of MP as follows:

\[
\lambda_t^{MP} = u_t'(c_t^h) - \xi_t^{MP} u_t''(c_t^h) q_t - \nu_t^{MP} u_t''(c_t^h) \Psi_{1,t}
\]

\[
\lambda_t^{MP} \Psi_{1,t} - \xi_t^{MP} G_{1,t} - \nu_t^{MP} I_{1,t} - u_t'(c_t^h) \Psi_{11,t}
\]

\[
= \beta E_t \left[ \lambda_{t+1}^{MP} (\theta_{t+1} - h - \Psi_{2,t+1}) - \nu_{t+1}^{MP} u_t'(c_{t+1}^h) \Psi_{12,t+1} \right]
\]

\[
\phi \mu_t^{MP} = \xi_t^{MP} u_t'(c_t^h)
\]

\[
\lambda_t^{MP} = \mu_t^{MP} + \xi_t^{MP} G_{2,t} + \nu_t^{MP} I_{2,t} + \beta (1 + r) E_t \left[ \lambda_{t+1}^{MP} \right]
\]
where $\Psi_{11,t} = \frac{\partial^2 \Psi(z_{t+1}, z_t)}{\partial z_{t+1}^2}$, $\Psi_{12,t+1} = \frac{\partial^2 \Psi(z_{t+2}, z_{t+1})}{\partial z_{t+2} \partial z_{t+1}}$, $G_{1,t} = \frac{\partial G(z_{t+1}, b_{t+1})}{\partial z_{t+1}}$, $G_{2,t} = \frac{\partial G(z_{t+1}, b_{t+1})}{\partial b_{t+1}}$, $I_{1,t} = \frac{\partial (z_{t+1}, b_{t+1})}{\partial z_{t+1}}$, and $I_{2,t} = \frac{\partial (z_{t+1}, b_{t+1})}{\partial b_{t+1}}$. $\lambda^{MP}_t$, $\mu^{MP}_t$, $\xi^{MP}_t$, and $\nu^{MP}_t$ are Lagrangian multipliers associated with the budget constraint, collateral constraint, and two implementation constraints, respectively.

**Wedge in Marginal Valuation of Wealth:** The main difference between CE and MP is reflected in the marginal valuation of wealth, $\lambda^{CE}_t$ and $\lambda^{MP}_t$. One can see that the wedge includes two terms due to the presence of implementation constraints: The first term is $-\xi^{MP}_t u''(c^h_t)q_t$, which captures pecuniary externalities in the economy, and the second term is $-\nu^{MP}_t u''(c^h_t)\Psi_{1,t}$, which captures the inability of the social planner to change $z_{t+1}$. Consistent with results in the literature, the first term is positive due to condition (17). Uniquely, I also have the second term with $\nu^{MP}_t$, which is the shadow price of implementation constraint (14). The value of $\nu^{MP}_t$ is given by the optimality condition (16). Quantitatively, it is small. Hence, the wedge $-\xi^{MP}_t u''(c^h_t)q_t - \nu^{MP}_t u''(c^h_t)\Psi_{1,t}$ is positive.

Due to this wedge, the competitive equilibrium is constrained inefficient, and the social planner chooses a different allocation than do private agents. However, the difference appears only when the constraint is slack. The reason is that the social planner cannot change the allocation when the constraint binds. In the period when the collateral constraint is slack, i.e., $\mu^{MP}_t = 0$, the social planner chooses a higher level of bond holding than do private agents due to a higher valuation of future wealth $E_t[\lambda^{MP}_{t+1}]$ (see the optimality conditions of bond holding in CE and MP, (12) and (18)).\(^{20}\) Hence, there is an over-borrowing issue in competitive equilibrium, consistent with the literature.

**A New Policy Trade-off:** Unlike previous literature, there is a new policy trade-off between the trend and cyclical consumption growth for the macroprudential social planner. Intuitively, the social planner internalizes the pecuniary externalities and addresses the over-borrowing issue in the decentralized economy. By constraining the external borrowing during normal periods, she increases welfare by reducing the frequency of crises and the resulting output losses. As a result, the volatility of cyclical consumption growth is reduced. However, this comes at a cost of lowering trend growth during normal periods since the marginal cost of choosing $z_{t+1}$ increases with lower borrowing. In the quantitative exercise below, I show that each channel has a significant welfare consequence.

**Implementation:** I assume that the planner has access to a macroprudential tax $\tau^{MP,b}_t$ on capital.

\(^{20}\)Quantitatively, the term $\nu^{MP}_t u''(c^h_t)\Psi_{1,t} + \nu^{MP}_t I_{2,t}$ is small.
flows and a lump-sum transfer $T_t^{MP}$. The budget constraint for private agents becomes

$$c_t^h + hz_t + \Psi (z_{t+1}, z_t) + q_t n_{t+1} + \left( 1 - \tau_t^{MP,h} \right) b_{t+1} = y_t + q_t n_t + (1 + r) b_t + T_t^{MP}$$

where $T_t^{MP} = -\tau_t^{MP,b} b_{t+1}$.

**Proposition 1. Decentralization with Macroprudential Policy**

The macroprudential social planner’s allocation can be implemented by a macroprudential tax $\tau_t^{MP,b}$ on capital flows that is rebated to private agents with a lump-sum transfer $T_t^{MP}$. Furthermore, the tax $\tau_t^{MP,b}$ is given by

$$\tau_t^{MP,b} = \frac{\beta g_{t+1} (1 + r) E_t \left[ \gamma \phi \mu_t^{MP} \hat{q}_{t+1} \left( \hat{c}_t^h \right)^{-1} + \gamma \hat{\nu}_t^{MP} \left( \hat{c}_t^h \right)^{-\gamma - 1} \Psi_{1,t+1} \right]}{(\hat{c}_t^h)^{-\gamma}} - \frac{\gamma \phi \mu_t^{MP} \hat{q}_t \left( \hat{c}_t^h \right)^{-1} + \gamma \hat{\nu}_t^{MP} \left( \hat{c}_t^h \right)^{-\gamma - 1} \Psi_{1,t} - \hat{\nu}_t^{MP} g_{t+1} \hat{G}_2, (\hat{c}_t^h)^{\gamma} - \hat{\nu}_t^{MP} g_{t+1} \hat{I}_2, (\hat{c}_t^h)^{-\gamma}}{(\hat{c}_t^h)^{-\gamma}}$$

**Proof.** See Appendix D.1.

Consistent with the literature, a macroprudential tax $\tau_t^{MP,b}$ is used to correct the wedge between $\lambda_t^{MP}$ and $\lambda_t^{CE}$. It is positive in the quantitative exercise, since the Lagrangian multiplier $\nu_t^{MP}$ is small. Hence macroprudential policy is also used to correct the over-borrowing issue in the economy.

### 5 Quantitative Results

In this section, I first compare the allocations of private agents and of the macroprudential social planner, and then analyze policy impacts on average growth. I also calculate welfare gains from macroprudential policy and compare these values with the literature. Lastly, I analyze the size of macroprudential taxes. In Appendix E, I conduct a sensitivity analysis with respect to the results.

#### 5.1 Comparing CE and MP Allocations

The difference between the macroprudential social planner and private agents is captured by policy functions. Figure 4 plots consumption $\hat{c}_t^h$, endogenous growth rate $g_{t+1}$, asset price $\hat{q}_t$, and bond holding $\hat{b}_{t+1}$ for the competitive equilibrium (red solid line) and the macroprudential
social planner (green dashed line) over the bond holding \( \hat{b}_t \) when \( \theta_t \) is 2 standard deviations below its long-run average.\(^{21}\)

Figure 4: Policy Functions: CE and MP

There are kinks in all policy functions due to the presence of the collateral constraint. When the economy starts from a lower bond holding \( \hat{b}_t \) (a higher debt to repay), the collateral constraint binds, and private agents must cut external borrowing and total spending. As a result, both consumption and growth are reduced.

Consistent with the literature, there is an over-borrowing phenomenon in the competitive

\(^{21}\)I choose \( \theta_t \) to be at 2 standard deviations below its long-run average because the economy in competitive equilibrium converges to a marginally unconstrained steady state in the absence of future shocks in \( \theta_t \). Hence, any small shock to \( \theta_t \) pushes the economy into a constrained state, i.e., a crisis episode.
equilibrium because the social planner chooses a higher bond holding $\hat{b}_{t+1}$ than do private agents. Unlike in the literature, the over-borrowing also has an implication for the endogenous growth rate. Due to the new policy trade-off, the social planner chooses a lower $g_{t+1}$ when the constraint is slack. Trend growth is lower with this policy, but the economy becomes more resilient.

Figure 5 displays the ergodic distributions of bond holding $\hat{b}_{t+1}$ and endogenous growth rate $g_{t+1}$. Compared with private agents, the macroprudential social planner borrows less and thus chooses more mass in the range of higher bond holdings. In terms of the ergodic distribution for $g_{t+1}$, the social planner has less mass at both extremely low and normal (around 2 percent) growth levels. One can see that the dispersion of growth for MP has been marginally reduced. However, it is unclear whether average growth has been increased or decreased.

To see the impact of macroprudential policy on average growth and the probability of crisis, Table 3 reports model moments for the social planner and private agents. With macroprudential policy, external borrowing is reduced from 27.18 percent to 25.78 percent, which lowers average growth from 2.315 percent to 2.307 percent. However, the policy also reduces the probability of crisis from 6.23 percent to 1.89 percent. Hence, the economy becomes more resilient.

Figure 6 reports the event window as before but also plots the dynamics of variables for the social planner given the same exogenous shock $\theta_t$. One can see that the probability of crisis has been reduced by the social planner in the last panel of Figure 6. Furthermore, the planner chooses a higher bond holding in normal periods and thus suffers less when a very large shock hits at time 0. As a result, the social planner cuts consumption and growth-enhancing
Table 3: Moments: CE and MP

<table>
<thead>
<tr>
<th>Moments</th>
<th>CE</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average GDP growth (%)</td>
<td>2.315</td>
<td>2.307</td>
</tr>
<tr>
<td>Probability of crisis (%)</td>
<td>6.23</td>
<td>1.89</td>
</tr>
<tr>
<td>NFA-GDP ratio (%)</td>
<td>−27.18</td>
<td>−25.78</td>
</tr>
<tr>
<td>Consumption-GDP ratio (%)</td>
<td>77.53</td>
<td>77.65</td>
</tr>
<tr>
<td>Correlation between current account and output</td>
<td>−0.22</td>
<td>−0.37</td>
</tr>
</tbody>
</table>

expenditures less during crises.

Figure 6: Event Window: CE and MP

![Graphs showing event window for CE and MP](image)

However, macroprudential policy also reduces borrowing and thus the endogenous growth in normal periods. To show its impact, Figure 7 plots the transition dynamics from competitive equilibrium to the equilibrium chosen by the social planner. The transition dynamics is constructed by first running 1,000 simulations of 1,020 periods for competitive equilibrium and then introducing the social planner from period 1,001.

On the whole, the macroprudential social planner borrows less than private agents, which reduces both consumption and
endogenous growth. However, the economy becomes more resilient and has a lower probability of crisis. Therefore, consumption converges on a higher level. But the endogenous growth rate $g_{t+1}$ only converges to a lower level because the economy borrows less in the long run.

Figure 7: Transition Dynamics: CE and MP

5.2 Policy Impacts on Average Growth

This model allows for an analysis of policy impacts on average growth. Clearly macroprudential policy increases the endogenous growth rate $g_{t+1}$ during crises but reduces it in normal periods. Even though the policy lowers the volatility of growth unambiguously, its impacts on average growth are theoretically ambiguous.

In the baseline calibration, there is a negative relationship between average growth and financial stability for macroprudential policy. A more general question is which parameters govern this relationship? To answer this question, I simplify the model so it can be solved mostly analytically.

Instead of using the existing log AR(1) process for $\theta_t$, I assume that $\theta_t = 1$ for all $t$, and
that it falls to 0.9 in the second period, with a probability \( p \in [0, 1] \). Furthermore, the economy is unconstrained in a steady state, and I need to change \( \beta \) such that \( \beta (1 + r) g_{ss}^{-\gamma} = 1 \), where \( g_{ss} = 1.023 \), as in the baseline calibration. I keep other parameter values the same as before. Hence, crisis occurs in the economy when \( \theta_2 = 0.9 \) and the collateral constraint binds.

I plot the average growth chosen by the private agents and by the social planner in Figure 8. Whether the social planner increases or decreases average growth depends on two parameters: The probability of negative shock \( p \) and the tightness of the collateral constraint \( \phi \). Intuitively, the macroprudential social planner can increase average growth because she reduces the cost of crisis and thus raises the growth rate during a crisis. However, a crisis occurs with probability \( p \), and its cost depends on the tightness of the collateral constraint. When \( p \) is higher or \( \phi \) is lower, macroprudential policy is very beneficial, since the expected cost of crisis is relatively large. Hence, the policy can increase average growth in these scenarios.

Figure 8: Policy Impacts on Average Growth: CE and MP

---

I also find that the magnitude of the impacts is small (see Figure 8 and Table 3). This is because there is an optimal rate of growth defined by the technology \( \Psi(z_{t+1}, z_t) \). Macroprudential policy does not change this function directly but only changes the marginal valuation of wealth. Furthermore, any changes in the growth rate have non-trivial effects on welfare (see Lucas (1987) and Barlevy (2004)). Hence, if the optimal policy must affect growth negatively

---

23 I run 100-period simulations in two separate states to calculate average growth: \( \theta_2 = 0.9 \) in state L and \( \theta_2 = 1 \) in state H. The growth rate for each simulation is calculated as follows:

\[
G^i = (\Pi_{t=1}^{100} g_{t+1})^{\frac{1}{100}}, \text{ where } i \in \{H, L\}
\]

Therefore, average growth is \( p \ast G^L + (1 - p) \ast G^H \).
in order to increase financial stability, a planner will tend to choose a policy that changes growth only by a small amount. Otherwise, it is too costly for social welfare.

5.3 Welfare Gains

To calculate the welfare gains from macroprudential policy, I define a variable $\Delta MP(\hat{b}_t, \theta_t)$, which compares two utilities and converts their difference into consumption equivalents:

$$\Delta MP(\hat{b}_t, \theta_t) = 100 \left[ \frac{\hat{V}^{MP}(\hat{b}_t, \theta_t)}{\hat{V}^{CE}(\hat{b}_t, \theta_t)} \right]^{\frac{1}{1-\gamma}} - 1$$  \hspace{1cm} (19)

where $\hat{V}^i(\hat{b}_t, \theta_t)$ is a normalized value function and $i \in \{CE, MP\}$.

$\Delta MP(\hat{b}_t, \theta_t)$ depends on state variables $\{\hat{b}_t, \theta_t\}$, and I plot it in Figure 9.\footnote{Like the policy functions, $\Delta MP(\hat{b}_t, \theta_t)$ is plotted over the bond space $\hat{b}_t$ when the shock $\theta_t$ is 2 standard deviations below its long-run average.} Consistent with the literature, it peaks in the region where the magnitude of externalities is at its maximum. It becomes smaller when the economy has a higher amount of bond holding, since the probability of future crisis is lower. It also becomes smaller when the economy has a lower amount of bond holding, i.e. when the constraint binds. The macroprudential social planner chooses the same allocation as the private agents in these regions. Hence, the welfare gains are small.

To understand the average benefit of macroprudential policy, I also define a variable $EV^{MP}$ as follows:

$$EV^{MP} = E[\Delta MP(\hat{b}_t, \theta_t)]$$  \hspace{1cm} (20)

where the expectation is taken using the ergodic distribution of $\hat{b}_t$ and $\theta_t$ in competitive equilibrium.

The unconditional welfare gains from the macroprudential social planner $EV^{MP}$ are equivalent to a 0.06 percent permanent increase in annual consumption, the same range as in the literature. Hence, endogenous growth does not fundamentally change the benefit of macroprudential policy. The benefit of the policy is a lower frequency of crises as well as a smaller drop in consumption and growth during crises. As I will show later, the welfare benefit from reducing the magnitude of crises is enhanced with endogenous growth. However, crisis is a rare event and its frequency is further reduced by the policy. Furthermore, there is policy trade-off between the trend and cyclical consumption growth. The welfare cost of lowering the trend growth in normal periods is also significant with endogenous growth. Overall, the macroprudential policy
still increases welfare. Its magnitude is comparable to that in the previous literature.

**Welfare Impact of the Policy Trade-off:** To understand the welfare channel of the new policy trade-off, I split the overall welfare gains into two channels: One is a cyclical component of consumption $c^h_t$, a traditional channel as in the literature, and the other is a trend component of consumption, i.e., productivity $z_t$, a new channel with endogenous growth. Specifically, utilities depend on the net consumption series $\{c^h_t\}_t=0^\infty$, which in turn is the product of the cyclical component of consumption $\{c^h_t\}_t=0^\infty$ and the trend component of consumption $\{z_t\}_t=0^\infty$. I will compare these two series for private agents and the social planner in order to understand the welfare impact of the policy trade-off.

To accomplish this, I run 1,000 simulations and get both cyclical and trend components of consumption for the competitive equilibrium and the social planner. To control for the trend (cyclical) component of the consumption channel, I multiply the trend (cyclical) component of consumption in competitive equilibrium by the cyclical (trend) component of consumption under the social planner to construct a counter-factual consumption. I then compare the utility of this counter-factual consumption with the utility of consumption in competitive equilibrium. The difference between these two is considered as gains through the cyclical (trend) component of consumption channel.
Table 4 reports the results. Indeed, gains through the cyclical component of consumption channel are reinforced by endogenous growth: a 0.40 percent permanent increase in annual consumption, which is much larger than those found in the literature. However, there are welfare losses through the trend component of the consumption channel, since the policy reduces average growth. Even if the magnitude of reduction is small, 0.01 percentage point, the cost in terms of welfare is large, a 0.34 percent permanent decrease in annual consumption. Overall, macroprudential policy is still desirable, but due to the new policy trade-off, the gains are no larger than those in the models with exogenous growth.

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Trend Consumption Channel</th>
<th>Cyclical Consumption Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP</td>
<td>0.06</td>
<td>−0.34</td>
<td>0.40</td>
</tr>
</tbody>
</table>

5.4 Policy Instruments

Figure 10 shows the macroprudential tax on capital flows $\tau_{t}^{MP,b}$. The tax rate varies from 0 to 5 percent, depending on the state variable $\hat{b}_{t}$, and I find that it is 1.28 percent on average. As explained before, the macroprudential social planner cannot change the allocation when the constraint binds, and I set the tax rate at zero in these regions. Consistent with the literature, the tax rate peaks in the region where the magnitude of externalities is at its maximum. The tax approaches zero when the economy has sufficient bond holdings $\hat{b}_{t}$.

6 Conclusion

This paper introduces endogenous growth into a model with occasionally binding collateral constraints of the type that has been used previously in the literature on macroprudential policy. In the previous literature, binding constraints did not have a long-run impact on output. By contrast, in my model, they do, which increases their cost and presumably might reinforce the case for macroprudential policy. My model thus lends itself to analyzing the role of macroprudential policy in the context of a tradeoff between growth and financial stability.

The impact of macroprudential policy on average growth is, in general, ambiguous. Macroprudential policy reduces the frequency of crises and their impact on growth but comes at the

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25 As before, I plot it over the bond holding $\hat{b}_{t}$ when the shock $\theta_{t}$ is 2 standard deviations below its long-run average.
cost of reducing borrowing and growth in good times. To resolve this ambiguity, I look at a calibrated version of the model.

In the quantitative analysis, I find that optimal macroprudential policy substantially reduces the frequency of crisis but has a very small negative effect on average growth. As is shown in the literature, changes in average growth have very large welfare impacts (see Lucas (1987) and Barlevy (2004)). Given that optimal macroprudential policy must lower average growth to increase financial stability, it does not change growth by a large amount, because even a small reduction in growth is costly in terms of welfare. Quantitatively, a 0.01 percentage point reduction in average growth leads to a welfare loss equivalent to a 0.34 percent permanent decrease in annual consumption.

Nevertheless, macroprudential policy is still desirable because it reduces the probability of crisis and smooths consumption. The benefits from consumption smoothing actually outweigh the welfare loss from the reduction in average growth. Overall, welfare gains are at the magnitude of a 0.06 percent permanent increase in annual consumption, which is in the same range as in the existing literature.

This paper is suitable for policymakers’ reflections about their policies’ impacts on average growth and financial stability. One takeaway is that macroprudential policy only marginally lowers average growth to enhance financial stability. Therefore, it is still desirable to use macroprudential policy, even considering its negative impact on average growth.
To the best of my knowledge, this is the first paper to analyze the impact of macroprudential policy on growth. Hence, there are many unsolved, interesting questions that I leave for future research. First and foremost, my paper is about the role of macroprudential policy in capital flows. However, many countries, including advanced economies, adopted macroprudential policies towards other financial markets after the 2008-09 Global Financial Crisis. It would be interesting to continue the research by looking at the effects of other macroprudential policies (leverage ratio, capital requirement, etc.). Second, my paper abstracts from the risk-taking behavior in the economy. In the model, macroprudential policy negatively affects growth because it restricts the amount of funding to productive projects. However, private agents might respond to the policy by taking on riskier projects. Such risk-taking behavior might be socially inefficient, even if it is privately optimal. In the end, excessive risk-taking behavior might lower average growth. Therefore, it may be interesting to see whether average growth is further driven down by this optimal policy.
References


A Data Source

The sample includes the following 55 countries:

<table>
<thead>
<tr>
<th>Algeria</th>
<th>Argentina</th>
<th>Australia</th>
<th>Austria</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Canada</td>
<td>Chile</td>
<td>China</td>
<td>Colombia</td>
</tr>
<tr>
<td>Cote d’Ivoire</td>
<td>Croatia</td>
<td>Czech Republic</td>
<td>Denmark</td>
<td>Dominican Republic</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Egypt, Arab Rep.</td>
<td>El Salvador</td>
<td>Finland</td>
<td>France</td>
</tr>
<tr>
<td>Germany</td>
<td>Greece</td>
<td>Hungary</td>
<td>Iceland</td>
<td>Indonesia</td>
</tr>
<tr>
<td>Ireland</td>
<td>Italy</td>
<td>Japan</td>
<td>Korea, Rep.</td>
<td>Lebanon</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Mexico</td>
<td>Morocco</td>
<td>Netherlands</td>
<td>New Zealand</td>
</tr>
<tr>
<td>Nigeria</td>
<td>Norway</td>
<td>Pakistan</td>
<td>Panama</td>
<td>Peru</td>
</tr>
<tr>
<td>Philippines</td>
<td>Poland</td>
<td>Portugal</td>
<td>Russian Federation</td>
<td>South Africa</td>
</tr>
<tr>
<td>Spain</td>
<td>Sweden</td>
<td>Thailand</td>
<td>Tunisia</td>
<td>Turkey</td>
</tr>
<tr>
<td>Ukraine</td>
<td>United Kingdom</td>
<td>United States</td>
<td>Uruguay</td>
<td>Venezuela, RB</td>
</tr>
</tbody>
</table>

The sources are as follows:

**GDP Per Capita Growth:** GDP per capita from World Development Indicators (WDI);

**TFP:** Pen World Table;

**Consumption Share of GDP:** calculated using final consumption expenditure and GDP data in WDI;

**Net Foreign Asset to GDP Ratio:** an updated dataset in Lane and Milesi-Ferretti (2007) (see http://www.philiplane.org/EWN.html).

B Empirical Results for KM episodes

I use sudden stop episodes as in Korinek and Mendoza (2014) to show the persistent output-level effects of crises. One can see that this effect is robust to identification of crises. Furthermore, TFP displays a similar pattern to output, as in Figure 2.
Note: The series are constructed using an 11-year window centering on the sudden stop episodes.

C Normalized Economy

I normalize the economy by the endogenous variable $z_t$ and denote normalized variables by a hat. The normalized competitive equilibrium conditions are given by

\[
\left( c_t^h \right)^{-\gamma} \Psi_{1,t} = \beta g_{t+1}^{1-\gamma} E_t \left[ \left( c_{t+1}^h \right)^{-\gamma} \left( \theta_{t+1} - h - \Psi_{2,t+1} \right) \right]
\]

\[
\left( c_t^h \right)^{-\gamma} \hat{q}_t = \beta g_{t+1}^{1-\gamma} E_t \left[ \left( c_{t+1}^h \right)^{-\gamma} \left( \alpha \theta_{t+1} + \hat{q}_{t+1} \right) \right]
\]

\[
\left( c_t^h \right)^{-\gamma} = \hat{\mu}_t^{CE} + \beta g_{t+1}^{1-\gamma} (1 + r) E_t \left[ \left( c_{t+1}^h \right)^{-\gamma} \right]
\]

\[
\hat{c}_t^h + \Psi(g_{t+1}) + \hat{b}_{t+1}g_{t+1} = \theta_t - h + (1 + r)\hat{b}_t
\]

\[
\hat{\mu}_t^{CE} (\hat{b}_{t+1}g_{t+1} + \phi \hat{q}_t) = 0, \text{ with } \hat{\mu}_t^{CE} \geq 0.
\]
For the macroprudential social planner, the normalized equilibrium conditions are

\[
\hat{\lambda}_{t}^{MP} = \left(\hat{c}_{t}^{h}\right)^{-\gamma} + \frac{\gamma \Phi_{t}^{MP} \hat{q}_{t}}{\hat{c}_{t}^{h}} + \hat{\gamma} \hat{\lambda}_{t}^{MP} \left(\hat{c}_{t}^{h}\right)^{-\gamma - 1} \Psi_{1,t}
\]

\[
\hat{\lambda}_{t}^{MP} \Psi_{1,t} = \frac{\Phi_{t}^{MP} \hat{q}_{t}}{\hat{c}_{t}^{h}} - \hat{\gamma} \hat{\lambda}_{t}^{MP} \left(\hat{c}_{t}^{h}\right)^{-\gamma} - \hat{\gamma} \hat{\lambda}_{t}^{MP} \left(\hat{c}_{t}^{h}\right)^{-\gamma - 1} \Psi_{1,t} - \hat{\gamma} \hat{\lambda}_{t}^{MP} \left(\hat{c}_{t}^{h}\right)^{-\gamma - 1} \Psi_{1,t}
\]

\[
= \beta g_{t+1}^{-\gamma} E_{t} \left[ \hat{\lambda}_{t+1}^{MP} (\theta_{t+1} - h - \Psi_{2,t+1}) - \hat{\lambda}_{t+1}^{MP} \left(\hat{c}_{t+1}^{h}\right)^{-\gamma} \hat{\Psi}_{12,t+1} \right]
\]

\[
\hat{\lambda}_{t}^{MP} = \hat{\mu}_{t}^{MP} + \frac{\Phi_{t}^{MP} g_{t+1}^{-\gamma} \hat{G}_{2,t}}{\left(\hat{c}_{t}^{h}\right)^{-\gamma}} + \hat{\gamma} \hat{\lambda}_{t}^{MP} g_{t+1}^{-\gamma} \hat{I}_{2,t} + \beta (1 + r) g_{t+1}^{-\gamma} E_{t} \left[ \hat{\lambda}_{t+1}^{MP} \right]
\]

where

\[
I(z_{t+1}, b_{t+1}) = z_{t+1}^{-\gamma} \hat{I} (\hat{b}_{t+1}),
\]

\[
I_{1,t} = (-\gamma) z_{t+1}^{-\gamma - 1} \hat{I} (\hat{b}_{t+1}) + z_{t+1}^{-\gamma} \hat{I} (\hat{b}_{t+1}) \frac{-b_{t+1}}{z_{t+1}^{2}} = -z_{t+1}^{-\gamma - 1} \left[ \gamma \hat{I} + \hat{I} \hat{b}_{t+1} \right],
\]

\[
I_{2,t} = z_{t+1}^{-\gamma - 1} \hat{I}.
\]

and

\[
G(z_{t+1}, b_{t+1}) = z_{t+1}^{1-\gamma} \hat{G} (\hat{b}_{t+1}),
\]

\[
G_{1,t} = (1 - \gamma) z_{t+1}^{-\gamma} \hat{G} (\hat{b}_{t+1}) + z_{t+1}^{1-\gamma} \hat{G} (\hat{b}_{t+1}) \frac{-b_{t+1}}{z_{t+1}^{2}} = z_{t+1}^{-\gamma} \left[ (1 - \gamma) \hat{G} \hat{G} \hat{b}_{t+1} \right],
\]

\[
G_{2,t} = z_{t+1}^{-\gamma} \hat{G}.
\]

D Proofs

D.1 Proof of Proposition 1

Proof. To implement the macroprudential social planner’s allocation, I compare the normalized optimality conditions of private agents and of the macroprudential social planner (see Appendix
and find that

\[ \tau_{MP, b} = \beta g_{t+1} (1 + r) E_t \left[ \gamma \hat{\phi}_{t+1}^{MP} \hat{q}_{t+1} (\hat{c}_t^{h})^{-1} + \gamma \hat{\nu}_{t+1}^{MP} (\hat{c}_t^{h})^{-1} \Psi_{1,t+1} \right] \]

E Sensitivity Analysis

I conduct sensitivity analysis for different parameters in the model. As with the baseline calibration, I first give values for seven parameters, i.e., \{\beta, \psi, r, \gamma, \alpha, \rho, \sigma\}: I only change the value of one parameter while keeping the other parameter values the same, as in the baseline calibration. Given these values, I choose \{\kappa, h, \phi\} to match average growth, the consumption to GDP ratio, and the NFA-GDP ratio. I follow this strategy because I want the model to match average growth, which is affected by consumption’s share of GDP and by the NFA-GDP ratio. The sensitivity analysis results are presented in Table 5, and I discuss the robustness of my results with respect to the parameters. One can see that the results do not change with \alpha, since in the calibration, I assume that the collateral constraint binds in steady state, and that \phi changes with \alpha.

Impacts on Growth: The negative relationship between average growth and financial stability for the macroprudential social planner is very robust to all the parameter values. Furthermore, the growth cost of the policy is very small.

Welfare Gains: The results on welfare gains are robust to various parameters. In particular, I find that the macroprudential social planner can generate welfare gains equivalent to a 0.06 percent permanent increase in annual consumption. In particular, the size of gains increases with parameters that affect the size of externalities, such as \phi. The gains also increase with parameters that make growth more sensitive to shocks, such as \{\psi, \gamma\}. Given that the social planners smooth the economy, welfare gains also increase with parameters that govern risk, such as \{\rho, \sigma\}.\footnote{Here, lower \rho implies a higher risk for the economy, since it is more likely to enter a bad state tomorrow conditional on a good state today.} The welfare gains are supposed to decrease with the discount rate \beta and the
Table 5: Sensitivity Analysis

<table>
<thead>
<tr>
<th></th>
<th>Welfare Gains (%)</th>
<th>Tax on Capital Flows (%)</th>
<th>Prob. of Crisis (%)</th>
<th>Average GDP Growth (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MP (overall)</td>
<td>MP (growth)</td>
<td>MP (consumption)</td>
<td>CE</td>
</tr>
<tr>
<td>baseline</td>
<td>0.06</td>
<td>−0.34</td>
<td>0.40</td>
<td>1.28</td>
</tr>
<tr>
<td>β = 0.93</td>
<td>0.01</td>
<td>−0.04</td>
<td>0.05</td>
<td>1.51</td>
</tr>
<tr>
<td>β = 0.95</td>
<td>0.03</td>
<td>−0.17</td>
<td>0.20</td>
<td>1.67</td>
</tr>
<tr>
<td>ψ = 0.94</td>
<td>0.12</td>
<td>−0.48</td>
<td>0.59</td>
<td>1.65</td>
</tr>
<tr>
<td>ψ = 0.96</td>
<td>0.03</td>
<td>−0.16</td>
<td>0.18</td>
<td>1.04</td>
</tr>
<tr>
<td>φ = 0.07</td>
<td>0.02</td>
<td>−0.12</td>
<td>0.13</td>
<td>0.81</td>
</tr>
<tr>
<td>φ = 0.08</td>
<td>0.02</td>
<td>−0.17</td>
<td>0.20</td>
<td>0.94</td>
</tr>
<tr>
<td>r = 3%</td>
<td>0.12</td>
<td>−0.56</td>
<td>0.69</td>
<td>2.59</td>
</tr>
<tr>
<td>r = 4%</td>
<td>0.10</td>
<td>−0.41</td>
<td>0.51</td>
<td>1.92</td>
</tr>
<tr>
<td>γ = 3</td>
<td>0.21</td>
<td>−1.13</td>
<td>1.40</td>
<td>2.38</td>
</tr>
<tr>
<td>γ = 4</td>
<td>0.43</td>
<td>−1.77</td>
<td>2.19</td>
<td>3.03</td>
</tr>
<tr>
<td>α = 0.3</td>
<td>0.06</td>
<td>−0.34</td>
<td>0.40</td>
<td>1.28</td>
</tr>
<tr>
<td>α = 0.4</td>
<td>0.06</td>
<td>−0.34</td>
<td>0.40</td>
<td>1.28</td>
</tr>
<tr>
<td>ρ = 0.80</td>
<td>0.05</td>
<td>−0.34</td>
<td>0.40</td>
<td>1.37</td>
</tr>
<tr>
<td>ρ = 0.90</td>
<td>0.03</td>
<td>−0.31</td>
<td>0.33</td>
<td>1.35</td>
</tr>
<tr>
<td>σ = 0.02</td>
<td>0.02</td>
<td>−0.08</td>
<td>0.10</td>
<td>0.93</td>
</tr>
<tr>
<td>σ = 0.03</td>
<td>0.03</td>
<td>−0.19</td>
<td>0.22</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Note: Welfare gains and taxes on debt are calculated by simulating the economy for 10,000 periods. Crises are defined as periods when the collateral constraint binds and the current account reversal exceeds 1 standard deviation of its long-run average.

interest rate \(r\), since they decide private agents’ impatience condition, given by \(\beta (1 + r) g^{-\gamma}\). Intuitively, when agents are more impatient, i.e., there is a lower \(\beta\) or \(r\), the economy borrows more and ends up with more crises. Policy interventions should have more benefits, since they mitigate the frequency and severity of crises. Indeed, I find larger gains with a lower interest rate. However, I also find that welfare gains increase with \(\beta\). This is because \(\beta\) decides the Euler equation of productivity. High \(\beta\) means that private agents care more about the reduction of growth during crisis. Hence, policy interventions can generate larger benefits by reducing this reduction.

Size of Interventions: In the baseline results, I find that the macroprudential social planner imposes a 1.28 percent capital flows tax. Generally speaking, the magnitude of the macroprudential capital flows tax varies with different parameters and depends on the size of externalities and the ergodic distribution of debt.

F Numerical Methods for Solving Policy Functions

I first create a grid space \(G_b = \{\hat{b}^0, \hat{b}^1, \cdots\}\) for the bond holding \(\hat{b}_t\) and a grid space \(\Theta = \{\theta_1, \cdots, \theta_5\}\) for the exogenous technology shock \(\theta_t\). The discretization method for the log AR
Given policy functions in iteration step \(j\) of the endogenous gridpoint method, as in Carroll (2006) to iterate first-order conditions in CE and MP, and the iteration stops until policy functions converge. Policy functions in competitive equilibrium include consumption \(C\), endogenous growth \(G\), asset price \(Q\), and bond holding \(B\). Denote the iteration step by \(j\) and start from arbitrary policy functions \(C^0(\hat{b}_t, \theta_t)\), \(G^0(\hat{b}_t, \theta_t)\), \(Q^0(\hat{b}_t, \theta_t)\), and \(B^0(\hat{b}_t, \theta_t)\), where 0 means the iteration step \(j = 0\). Given policy functions in iteration step \(j\), I solve policy functions for iteration \(j + 1\) as follows:

1. For any \(\theta_t \in \Theta\) and \(\hat{b}_{t+1} \in G_t\), I can solve \(\{\hat{c}_t, g_{t+1}, \hat{q}_t\}\) using equilibrium conditions. Using the budget constraint, these allocations imply a unique \(\hat{b}_t\). Then I have a combination of \(\{\hat{b}_t\}\) and corresponding allocations \(\{\hat{c}_t, g_{t+1}, \hat{q}_t, \hat{b}_{t+1}\}\). I can update policy functions using these combinations. In this process, I need to deal with the collateral constraint. Specifically, I assume that the constraint is slack and then check whether this condition is satisfied.

2. I first assume that the constraint is slack and allocations \(g_{t+1}, \hat{c}_t, \hat{q}_t\) can be solved using the following conditions:

\[
\Psi_t(g_{t+1}) = \frac{E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} (\theta_{t+1} - h - \Psi_2(G^j(\hat{b}_{t+1}, \theta_{t+1}))) \right]}{(1 + r) E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} \right]} \\
\hat{c}_t = g_{t+1} \left[ \beta (1 + r) E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} \right] \right]^{-\frac{1}{\gamma}} \\
\hat{q}_t = \left( \hat{c}_t \right)^\gamma g_{t+1}^{-\gamma} E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} (\alpha \theta_{t+1} + Q(\hat{b}_{t+1}, \theta_{t+1})) \right] \\
\]

3. If the collateral constraint \(-\hat{b}_{t+1} g_{t+1} \leq \phi \hat{q}_t\) is satisfied, I proceed to solve \(\hat{b}_t\) using the budget constraint:

\[
\hat{b}_t = \frac{\hat{c}_t + h + \Psi(g_{t+1}) + \hat{b}_{t+1} g_{t+1} - \theta_t}{1 + r} \\
\]

4. If the constraint is violated, I can solve allocations \(\{\hat{q}_t, \hat{c}_t, g_{t+1}\}\) using the following equations:

\[
\left( \hat{c}_t \right)^{-\gamma} \Psi_t(g_{t+1}) = \beta g_{t+1}^{-\gamma} E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} (\theta_{t+1} - h - \Psi_2(G^j(\hat{b}_{t+1}, \theta_{t+1}))) \right] \\
-\hat{b}_{t+1} g_{t+1} = \phi \hat{q}_t \\
\hat{q}_t = \left( \hat{c}_t \right)^\gamma g_{t+1}^{-\gamma} E_t \left[ (C^j(\hat{b}_{t+1}, \theta_{t+1}))^{-\gamma} (\alpha \theta_{t+1} + Q(\hat{b}_{t+1}, \theta_{t+1})) \right] \\
\]

5. I can update policy functions using the combinations of \(\hat{b}_t\) and \(\{g_{t+1}, \hat{c}_t, \hat{q}_t, \hat{b}_{t+1}\}\).
6. I keep iterating until policy functions in two consecutive iterations are close enough.

To solve policy functions for the social planner, I need to solve additional policy functions of Lagrangian multipliers, i.e. $\mu(\hat{b}_t, \theta_t)$ and $\nu(\hat{b}_t, \theta_t)$, using equilibrium conditions described in Appendix C. Otherwise, the procedure is the same as above.