Sudden Stop with Local Currency Debt *

Siming Liu† Chang Ma‡ Hewei Shen§

January 9, 2024

Abstract

Over the past two decades, emerging market economies have improved their liability structures by increasing the share of their debt denominated in local currency. This paper introduces a local currency debt (i.e., in units of aggregate consumption) into a sudden stop model and explores how this alternative structure sheds new perspectives on financial regulations. Decentralized agents do not internalize the effects of their portfolio decisions on financial amplification and undervalue the insurance benefit of using local currency debt. However, due to debt-deflation incentives and the cost of buying insurance, a discretionary planner is reluctant to issue local currency debts, and capital controls are primarily used to restrict credit volumes. In contrast, a social planner with commitment would promise a higher future payoff to obtain a more favorable bond price. The capital control under commitment encourages borrowing in local currency, mitigates the severity of crises, and improves welfare relative to laissez-faire.

Keywords: Sudden stop, Pecuniary externality, Local currency debt, Time inconsistency, Capital control tax.

JEL Classification: F38, F41, G18

*We thank the co-editor Javier Bianchi and two anonymous referees for their helpful comments. We are indebted to Grey Gordon, Juan Carlos Hatchondo, Olivier Jeanne, Alessandro Rebucci, Eugenio Rojas, and Jian Wang for their suggestions. We are also grateful to the participants at the 2021 China International Conference in Macroeconomics (CICM), 2021 Western Economic Association International (WEAI) Conference, 2021 CCER Summer Institute, 2022 Fall Midwest Macro Meeting, 2023 CES North American Conference, the 29th Conference of Computing in Economics and Finance (CEF), and the seminar at the IMF. Chang Ma gratefully acknowledges financial support from the National Natural Science Foundation of China (grant no. 72003043) and was also sponsored by the Shanghai Pujiang Program. Hewei Shen is thankful for the Junior Faculty Summer Fellowship from the Dodge Family College of Arts and Sciences at the University of Oklahoma. All errors are our own.

†Department of Economics, Binghamton University (SUNY). Email: simliu86@gmail.com.

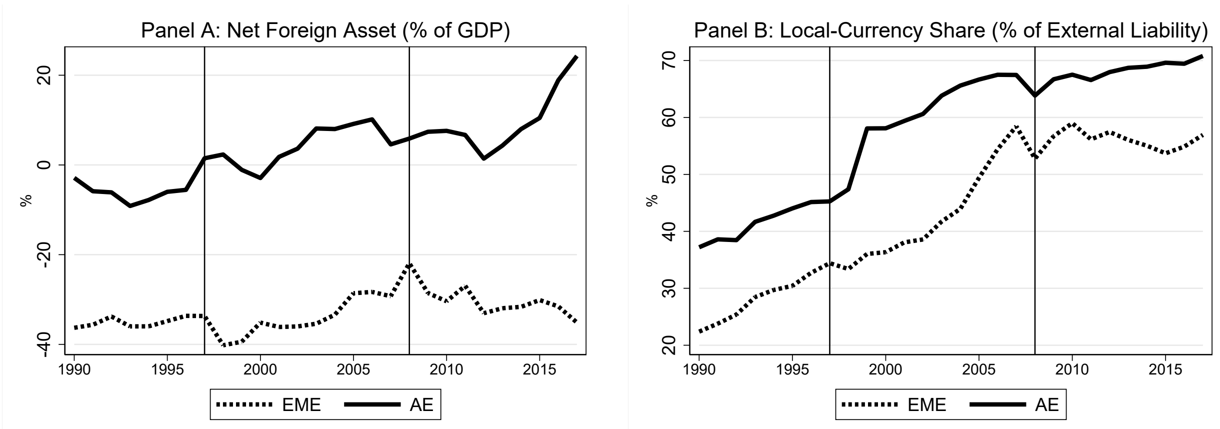
‡Fanhai International School of Finance (FISF), Fudan University. Email: changma@fudan.edu.cn.

§Department of Economics, University of Oklahoma. Email: hewei.shen@ou.edu.

1 Introduction

Financial fragility within emerging markets is closely related to countries' liability structures and their foreign currency exposure. In economic downturns, a country's dollar-denominated debt amplifies the adverse effect of negative shocks, creating a large devaluation of the domestic currency. Under certain financial frictions, the reduction in income further constrains a country's ability to borrow and its real absorption. As in the sudden stop literature, the endogenous feedback between consumption collapse, real depreciation, and amplification through financial constraints leads to a Fisher's debt-deflation mechanism (e.g., Mendoza, 2002; Bianchi, 2011; Benigno et al., 2016; Schmitt-Grohé & Uribe, 2021). Therefore, sudden stops are usually characterized by large current account reversals, currency devaluations, and a sudden freeze in financial intermediation.[1]

Figure 1: Net Foreign Assets and Local Currency Share of External Liabilities



NOTE: The vertical lines mark the 1997 Asian Crisis and 2009 Global Financial Crisis. The data comes from Bénétrix et al. (2019). See appendix A for the list of countries in the sample.

The financial instability that arises from currency mismatch also has welfare consequences and calls for appropriate financial regulations. While the existing literature (e.g., Bianchi, 2011) has argued for a restriction on debt volumes, many ignore the composition of credit flows. The capital structures of emerging countries, however, have evolved drastically over the past two decades. As shown in figure 1, although emerging economies' net foreign asset position remains largely negative, their external liability has steadily increased toward local currency. This non-negligible change in capital structures suggests that countries have obtained a safer foreign exchange position and warrants a renewed perspective on financial regulations. How does a country's rising local currency share affect its economic resilience during a financial crisis? Are the existing policy prescriptions still effective in today's environment? What are the optimal capital control policies under the

[1] The problem of excessive reliance on foreign currency debt has been described as the "original sin" by Eichengreen & Hausmann (1999).

presence of local currency debt?[2]

We answer these questions by embedding the issuance of local currency debt into a sudden stop model with a flow collateral constraint. The model is a two-sector small open economy with limited access to international financial markets. Domestic households can trade two types of financial assets with foreign investors: a one-period bond denominated in the tradable consumption good (referred to as foreign currency debt, or FCD henceforth) and a one-period bond denominated in the domestic consumption bundle (referred to as local currency debt, or LCD henceforth). The LCD invites a state-contingent payment schedule from the foreign investors' perspective, and its price is endogenously determined by the foreign investors' expectation of exchange rate fluctuations.[3] The form of the collateral constraint follows Mendoza (2002), which says that a country's total proceeds of borrowing is restricted by the market value of its income. This constraint has been widely adopted by other works that study the sudden stop phenomenon and associated pecuniary externalities.

Relative to a standard sudden stop economy with only FCD, the introduction of LCD changes the state contingency in the payoff of a country's liability. It improves risk sharing of the indebted economies with the rest of the world and, more importantly, adds economic resilience during a financial crisis. In particular, the local currency bond provides a buffer against large adverse shocks and mitigates the severity of sudden stop crises. However, risk-averse foreign investors perceive procyclical movements in the real exchange rate and charge a risk premium on their holding of local currency bonds. Therefore, in equilibrium, the domestic agent chooses debt denominations based on the hedging benefit of using LCD and the cost of buying this insurance.[4]

We then investigate policy implications in this new environment. Similar to Bianchi (2011), the pecuniary externality arising from the financial constraint leads to an overborrowing phenomenon and an inefficiently high probability of a financial crisis. On top of that, the introduction of LCD adds inefficiencies in the denomination choice. This occurs because private agents do not internalize the effects of their portfolio decisions on the financial amplification and therefore undervalue the insurance benefit of LCD. If allowed to control debt denominations, a social planner would have incentives to issue more debts in local currency because it generates a more favorable payoff schedule.

[2]The theoretical literature (e.g., Caballero & Krishnamurthy, 2003; Drenik et al., 2022; Engel & Park, 2022) has argued that the trend in the change of debt denomination can be attributed to long-run factors such as more disciplined monetary policies, development in financial institutions and financial markets, or lowered policy risk. The purpose here is not to explain this trend but to consider its implications for capital control policies.

[3]Ottonello & Perez (2019) and Du et al. (2020) have argued that governments that lack the commitment to monetary policy would tilt the composition of debt toward foreign currency. Compared to these papers, we abstract from monetary policy and only consider real bonds that are denominated in consumption goods. The associated time-inconsistency problems are also different.

[4]The insurance role of LCD in financial crises is supported by our empirical analysis using cross-country data. Figures A.1-A.2 in appendix A compare the crisis event windows between two groups of countries and show that countries with higher local currency shares in their liabilities experienced milder recessions during sudden stops. This result holds in both the advanced and emerging country groups. In addition, panel estimation in table A.2 shows that an increased share of local currency debt improves a country's economic resilience during a financial crisis. Table A.1 shows the list of sample countries and the identified sudden stop episodes.

During a financial crisis, the improved capital structure eases a country's debt burden when the real exchange rate depreciates. The reduction in debt burden mitigates the financial amplification caused by a negative shock and improves the country's borrowing opportunity. The relaxed financial constraint also increases the consumption demand and alleviates the real depreciation during the financial crisis. In addition, the smaller depreciation in expectation makes the issuance of LCD less costly ex ante.

However, we find that the efficient use of LCD requires policy commitment. We analyze the optimal capital control policies by solving two social planning problems: a Markov (discretionary) planner and a social planner under commitment. The discretionary planner solves a recursive problem and has strong incentives to lower the real exchange rate to deflate the debt burden in local currency. This ex post debt-deflating incentive reduces the ex ante bond price and, in equilibrium, makes borrowing in LCD undesirable. On the other hand, the commitment planner can flexibly manipulate the future consumption profile to obtain a more favorable debt payoff schedule. Specifically, she would commit to increasing consumption in good states of the world in order to ensure a better bond price. As a result, the improvement in bond price leads to the greater issuance of LCD. When a sudden stop occurs, the larger debt share in local currency alleviates the country's consumption decline and exchange rate depreciation.

Our model calibration reveals that commitment to capital control policies is quantitatively important. In the absence of commitment power, the Markov planner's primary policy objective is to control total credit volumes. The stringent capital regulation leads to significantly less borrowing, and due to a debt-deflating motive, a lower share of LCD in its capital inflows. The model then becomes similar to a sudden stop economy with only dollar debt. The policy implication under commitment is quite different: the planner aims to change the composition of credit flows by tilting more debts toward the local currency. The improved bond price and capital structure ease the pain of a sudden stop crisis and simultaneously create better borrowing opportunities. Therefore, while both social planners obtain a more stabilized financial market, the commitment planner's policy allows the economy to sustain a higher debt level than the competitive equilibrium and achieves a larger welfare gain.

We also compare our baseline model with an FCD-only economy à la Bianchi (2011) and its constrained-efficient outcome. Our analysis shows that even without any capital control policies, the ability to issue LCD alone improves the economy's risk-sharing and eases the severity of a financial crisis. Meanwhile, as discussed in the sudden stop literature, the main objective of prudential regulations is to target crisis episodes and reduce the probability of crises. The simulation result shows that introducing LCD alone can deliver a sizable welfare gain that is quantitatively comparable to imposing prudential regulations in an FCD-only economy. This result suggests that financial integration has a strong foothold on welfare consequences and could be a substitute for financial regulations in a dollar-debt economy.

The policy implications from our model suggest that an ideal capital control policy would deliver a less risky liability structure for a country, i.e., encouraging the issuance of LCD relative to FCD. Such policy implication is consistent with Ostry et al. (2012), who suggest using capital controls to alter the composition of capital inflows. Furthermore, our quantitative analysis also highlights the importance of policy commitment to achieve this goal.

Related Literature. First, the paper contributes to the sudden stop literature with pecuniary externalities (e.g., Benigno et al., 2013, 2016, 2019; Bianchi, 2011; Jin & Shen, 2020; Ma, 2020; Schmitt-Grohé & Uribe, 2021).[5] The standard sudden stop model assumes that debts are only denominated in hard currency while a significant proportion of collateral income comes from the domestic sector. This currency mismatch generates an inefficient amount of borrowing and results in financial vulnerability. Based on these studies, we introduce the LCD into a sudden stop model and consider the policy adjustments that change countries' debt denomination. In our model, the pecuniary externalities from the collateral and budget constraints lead to an inefficient denomination choice. Moreover, the LCD also creates a time-inconsistency issue. As a result, policy implications are rather different. Our paper demonstrates the importance of a nonuniform capital control tax that changes the composition of capital flows.

Mendoza & Rojas (2017, 2019) introduce the external local currency debt (denominated in the aggregate consumption bundle) into a sudden stop model and consider the new policy implications. They also highlight the time-inconsistency issue in the design of optimal policies, which is due to the endogenous payoff of LCD and its endogenous bond price. Our paper differs from theirs in two important ways. First, we consider the portfolio choice between LCD and FCD. By doing so, we contribute to the recent policy discussion of capital controls in reshaping the composition of capital inflows. Second, we compare the policy assignments involving both commitment and discretionary planners' problems. Mendoza & Rojas (2019), instead, only analyze welfare implications if the government commits to simple policy rules. Mendoza & Rojas (2017), on the other hand, solves for the conditionally efficient allocations, which requires the social planner's commitment to support the pricing function in the competitive equilibrium. Throughout our paper, we follow the method used in Bianchi & Mendoza (2018) to solve for social planners' constrained-efficient allocations and characterize the associated capital control policies.[6]

Our paper relates to the existing studies on countries' currency portfolio of external debts (e.g., Bohn, 1990; Drenik et al., 2022; Du et al., 2020; Korinek, 2011; Ottonello & Perez, 2019). In a pioneer work, Bohn (1990) analyzes the benefits of foreign currency debt relative to domestic currency debt. In particular, FCD is more desirable when domestic inflation is relatively more

[5] See Bianchi & Mendoza (2020), Erten et al. (2021), and Rebucci & Ma (2020) for a comprehensive review on the theoretical framework of capital controls and the empirical evidence.

[6] Bianchi & Mendoza (2018)'s model also features a time-inconsistency problem in the macroprudential regulation. But, different from our paper, their time-inconsistency issue is due to the forward-looking nature of asset price (such as land or capital price) in the collateral constraint.

uncertain and the time-inconsistency problem with LCD is more severe. Korinek (2011) builds a small open economy model with debts denominated in tradable and nontradable goods and studies the mutual feedback between currency denomination, exchange rate risk, and macro-volatility. However, his analysis abstracts from the endogenous collateral constraint and associated pecuniary externalities. In contrast, we build a richer framework to study the interaction between the portfolio choice and the pecuniary externality caused by collateral constraints.

Ottonello & Perez (2019) investigate the government's debt denomination choice when the monetary policy lacks commitment. The incentive to dilute debt payment through currency depreciation induces the government to issue a larger fraction of debt in foreign currency and forgo the hedging benefit of LCD. Consistent with this channel, Du et al. (2020) provide data evidence showing that governments whose LCD provides stronger hedging benefits actually borrow more in foreign currency. In a similar vein, Drenik et al. (2022) set up an optimal contract model to investigate the interaction between the currency choice of private debt and optimal monetary policy. Different from these papers, we abstract from monetary policies and the mechanism in this paper is generated by the real exchange rate risk.

This paper also belongs to the literature that studies externalities associated with risk-sharing and portfolio decisions (e.g., Bocola & Lorenzoni, 2020, 2023). In a generalized model with state-contingent claims, Bocola & Lorenzoni (2023) show that entrepreneurs demand an insufficient amount of risk-sharing because the aggregate risk makes risk-averse consumers charge a high insurance premium. In an analytical setup, Bocola & Lorenzoni (2020) show that liability dollarization is a self-fulfilling equilibrium due to the feedback between denomination choice and the risk premium of LCD. The lending of last resort can eliminate this fragile equilibrium and guide the economy toward a better one without currency mismatch. While these papers focus on the domestic debt, our paper emphasizes external liability dollarization and, therefore, has different policy implications. We study the role of ex ante capital control tax in restoring social efficiency, whereas they study ex post bailout policies.

Another related paper is Farhi & Werning (2016), who study a currency portfolio problem in an environment with aggregate demand externalities. However, their model environment abstracts from the time-inconsistency issue, which is an important aspect of our analysis. Bianchi & Sosa-Padilla (2020) also feature a portfolio problem in the presence of demand externality. They show that the interaction between sovereign default risk and nominal rigidity can account for a macro-stabilization role of international reserves in tranquil times.

Road Map. The rest of the paper is organized as follows. Section 2 lays out our benchmark model and builds the environment to study optimal policies. Section 3 considers a simplified environment and characterizes some analytical properties. Section 4 calibrates the quantitative model and presents simulation results. Section 5 compares our framework with an FCD-only economy to discuss the relationship between financial integration and financial regulation. Section 6 concludes the paper.

2 The Model

This section builds a small open economy model with a flow collateral constraint. Unlike standard sudden stop models in the literature (e.g., Mendoza, 2002; Bianchi, 2011), we include the local currency debt into the environment and consider agents' debt denomination decisions.

2.1 Economic Environment

There are two types of agents in the economy: domestic households and foreign investors. Time is discrete and lasts for infinite horizons: $t = 0, 1, 2, 3 \cdots$. Households are identical infinitely-lived agents that consume both tradable ($c_{T,t}$) and nontradable ($c_{N,t}$) goods to maximize their lifetime utility as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $\mathbb{E}_t(\cdot)$ is the expectation operator conditional on information at time t. β is the subjective discount factor. The per-period utility takes on the CRRA form: $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$, where the final consumption c_t is a composite product that comes from both tradable and nontradable sectors with a CES aggregator,

$$c_t = \left[\omega c_{T,t}^{\frac{\theta-1}{\theta}} + (1-\omega) c_{N,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

σ is the coefficient of relative risk aversion, $\omega \in (0,1)$ is the weight on tradable consumption in the composite consumption basket, and $\theta > 0$ is the elasticity of substitution between the two sectors.

Throughout the paper, the tradable good serves as the numeraire, and its price is normalized to 1. We denote p_t^N and p_t^C as the relative prices of the nontradable good and composite consumption, respectively.[7] In each period, the household receives tradable and nontradable endowments. We assume that the supply of nontradable goods is a constant $\{y_{N,t} = \overline{y}_N\}_{t=0}^{\infty}$, and that the tradable endowment follows a log AR(1) process of

$$\log(y_{T,t}) = (1-\rho)\mu + \rho \log(y_{T,t-1}) + \epsilon_t, \tag{3}$$

where $\rho \in (0,1)$ and μ are the persistence and mean of the endowment process, respectively. ϵ_t is an i.i.d. random variable that follows a normal distribution: $\epsilon_t \sim N(0, \sigma_\epsilon^2)$.

The financial market is both incomplete and imperfect. There are two types of financial assets households can trade with foreign investors: b_{t+1}^T denotes the units of one-period bonds denominated

[7] As we will show later, p_t^C is monotonically increasing in p_t^N. Throughout the paper, p_t^C is interpreted as the real exchange rate.

in tradable consumption (referred to as FCD), while b_{t+1}^C denotes the units of one-period bonds denominated in aggregate consumption bundles (referred to as LCD). The bond prices (q_t^T and q_t^C) are determined by the international investors' problem, which will be specified below.

The household's sequential budget constraint is given by

$$c_{T,t} + p_t^N c_{N,t} + p_t^C b_t^C + b_t^T = y_{T,t} + p_t^N \overline{y}_N + q_t^C b_{t+1}^C + q_t^T b_{t+1}^T. \tag{4}$$

In period t, the household receives tradable and nontradable incomes, decides consumption allocations, and issues bonds in the international financial market.

As we will discuss later, the portfolio in a country's liability is uniquely pinned down by investors' risk aversion. Presumably, the domestic agents will have incentives to borrow in local currency because its payment structure allows them to enjoy risking-sharing benefits relative to the foreign currency borrowing. However, many countries find it difficult to issue LCDs and excessively rely on foreign currency borrowings in the international market due to some institutional distortions (e.g., a less disciplined monetary policy and incomplete financial integration).[8] While many papers have tackled this issue, we abstract from these institutional costs of using LCD and instead focus on the suboptimality in private agents' portfolio decisions that arises from pecuniary externality.

The household's borrowing capacity is restricted by a collateral constraint, saying that the maximum amount of total borrowings cannot exceed a fraction of the current income:

$$q_t^C b_{t+1}^C + q_t^T b_{t+1}^T \leq \kappa(y_{T,t} + p_t^N \overline{y}_N), \tag{5}$$

with the parameter $\kappa \in (0, 1)$. This constraint is similar to what has been used by many papers to capture important aspects of sudden stop episodes (e.g., Mendoza, 2002; Bianchi, 2011; Korinek, 2018), and like many of them, we do not explicitly derive the credit constraint as the outcome of an optimal contract between lenders and borrowers. Instead, this collateral constraint is the reduced form representation of an environment where informational and institutional frictions affect the credit relationship between domestic and foreign agents.

In economic downturns, the depreciating real exchange rate restricts international borrowing and makes private agents reduce their consumption demands. The lower consumption, in turn, reduces the value of the collateral and brings about a Fisher's debt-deflation mechanism. From the quantitative side, the severe economic recession driven by asset price deflation is a desirable feature of the model and provides a good laboratory to study the welfare effect of financial interventions.

In addition to the collateral constraint, we assume that from emerging countries' perspectives, there cannot be any positive net supply of local currency bonds in the international market. In the

[8] For example, Engel & Park (2022) show that the local government's temptation to deflate the LCD would make the LCD more expensive than the FCD in equilibrium. Ma & Wei (2020) show that poor institutional quality makes international investors suffer more expropriation risk when holding LCD, which also makes it more expensive.

model, this means that households' issuance of LCDs cannot be negative; that is

$$b_{t+1}^C \geq 0. \tag{6}$$

Generally, this restriction is consistent with the fact that most international reserve assets are denominated in the US dollar or euro rather than any emerging market currencies. In contrast to the restriction on local currency borrowing, we do not make any restriction on the borrowing in foreign currency and assume that $b_{t+1}^T \in \mathbb{R}$.

The household's problem is to choose $\{c_{T,t}, c_{N,t}, b_{t+1}^C, b_{t+1}^T\}_{t=0}^{\infty}$ so as to maximize lifetime utility (1) subject to constraints (4)-(6) while taking the initial conditions $\{b_0^C, b_0^T\}$, the exogenous process of $\{y_{T,t}\}_{t=0}^{\infty}$, and the paths of equilibrium prices $\{p_t^N, p_t^C, q_t^C, q_t^T\}_{t=0}^{\infty}$ as given. The solution is characterized by the following optimality conditions:

$$\lambda_t = u_T(t), \tag{7}$$

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{c_{T,t}}{c_{N,t}}\right)^{\frac{1}{\theta}}, \tag{8}$$

$$q_t^T(\lambda_t - \mu_t) = \beta \mathbb{E}_t \lambda_{t+1}, \tag{9}$$

$$q_t^C(\lambda_t - \mu_t) + \eta_t = \beta \mathbb{E}_t \left[\lambda_{t+1} p_{t+1}^C\right], \tag{10}$$

where λ_t and μ_t are the Lagrange multipliers associated with the budget and collateral constraints, respectively. η_t is the Lagrange multiplier on non-negativity constraint (6). The relative price of the final consumption good is defined as follows:

$$p_t^C = \left[\omega^\theta + (1-\omega)^\theta \left(p_t^N\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{11}$$

Equation (7) indicates that the private agents' marginal valuation of wealth equals their marginal utility. Equation (8) links the price of the nontradable good to the consumption ratio of tradable and nontradable sectors. Equations (9) and (10) are the Euler equations for the issuance of FCD and LCD, respectively. In both equations, the left-hand side represents the marginal benefit of borrowing, while the right-hand side represents the expected marginal cost. The presence of a collateral constraint ($\mu_t > 0$) creates a wedge in the two bond Euler equations, implying that the marginal benefit of borrowing declines when the financial constraint binds.

International Investors. There is a continuum of deep-pocketed risk-averse international investors (lenders) who purchase bonds issued by domestic agents. Following Bianchi et al. (2018), we assume that the investors' pricing kernel is given by

$$\mathcal{M}_{t,t+1} = e^{-r-\sigma^m(\epsilon_{t+1}+0.5\sigma^m\sigma_\epsilon^2)}, \tag{12}$$

where r is the international risk-free rate and $\sigma^m > 0$ is a parameter that governs the degree of the investors' risk aversion. A strictly positive σ^m implies a negative correlation between the pricing kernel and the tradable endowment shock: $cov(\mathcal{M}_{t,t+1}, \epsilon_{t+1}) < 0$. The economic meaning is that investors would place more weight on the bond's payoff in recessionary states of the local economy than in expansionary states.

The lenders' zero-profit condition leads to the following bond-pricing equations,

$$q_t^T = \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\right] = e^{-r}, \tag{13}$$

$$q_t^C = \mathbb{E}_t\left[\mathcal{M}_{t,t+1} p_{t+1}^C\right] = \mathbb{E}_t[\mathcal{M}_{t,t+1}]\mathbb{E}_t[p_{t+1}^C] + cov(\mathcal{M}_{t,t+1}, p_{t+1}^C), \tag{14}$$

where the second equality in (13) is due to the log-normality property of the pricing kernel. Let $R_{t+1}^C = p_{t+1}^C / q_t^C$ and $R_t^T = 1/q_t^T$ be the realized returns on LCD and FCD, respectively. Some simple algebra leads to the expression of risk premium on the return of LCD,

$$\rho \equiv \mathbb{E}_t\left[R_{t+1}^C - R_t^T\right] = -\frac{cov(\mathcal{M}_{t,t+1}, R_{t+1}^C)}{\mathbb{E}_t[\mathcal{M}_{t,t+1}]}. \tag{15}$$

The risk premium depends on the covariance between the lenders' pricing kernel and the real exchange rate fluctuations. In our calibration, the lenders' pricing kernel negatively correlates with the economy's tradable endowment shock ($\sigma_m > 0$). It indicates that the price of q_t^C is discounted by a risk premium to compensate lenders' loss in downturns for holding LCD.

2.2 Competitive Equilibrium

The market-clearing conditions in the tradable and nontradable sectors are given by

$$c_{T,t} + p_t^C b_t^C + b_t^T = y_{T,t} + q_t^C b_{t+1}^C + q_t^T b_{t+1}^T, \tag{16}$$

$$c_{N,t} = \overline{y}_N, \tag{17}$$

respectively. Then, we define the decentralized equilibrium as follows.

Definition 1 (**Competitive Equilibrium**). Given the initial conditions on the debt position $\{b_0^C$, $b_0^T\}$ and the sequence of tradable endowments $\{y_{T,t}\}_{t=0}^\infty$, a *competitive equilibrium* is defined as the sequence of allocations $\{c_{T,t}$, $c_{N,t}$, b_{t+1}^C, $b_{t+1}^T\}_{t=0}^\infty$ and prices $\{q_t^T$, q_t^C, p_t^N, $p_t^C\}_{t=0}^\infty$ such that: (i) taking prices as given, the representative household chooses $\{c_{T,t}, c_{N,t}, b_{t+1}^C, b_{t+1}^T\}_{t=0}^\infty$ so as to maximize lifetime utility (1) subject to budget constraint (4), collateral constraint (5), and non-negativity constraint (6); (ii) nontradable good price and real exchange rate $\{p_t^N$, $p_t^C\}_{t=0}^\infty$ are determined by equations (8) and (11); (iii) bond prices $\{q_t^T$, $q_t^C\}_{t=0}^\infty$ are determined by the foreign lenders' problem in equations (13) and (14); and (iv) the market-clearing conditions (16)-(17) hold.

2.3 Optimal Policy Intervention

Similar to the framework used in the sudden stop literature (e.g., Bianchi, 2011; Korinek, 2018; Schmitt-Grohé & Uribe, 2021), models with an endogenous collateral constraint feature a pecuniary externality where atomistic agents fail to consider the effect of their collective borrowing decisions on the nontradable good price and the collateral value. As a result, excessive borrowings in the decentralized market lead to a severe credit contraction when a large negative shock hits and the collateral constraint binds. The externality also creates a role for financial interventions in normal times, such as taxing capital inflows.

We now consider social planning problems: the planners directly choose the issuance of FCD and LCD and let the prices be determined competitively. In the following sections, we consider the problem of a Markov planner who makes discretionary choices, taking future policies as given (denoted as DP), and the problem of a social planner who can commit to her future policy decisions (denoted as CP). The competitive equilibrium is denoted as CE.

2.3.1 Discretionary Planner

We describe the discretionary planner's (DP's) problem recursively. Unlike private agents, the social planner internalizes the effect of her time-t decisions on the prices of p_t^N and p_t^C in budget and collateral constraints. However, because the domestic bond price (q_t^C) is determined by the real exchange rate in the future (p_{t+1}^C for all $y_{T,t+1} \in \mathcal{Y}$), a discretionary planner at time t cannot discipline her future decisions to benefit the current-period bond price.

To simplify notation, we omit the time subscript and use a prime to denote variables in the next period. Let $\mathcal{S} = (b^C, b^T, s)$ denote the aggregate state of the economy, where $s = \{y_T\}$ is the exogenous state, and $f(s'|s)$ is the transition density. The DP's recursive problem is given by:

$$V^{DP}(b^C, b^T, s) = \max_{\{b^{C'},\ b^{T'},\ c_T\}} \left\{ c^{1-\sigma}/(1-\sigma) + \beta \mathbb{E}_{s'|s} V^{DP}(b^{C'}, b^{T'}, s') \right\}, \tag{18}$$

$$\text{s.t.} \quad c_T + p^C b^C + b^T = y_T + q^C(b^{C'}, b^{T'}, s) b^{C'} + q^T(s) b^{T'}, \tag{19}$$

$$q^C(b^{C'}, b^{T'}, s) b^{C'} + q^T(s) b^{T'} \leq \kappa(y_T + p^N \overline{y}_N), \tag{20}$$

$$b^{C'} \geq 0, \tag{21}$$

$$q^T(s) = \int_{\mathcal{S}} \mathcal{M}(s, s') f(s'|s) ds', \tag{22}$$

$$q^C(b^{C'}, b^{T'}, s) = \int_{\mathcal{S}} \mathcal{M}(s, s') p^C(b^{C'}, b^{T'}, s') f(s'|s) ds', \tag{23}$$

$$p^N = \frac{1-\omega}{\omega} \left(\frac{c_T}{\overline{y}_N} \right)^{\frac{1}{\theta}}, \tag{24}$$

$$p^C = \left[\omega^\theta + (1-\omega)^\theta \left(p^N\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{25}$$

where $c=\left[\omega c_T^{\frac{\theta-1}{\theta}}+(1-\omega)\overline{y}_N^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$. Then, we can define the recursive equilibrium as follows.

Definition 2 (Recursive Equilibrium for the Discretionary Planner [DP]). Given the transition matrix of the exogenous state $f(s'|s)$, the *recursive equilibrium for a discretionary planner* is defined as a set of decision rules $\{c_T(\mathcal{S}), b^{C'}(\mathcal{S}), b^{T'}(\mathcal{S})\}_{t=0}^{\infty}$, price functions $\{q^T(s), q^C(b^{C'}, b^{T'}, s)$, $p^N(\mathcal{S})$, $p^C(\mathcal{S})\}_{t=0}^{\infty}$, and a value function $\{V^{DP}(\mathcal{S})\}$ that solve the problem in equations (18)-(25).

The social planner's problem differs from the CE in three ways. First, since p^N is increasing in c_T, any negative shock that reduces collateral value and tightens the collateral constraint will cause a collapse in tradable consumption and drive a Fisherian debt-deflation spiral. So, the social planner is incentivized to restrict borrowing in states outside the financial crisis to mitigate the future unfavorable effect on the collateral constraint. Second, the social planner also understands that p^C is increasing in c_T. So, she has incentives to lower the consumption to lessen the LCD burdens.

Finally, a time-consistency problem arises because the domestic bond price (q^C) depends on the next-period real exchange rate ($p^{C'}$). The current planner prefers that the next-period planner chooses a higher level of consumption because that will lead to a more favorable bond price and a smaller cost of issuing local currency debts. However, when the next period arrives, the social planner disregards this prior benefit and has strong incentives to deflate the debt by lowering the current consumption. The debt-deflating motive results in unfavorable bond prices and the inefficient use of LCD in equilibrium.

These incentives are illustrated by the following optimality conditions:

$$\lambda_t^{DP}\left(1+\underbrace{b_t^C\frac{\partial p_t^C}{\partial c_{T,t}}}_{\substack{\text{Additional cost from}\\ \text{higher LCD repayment}}}\right)=\underbrace{u_T^{DP}(t)}_{\text{Direct utility gain}}+\underbrace{\mu_t^{DP}\kappa\overline{y}_N\frac{\partial p_t^N}{\partial c_{T,t}}}_{\substack{\text{Indirect gain from relaxing}\\ \text{the collateral constraint}}}, \tag{26}$$

$$\left(\lambda_t^{DP}-\mu_t^{DP}\right)\left[q_t^T(b_{t+1}^C,b_{t+1}^T,s_t)+\frac{\partial q_t^C(b_{t+1}^C,b_{t+1}^T,s_t)}{\partial b_{t+1}^T}b_{t+1}^C\right]=\beta\mathbb{E}_t\lambda_{t+1}^{DP}, \tag{27}$$

$$\left(\lambda_t^{DP}-\mu_t^{DP}\right)\left[q_t^C(b_{t+1}^C,b_{t+1}^T,s_t)+\frac{\partial q_t^C(b_{t+1}^C,b_{t+1}^T,s_t)}{\partial b_{t+1}^C}b_{t+1}^C\right]+\eta_t^{DP}=\beta\mathbb{E}_t\lambda_{t+1}^{DP}p_{t+1}^C, \tag{28}$$

where λ_t^{DP}, μ_t^{DP}, and η_t^{DP} are the Lagrange multipliers to the budget, collateral, and non-negativity constraints in the DP's problem, respectively. Equation (26) describes the marginal valuation of wealth from the social planner's perspective (λ_t^{DP}). Compared with the CE, two additional terms show up. First, the social marginal wealth includes an indirect utility gain of increasing consumption that is not present in the private marginal wealth. The term is positive if the collateral constraint binds: $\mu_t^{DP}>0$. This expression indicates that private agents undervalue the benefit of raising consumption in relaxing the collateral constraint during a financial crisis. Second, the social planner realizes that raising consumption has an additional cost on the debt burden when the existing share of LCD is positive ($b_t^C > 0$). This incentive induces her to lower consumption to reduce debt repayment if LCD exists.

Equations (27)-(28) are the Euler equations with respect to bond issuance. Compared to the standard Euler equations (9)-(10) under the CE, the planner now realizes that the domestic bond price is elastic to her debt issuance and portfolio decisions. In particular, the planner, while not being able to commit to future consumption paths, recognizes that today's denomination choices will affect tomorrow's consumption profile and thus impact the bond price schedule in the current period.[9]

2.3.2 Commitment Planner

In our model, the presence of LCD offers the hedging benefit to the small open economies, and at the same time, creates a time-inconsistency issue. Since the price of LCD is defined recursively, the social planner would have incentives to manipulate the consumption profile in the next period to obtain a favorable bond price today. As we will show below, this incentive leads to better ex ante portfolio decisions.

Next, we describe the problem of a social planner who can commit to future consumption paths while taking the CE's equilibrium conditions as given. In the quantitative exercise in section 4, we show that the discretionary and commitment problems necessitate different policy toolkits from financial regulators. Specifically, the commitment planner's problem is described as follows:

$$\max_{\{b_{t+1}^C,\ b_{t+1}^T,\ c_{T,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma}/(1-\sigma) \tag{29}$$

$$\text{s.t.} \quad c_{T,t} + p_t^C b_t^C + b_t^T = y_{T,t} + q_t^C b_{t+1}^C + q_t^T b_{t+1}^T, \tag{30}$$

$$q_t^C b_{t+1}^C + q_t^T b_{t+1}^T \leq \kappa \left(y_{T,t} + p_t^N \overline{y}_N \right), \tag{31}$$

$$b_{t+1}^C \geq 0, \tag{32}$$

$$q_t^T = \int_{\mathcal{S}} \mathcal{M}(s_t, s_{t+1}) f(s'|s) ds', \tag{33}$$

$$q_t^C = \int_{\mathcal{S}} \mathcal{M}(s_t, s_{t+1}) p_{t+1}^C f(s'|s) ds', \tag{34}$$

$$p_t^N = \frac{1-\omega}{\omega} \left(\frac{c_{T,t}}{\overline{y}_N} \right)^{\frac{1}{\theta}}, \tag{35}$$

$$p_t^C = \left[\omega^\theta + (1-\omega)^\theta \left(p_t^N \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{36}$$

[9] In the quantitative analysis, our numerical results verify that the bond price tends to improve when agents denominate a larger fraction of debts in local currency.

We now define the commitment planner's equilibrium as the following.[10]

Definition 3 (Equilibrium for the Commitment Planner [CP]). Given the initial debt $\{b_0^C, b_0^T\}$ and the sequence of tradable endowments $\{y_{T,t}\}_{t=0}^{\infty}$, the *equilibrium for a commitment planner* is defined as the sequence of allocations $\{c_{T,t},\ b_{t+1}^C,\ b_{t+1}^T\ \}_{t=0}^{\infty}$ and prices $\{q_t^T,\ q_t^C,\ p_t^N,\ p_t^C\}_{t=0}^{\infty}$ that maximize the representative agent's lifetime utility (29) subject to budget constraint (30), collateral constraint (31), non-negativity constraint (32), bond-pricing equations (33)-(34), and implementability conditions (35)-(36).

We denote λ_t^{CP}, μ_t^{CP}, and η_t^{CP} as the Lagrange multipliers to the budget, collateral, and non-negativity constraints, respectively. The solution to the CP's problem is characterized by the following optimality conditions,

$$\lambda_t^{CP}\left(1+\underbrace{\frac{\partial p_t^C}{\partial c_{T,t}}b_t^C}_{\substack{\text{Additional cost from}\\ \text{higher LCD repayment}}}\right)=\underbrace{u_T(t)}_{\text{Direct utility gain}}+\underbrace{\mu_t^{CP}\kappa\overline{y}_N\frac{\partial p_t^N}{\partial c_{T,t}}}_{\substack{\text{Indirect gain from}\\ \text{relaxing the collateral constraint}}}$$

$$+\underbrace{(\lambda_{t-1}^{CP}-\mu_{t-1}^{CP})b_t^C\mathcal{M}(s_{t-1},s_t)\frac{\partial p_t^C}{\partial c_{T,t}}\frac{1}{\beta}}_{\substack{\text{Indirect gain from committing a higher consumption}\\ \text{to improve the previous-period bond price }(h_t)}}, \tag{37}$$

$$\left(\lambda_t^{CP}-\mu_t^{CP}\right)q_t^T=\beta\mathbb{E}_t\lambda_{t+1}^{CP}, \tag{38}$$

$$\left(\lambda_t^{CP}-\mu_t^{CP}\right)q_t^C+\eta_t^{CP}=\beta\mathbb{E}_t\lambda_{t+1}^{CP}p_{t+1}^C. \tag{39}$$

Equation (37) defines the marginal valuation of wealth (λ_t^{CP}) for the commitment planner. Like the discretionary planner, the commitment planner also considers the indirect utility gain of raising consumption in relaxing the collateral constraint. In addition, she internalizes the positive effect of consumption on debt burdens when the LCD exists.

The last term in equation (37) describes how the increased consumption affects the previous-period bond price. This term arises due to the planner's commitment power. Promising a higher tradable consumption in period t can appreciate the current exchange rate (p_t^C) and increase the price of LCD in period $t-1$ (q_{t-1}^C). The higher bond price, in turn, allows agents to issue LCD more easily, which offers additional hedging benefits when an adverse shock hits the economy in period t.

The presence of μ_{t-1}^{DP} and λ_{t-1}^{DP} in the period-t optimality condition confirms that the commitment planner's problem is indeed time inconsistent. During time $t-1$, the planner has the incentive to pledge a higher consumption in period t to inflate its future exchange rate, thus improving the

[10] Appendix C describes our solution algorithm. We adopt the method discussed in Marcet & Marimon (2019) and reformulate the commitment planner's problem recursively after introducing an auxiliary state variable. The auxiliary state summarizes the history of commitment made by the social planner in previous periods. Kehoe & Perri (2002) applies a similar method to an open economy environment.

ex ante bond price charged by international lenders. However, once period t arrives, implementing this pledge becomes suboptimal because higher consumption leads to a greater debt payment if the share of LCD is positive. Our model simulation (in section 4.3) shows that this time-inconsistency issue is quantitatively important, so the commitment planner always values wealth more than the discretionary social planner.

In the commitment planner's problem, the bond Euler equations (38)-(39) take on the same form as equations (9)-(10) under the CE. This, however, does not imply that the commitment planner should make the same portfolio decision as the private agents. In both equations, the benefit and cost of issuing bonds are evaluated by the social planner's marginal wealth (λ^{CP}) but not the private agents' (λ^{CE}).

2.3.3 Decentralization

In this section, we consider how a pair of state-contingent debt taxes can implement the social planners' allocations. Let τ_t^T and τ_t^C be the capital control tax rates levied on the issuances of foreign and local currency debts, respectively. We assume that the tax revenue from financial regulation is rebated back to households in a lump-sum fashion. In a tax-regulated economy, the optimality conditions with respect to the bond issuance are

$$(u_T(t) - \mu_t)q_t^T = \beta(1 + \tau_t^T)\mathbb{E}_t u_T(t+1), \tag{40}$$

$$(u_T(t) - \mu_t)q_t^C + \eta_t = \beta(1 + \tau_t^C)\mathbb{E}_t \left[u_T(t+1)p_{t+1}^C\right]. \tag{41}$$

The following proposition characterizes the optimal capital control taxes that restore the social planners' allocations.

Proposition 1 (Decentralization).

The allocations achieved by the discretionary planner and commitment planner can be implemented with two distinct tax schedules on FCD and LCD, with tax revenues rebated back to the households as a lump-sum transfer.

Proof. See appendix B.1. □

The expressions of capital control taxes are displayed in the proof of proposition 1 in appendix B.1. We focus on capital control policies in normal times when the collateral constraint does not bind: $\mu_t = 0$. There are two features we notice from the expressions of tax rates. First, in the discretionary planner's problem, the expressions of $\tau^{C,DP}$ and $\tau^{T,DP}$ (in equations B.2-B.3) indicate that the planner understands the effect of her portfolio decisions on the next-period consumption and how the expected consumption affects the current bond price. Second, the expressions of $\tau^{C,CP}$ and $\tau^{T,CP}$ (in equations B.4-B.5) contain a term that represents the commitment made in the previous periods. Such a commitment state increases the social planner's marginal benefit of borrowing and improves the domestic bond prices in equilibrium.

3 A Simple Model Illustration

In this section, we use a simplified model to illustrate the externalities that arise from the introduction of LCD. We assume there is fixed financing need $\bar{I}$ in period 1. The agents' only problem in the first period is to choose the debt denomination of external borrowings. A fraction δ_2 of debt is denominated in local currency while the remaining $1-\delta_2$ is denominated in foreign currency.[11] Beginning in the second period, the agent only borrows in FCD. Moreover, we assume uncertainty only happens in the second period with the tradable endowment taking a binary distribution: $\mathbb{P}(y_{T,2} = y^H_{T,2}) = 1-p$ and $\mathbb{P}(y_{T,2} = y^L_{T,2}) = p$. The collateral constraint only binds in the low-income state. There is no financial constraint starting from the third period, and endowments are constant: $y_{T,t} = \bar{y}_T$ for $t \geq 3$. We also assume the consumption aggregator takes on a Cobb-Douglas form; that is $\theta = 1$.

To derive an analytical result, we assume that agents pay the accrued interests of the first-period-issued bonds in the second period. The ex ante interest rates on foreign and local currency bonds are denoted by R_2^* and R_2, respectively. In period 2, the ex post return on FCD is non-contingent, while the return on LCD, $R_2 p_2^N$, is contingent on the income realizations.

From the second period onward, all borrowings are denominated in foreign currencies: $\{b_t\}_{t=3}^{\infty}$. Agents choose the optimal level of borrowing to maximize the lifetime utility: $\mathbb{E}\sum_{t=2}^{\infty}\beta^{t-2}u(c_{T,t}, c_{N,t})$, where $u(c_{T,t}, c_{N,t}) = \left(c_{T,t}^{\omega} c_{N,t}^{1-\omega}\right)^{1-\sigma}/(1-\sigma)$. The budget and collateral constraints beginning in the second period are given by

$$(\lambda_2) \quad c_{T,2} + p_2^N c_{N,2} + (1-\delta_2)\bar{I}R_2^* + \delta_2 \bar{I} R_2 p_2^N = \frac{1}{R^*} b_3 + y_{T,2} + p_2^N \bar{y}_N, \tag{42}$$

$$(\mu_2) \quad \frac{1}{R^*} b_3 \leq \kappa(y_{T,2} + p_2^N \bar{y}_N), \tag{43}$$

$$(\lambda_t) \quad c_{T,t} + p_t^N c_{N,t} + b_t = \frac{1}{R^*} b_{t+1} + \bar{y}_T + p_t^N \bar{y}_N, \quad \forall\ t \geq 3, \tag{44}$$

where λ_t and μ_2 are the corresponding Lagrange multipliers. We assume $\beta R^* = 1$ such that the economy can perfectly smooth consumption after the second period: $c_{T,t} = c_{T,3}$ for any $t \geq 3$.

Asset-pricing equations imply that

$$R_2^* = \frac{1}{\mathbb{E}\mathcal{M}_2}, \qquad R_2 = \frac{1}{\mathbb{E}\left[\mathcal{M}_2 p_2^N\right]}, \tag{45}$$

where $\mathcal{M}_2$ is the foreign lenders' pricing kernel. We assume the pricing kernel takes the values of

[11] For simplicity, in this section, we assume the payoff of LCD depends on the nontradable price (p^N) rather than the aggregate consumption price (p^C) as in the full model.

$\mathcal{M}_2^H$ and $\mathcal{M}_2^L$ in the high- and low-income states, respectively, and has a relationship $\mathcal{M}_2^H < \mathcal{M}_2^L$. The expected return on LCD is $\mathbb{E}[R_2 p_2^N]$. Then the risk premium on LCD can be expressed as $\rho = \frac{\mathbb{E}[R_2 p_2^N] - R_2^*}{\mathbb{E}[R_2 p_2^N]} = -R_2^* cov(\mathcal{M}_2, \frac{p_2^N}{\mathbb{E}p_2^N})$. The risk premium is positive when the pricing kernel negatively correlates with the nontradable good price.[12]

Hedging Benefit of LCD. We begin by solving the competitive equilibrium recursively from the second period. Conditional on a certain local currency share (δ_2), the solution to the second-period problem is characterized by a triplet $\{c_{T,2}^H, c_{T,2}^L, R_2\}$ that solves the following three equations:

$$c_{T,2}^H = \mathcal{C}^H(R_2, \delta_2) \equiv \frac{\frac{1}{R^*}\bar{y}_T + (1 - \frac{1}{R^*})y_{T,2}^H - (1 - \frac{1}{R^*})(1 - \delta_2)\bar{I}R_2^*}{1 + (1 - \frac{1}{R^*})\delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}, \tag{46}$$

$$c_{T,2}^L = \mathcal{C}^L(R_2, \delta_2) \equiv \frac{(1+\kappa)y_{T,2}^L - (1 - \delta_2)\bar{I}R_2^*}{1 - (\kappa \bar{y}_N - \delta_2 \bar{I} R_2)\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}, \tag{47}$$

$$R_2 = \mathcal{R}(c_{T,2}^H, c_{T,2}^L) \equiv \frac{1}{(1-p)\mathcal{M}_2^H \frac{1-\omega}{\omega}\frac{c_{T,2}^H}{\bar{y}_N} + p\mathcal{M}_2^L \frac{1-\omega}{\omega}\frac{c_{T,2}^L}{\bar{y}_N}}. \tag{48}$$

The first two equations characterize the consumption profiles in the high- and low-income states, while the third equation determines the domestic bond interest rate. From the first two equations, we notice that the consumption is less sensitive to tradable endowment shock if an economy has a larger fraction of LCD, and this is especially true at the low-income state ($\frac{\partial^2 \mathcal{C}^L}{\partial y_T^L \partial \delta_2} < 0$). Because the payoff of LCD depends on the nontradable price, it is a better hedging device against the tradable endowment shock compared to FCD.[13] We can also see that a higher interest rate on LCD increases the overall debt burden and reduces second-period consumption in both the high- and low-income states ($\frac{\partial \mathcal{C}^H(R_2,\delta_2)}{\partial R_2} < 0$, $\frac{\partial \mathcal{C}^L(R_2,\delta_2)}{\partial R_2} < 0$).

In the first period, the optimal portfolio δ_2 is pinned down by the following Euler equation:

$$\frac{R_2 \mathbb{E}p_2^N - R_2^*}{R_2 \mathbb{E}p_2^N} = \mathbb{E}\left[\frac{\lambda_2^{CE}}{\mathbb{E}\lambda_2^{CE}}\left(1 - \frac{p_2^N}{\mathbb{E}p_2^N}\right)\right]. \tag{49}$$

The left-hand side represents the *insurance cost of using LCD* (relative to FCD) that is equal to the risk premium: $\rho = -R^* cov(\mathcal{M}_2, \frac{p_2^N}{\mathbb{E}p_2^N})$. The right-hand side represents the *hedging benefit of using LCD*, which depends on how people's marginal value of wealth (λ_t^{CE}) covariates with LCD's relative payoff.

[12] Appendix D provides the detailed description of the simplified model and the associated optimality conditions.

[13] For relevant parameter values, we find that $\frac{\partial \mathcal{C}^H(R_2,\delta_2)}{\partial \delta_2} < 0$ and $\frac{\partial \mathcal{C}^L(R_2,\delta_2)}{\partial \delta_2} > 0$. This means that increasing the share of LCD in the first period produces a smaller consumption dispersion in the second period.

3.1 Inefficiency in the LCD Issuance

Discretionary Planner. We start by analyzing the discretionary planner's choice in the second period. In the low-income state, consumption is uniquely pinned down by the collateral constraint. As a result, the DP 's consumption is the same as in equation (47); that is $c_{T,2}^{L,DP}(R_2, \delta_2) = \mathcal{C}^L(R_2, \delta_2)$. In the high-income state, the DP has incentives to choose a different consumption function because she realizes that the lower exchange rate will reduce the debt burden if the share of LCD is positive. The DP's high-state consumption function is given by

$$c_{T,2}^{H,DP} = \mathcal{C}^{H,DP}(R_2, \delta_2) \equiv \frac{\bar{y}_T + (R^* - 1)y_{T,2}^H - (R^* - 1)(1 - \delta_2)\bar{I}R_2^*}{\varphi(R_2, \delta_2) + R^* - 1 + (1 - \frac{1}{R^*})\delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}, \tag{50}$$

where $\varphi(R_2, \delta_2) = [1 + \phi(R_2, \delta_2)]^{\frac{1}{1-(-\sigma+1)\omega}}$ and $\phi(R_2, \delta_2) = \delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}$. $\varphi(R_2, \delta_2) > 1$ indicates that the DP has an incentive to deflate the LCD burden in states outside the financial crises.

For each value of δ_2, the DP's second-period problem is charactered by a triplet $\{c_{T,2}^{H,DP}$, $c_{T,2}^{L,DP}$, $R_2^{DP}\}$ that jointly solves the equations of (47), (48), and (50). The following proposition compares the second-period solutions of the CE and DP.

Proposition 2 (Ex post Debt-Reduction Incentive of the DP).

For each value of $\delta_2 > 0$, the DP's incentive to deflate the LCD in the high-income state results in lower consumption in both states and a higher domestic interest rate than that of the competitive equilibrium. Specifically, we have

$$c_{T,2}^{H,DP}(\delta_2) < c_{T,2}^H(\delta_2), \quad c_{T,2}^{L,DP}(\delta_2) < c_{T,2}^L(\delta_2), \quad R_2^{DP}(\delta_2) > R_2(\delta_2), \quad \textit{for any} \ \ \delta_2 > 0.$$

Proof. See appendix B.2. □

Proposition 2 shows that the social planner's debt-reduction incentive in the high-income state also translates into a lower consumption in the low-income state because the endogenous interest rate is determined by the weighted average of consumption, as in equation (48).

What does it imply for the planner's first-period portfolio decision? The portfolio Euler equation in the first period is given by

$$\frac{R_2^{DP}\mathbb{E}p_2^{N,DP} - R_2^*}{R_2^{DP}\mathbb{E}p_2^{N,DP}} = \mathbb{E}\left[\frac{\lambda_2^{DP}}{\mathbb{E}\lambda_2^{DP}}\left(1 - \frac{p_2^{N,DP}}{\mathbb{E}p_2^{N,DP}}\right)\right] - \frac{\delta_2}{R_2^{DP}}\frac{\partial R_2^{DP}(\delta_2)}{\partial \delta_2}\mathbb{E}\left[\frac{\lambda_2^{DP}}{\mathbb{E}\lambda_2^{DP}}\frac{p_2^{N,DP}}{\mathbb{E}p_2^{N,DP}}\right], \tag{51}$$

where $\{R_2^{DP}$, $p_2^{N,DP}$, $\lambda_2^{DP}\}$ are the DP's equilibrium values in the second period at a certain value of δ_2.[14] Like before, the left-hand side of equation (51) represents the *insurance cost* of using LCD from the social planner's perspective, while the right-hand side is the *hedging benefit*. Unlike the

[14]The DP's marginal valuation of wealth in the second period is given by: $\lambda_2^{DP} = \frac{u_T(c_{T,2}) + \mu_2^{DP}\kappa\frac{1-\omega}{\omega}}{1+\phi(R_2,\delta_2)}$.

CE's condition in equation (49), the hedging benefit is evaluated by the social planner's marginal value of wealth λ_2^{DP}, which incorporates pecuniary externalities. Meanwhile, the second term on the right-hand side, arising from the lack-of-commitment issue, represents how the endogenous interest rate varies with the local currency share.

Commitment Planner. In the discretionary planner's problem, the debt-reduction incentive prohibits agents from using LCD to hedge downside risk during financial crises. We address this issue by assuming a social planner who commits to future policies. In particular, the planner makes the portfolio decision (δ_2) in period 1, and simultaneously commits to a specific consumption profile in the second period ($c_{T,2}^H$, $c_{T,2}^L$). When the planner enters period 2, she chooses the exact level of borrowing (b_3^H, b_3^L) to implement the pre-committed consumption profile that satisfies the budget and collateral constraints.[15]

The full set of equilibrium conditions are given by equations (B.17)-(B.22) in appendix B. Since the low-state consumption (c_2^L) is pinned down by the collateral constraint, the planner is only flexible in choosing consumption in the high state (c_2^H). The following proposition shows that the planner would like to commit to a higher level of consumption in the high-income state (relative to the DP) to lower the ex ante interest rate, which allows herself to borrow LCD more easily in the first period. This larger fraction of LCD improves welfare in the crisis state.

Proposition 3 (Ex Ante Hedging Incentive of the CP).

Suppose the CP's optimal portfolio is δ_2^{CP*} and there exists a θ^* such that $\theta^* = \frac{\lambda_2^{H,CP}}{\mathcal{M}_2^H} = \frac{\lambda_2^{L,CP}}{\mathcal{M}_2^L}$. Then, the CP's decision satisfies

$$
\begin{aligned}
&c_{T,2}^{H,CP} = c_{T,2}^H(\delta_2^{CP*}), \quad c_{T,2}^{L,CP} = c_{T,2}^L(\delta_2^{CP*}), \quad R_2^{CP} = R_2(\delta_2^{CP*}), \\
&\lambda_2^{H,CP} = \lambda_2^H(\delta_2^{CP*}), \quad \lambda_2^{L,CP} > \lambda_2^L(\delta_2^{CP*}), \quad \mu_2^{L,CP} > \mu_2^L(\delta_2^{CP*}).
\end{aligned}
$$

Furthermore, we assume: (i) the CE's optimal portfolio is denoted as δ_2^{CE*}; and (ii) in the relevant range of δ_2 around δ_2^{CE*}, the CE's policy functions have the property that $\frac{\mathbb{E}[\lambda_2^{CE}(\delta_2)p_2^N(\delta_2)]}{\mathbb{E}\lambda_2^{CE}(\delta_2)} - \frac{R^*}{R_2(\delta_2)}$ is monotonically increasing in δ_2. Then, we have $\delta_2^{CP*} > \delta_2^{CE*}$.[16]

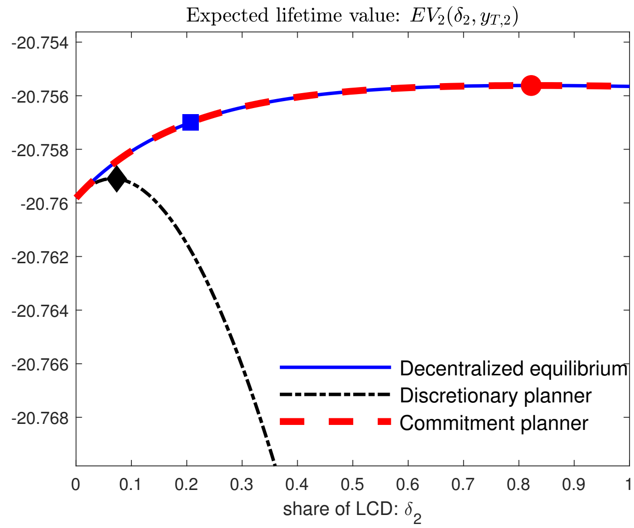
Proof. See appendix B.3. □

The first part of proposition 3 says that the commitment planner would prefer to maintain the same consumption allocations as the ones chosen by the decentralized agents at their optimal portfolio share of δ_2^{CP*}. In addition, since the social marginal value of wealth is larger than the

[15]In our simplified model, there is no scope for making commitments beginning in the third period. Therefore, we assume that the social planner in the first period only needs to commit to the second-period allocations. The commitment planner jointly solves the values of $\{c_{T,2}^{L,CP}, c_{T,2}^{H,CP}, \lambda_2^{L,CP}, \lambda_2^{H,CP}, \mu_2^{L,CP}, R_2^{CP}, \delta_2^{CP}\}$ to maximize her lifetime utility.

[16]It is usually inappropriate to make assumptions about endogenous objects. However, these assumptions here allow us to characterize portfolio decisions, and we confirm they hold in our numerical example.

Figure 2: Lifetime Utilities and the Optimal Portfolio Decisions in the Simple Model



NOTE: This figure shows the expected lifetime utility functions in the three equilibria. The expected lifetime utility is defined as $\mathbb{E}V_2(\delta_2, y_{T,2}) = (1-p)V_2(\delta_2, y^H_{T,2}) + pV_2(\delta_2, y^L_{T,2})$. The squared/diamond-shaped/round point refers to the optimal portfolio decision in the problem of DE/DP/CP, respectively. We provide details for this illustration in appendix D.

private agents' in the low-income state, the planner has a stronger incentive to mitigate the financial risk by issuing more LCDs, as shown by the second half of the proposition.[17]

The Euler equation that determines her portfolio decision is given by

$$\frac{R_2^{CP}\mathbb{E}p_2^{N,CP} - R_2^*}{R_2^{CP}\mathbb{E}p_2^{N,CP}} = \mathbb{E}\left[\frac{\lambda_2^{CP}}{\mathbb{E}\lambda_2^{CP}}\left(1 - \frac{p_2^{N,CP}}{\mathbb{E}p_2^{N,CP}}\right)\right]. \tag{52}$$

Notice that this equation takes exactly the same form as the one under the competitive equilibrium (equation 49) where the social planner chooses a portfolio equalizing the *insurance cost* of using LCD to its *hedging benefit*. The only difference is that the CP's hedging benefit is evaluated by the social marginal value of wealth (λ_2^{CP}) instead of the private one (λ_2).

The welfare implication of the two social planners also differs. Figure 2 provides a numerical illustration. Compared to the CE, the DP's strong debt-reduction incentive raises up the domestic interest rate and shrinks the resource frontier of the whole economy. In the first period, this incentive makes borrowing in LCD very costly. In the end, the lack of commitment leads to a welfare loss. On the other hand, the CP is cognizant that promising a higher consumption in the good state will benefit the domestic interest rate, and she also internalizes the benefit of using LCD in mitigating the exchange rate drop during a financial crisis. As a result, she increases the local

[17] In appendix D, we use a numerical example to compare the second-period consumption schedules under the CE and CP. Also, we show their first-period portfolio choices in the two environments.

currency share to fully utilize its hedging benefit and obtains the maximum welfare among the three equilibria. Proposition 4 in appendix D provides the expressions of capital control taxes that restore the social planner's allocations. We find that the tax rate that is used to adjust the first-period portfolio is a composite measure that depends on the crisis probability, the crisis severity, and a term representing the relative benefit of using LCD.

4 Quantitative Analysis

This section calibrates the full model and analyzes its quantitative implications. In appendix E, we show that our results are robust to alternative parameter values.

4.1 Calibration and Solution Method

We use the global solution method with time iteration. Specifically, the solution method involves iterating a set of decision rules based on the first-order conditions until convergence is achieved.[18] For the competitive equilibrium and the discretionary planner's problem, the vector of state variables is $\mathcal{S} = (b^C, b^T, s)$. For the commitment planner, we introduce an additional state variable h to capture the history of commitment made in the prior periods (the last term in equation 37). The CP's solution is defined on the extended state vector $\tilde{\mathcal{S}} = (b^C, b^T, h, s)$. After including the additional state variable, we formulate the commitment planner's problem in a recursive form that can be solved using the standard Euler equation iteration method. More details on the solution method are provided in appendix C.

We calibrate the model under the competitive equilibrium using Mexican annual data because Mexico is a standard sudden stop economy frequently used in the literature. Part of the parameters is borrowed from the literature or found by simply matching moments. The risk-free world interest rate is $r = 0.04$. The relative risk aversion σ is set to a standard value of 3.[19] The weight on tradable goods in the aggregate consumption basket ($\omega = 0.39$) is used to target the sectoral consumption ratio $c_N/c_T = 1.643$, as computed by Benigno et al. (2013). The elasticity of substitution (θ) is an important parameter because it governs the real exchange rate fluctuations, which implicitly determines the relative benefits of using LCD. In the baseline calibration, we use a conservative value of 0.83 following Bianchi (2011). The parameters in the income process are estimated by running a regression using the tradable output data between 1970 and 2021. As in Bianchi (2011), we consider the value added of the manufacturing and agriculture industries as tradable output, while the rest of the industrial production is considered nontradable output.

[18] By solving the models using first-order conditions, we implicitly assume that they are both necessary and sufficient for the models' solutions.

[19] We also solve a model with $\sigma = 2$ in the sensitivity analysis in appendix E. All the quantitative results in the baseline calibration remain.

Table 1: Parameter Values

Description	Parameter	Value	Target/Source
From literature or simple moment match:			
Tradable good weight	ω	0.39	Benigno et al. (2013)
Elast. of substitution	θ	0.83	Bianchi (2011)
Risk aversion	σ	3	DSGE literature
International interest rate	r	0.04	Bianchi (2011)
Income process: autocorrelation	ρ	0.81	Mexican tradable income data
Income process: std. dev.	σ_ϵ	0.064	Mexican tradable income data
Income process: mean	μ	$-\frac{1}{2}\sigma_\epsilon^2$	Mean tradable income =1
Calibrated to fit targets:			
Subjective discount factor	β	0.86	Prob. of crisis $\approx$ 5.5%
Maximum leverage ratio	κ	0.334	Mean NFA-to-GDP = -33.3%
Lenders' risk aversion	σ^m	3.83	Mean LCD share = 13.7%

NOTE: Mexico's tradable income process comes from the World Development Indicators and covers from 1970 to 2021. The net foreign asset (NFA) is constructed by Lane & Milesi-Ferretti (2018), and the data on the currency denomination of external debt liabilities comes from Bénétrix et al. (2019). To estimate the tradable income process, we linearly detrend the log tradable sector outputs measured in constant U.S. dollars. We include agriculture, manufacturing industries, and natural resources as the tradable outputs.

The remaining three parameters (β, κ, σ^m) are jointly calibrated to target data moments: the probability of a crisis, the average net foreign asset (NFA)-to-GDP ratio, and the average share of external debts denominated in local currency. In the model, a sudden stop crisis is defined as the period when (i) the collateral constraint binds; and (ii) the current account exceeds two standard deviations above its mean.[20] In the model simulation, the crisis happens with a probability of 5.5%, the same as the data counterpart found in Bianchi (2011). The dataset used by Lane & Milesi-Ferretti (2018) suggests that Mexico has an average NFA-to-GDP ratio of -33.3% between 1970 and 2015, while our model predicts an average debt burden-to-GDP ratio of 33.2%. To target the share of LCD, we use the cross-border currency exposure data of Bénétrix et al. (2019). For Mexico, the average share of external debt denominated in local currency between 1990 and 2017 is 13.7%, and that the local currency share has a standard deviation of 7.3%. In the model simulation, the average and standard deviations of local currency share are respectively 13.6% and 10.7%.

4.2 CE Policy Functions

We begin by considering policy functions in the decentralized equilibrium. We define the total debt level as $b_t \equiv b_t^C + b_t^T$ and the share of debt in local currency as $\delta_t \equiv b_t^C/(b_t^C + b_t^T)$. Figure 3 plots decision rules for a variation of the debt levels (b_t), while keeping the states of δ_t and $y_{T,t}$ at their

[20]We confirmed that our quantitative evaluation of optimal policies does not depend on alternative definitions of the sudden stop used in the existing studies.

relative high and low levels. A higher b_t with constant values of δ_t and $y_{T,t}$ means a higher debt burden $(b_t \delta_t p_t^C + b_t(1-\delta_t))$ in the relevant ranges of the state variables. First, we notice that all the decision rules feature strong nonlinearities. For example, in the low-debt state where the collateral constraint does not bind ($\mu_t = 0$), tradable consumption decreases and the debt issuance increases in the debt balance. As b_t gradually approaches the crisis threshold, the probability of hitting a financial constraint in the next period becomes relevant. In response, the private agent starts to issue positive shares of LCD to provide insurance for the upcoming financial crises. This occurs until the collateral constraint becomes binding ($\mu_t > 0$).[21]

In addition, panel E shows that when the financial constraint is nonbinding, the price of LCD decreases in the debt balance. This is because q_t^C is determined by the next-period real exchange rate (p_{t+1}^C), and the greater amount of borrowing lowers the exchange rate in expectation. Panel D shows that the risk premium (for holding LCD) increases when the debt goes up. The reason is that as the higher debt balance drives up the chance of a financial crisis in the next period, the expectation of currency depreciation during a crisis necessitates a larger risk premium for lenders' holding of LCD.

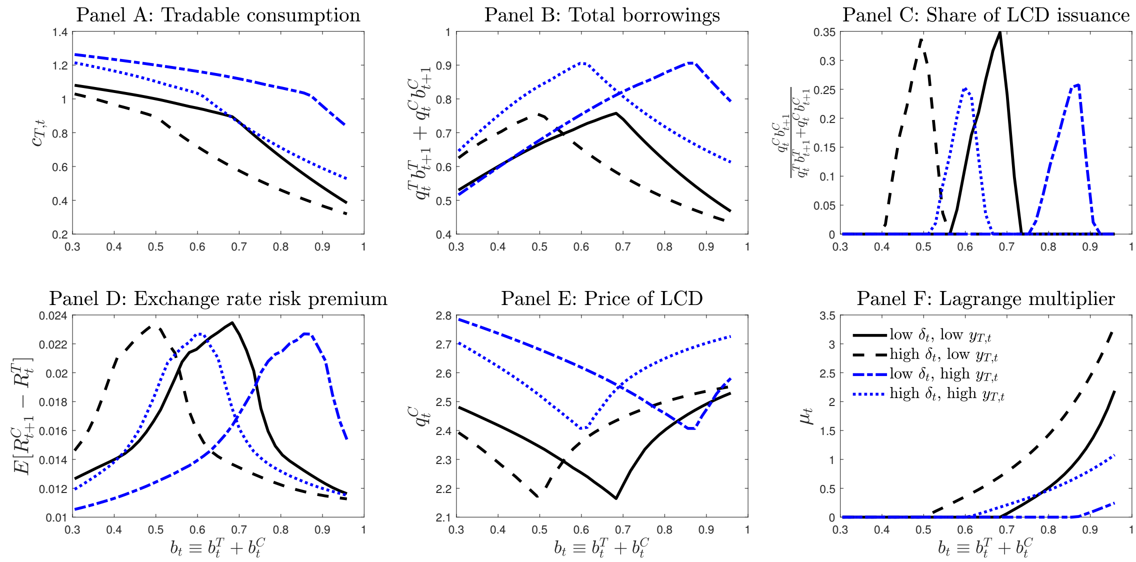
When the debt level is sufficiently high, the collateral constraint starts to bind, triggering the Fisherian debt-deflation channel. In the binding region, tradable consumption sharply declines as the debt increases. The collapse in consumption, in turn, further exacerbates the reduction in the collateral value and borrowing opportunities. Because the reduced borrowing in the current period implies a smaller debt in the next period, the agent has fewer incentives to issue LCD for the insurance benefit. Therefore, as shown by panel C, the share of LCD quickly declines to zero once the constraint binds. Finally, panels D and E show that in the binding region, a higher debt balance results in the lower risk premium and the recovered bond price.

Figure 3 also compares policy functions for different levels of endowment ($y_{T,t}$) and existing shares of LCD (δ_t). Recall that δ_t is defined as $\delta_t = b_t^C/(b_t^C + b_t^T)$. A higher δ_t indicates (1) a higher fraction of debt denominated in the local currency; and (2) a higher debt burden since the real exchange rate value (p_t^C) is greater than one in our simulations.[22] Therefore, from the upper panel of figure 3, we find that a higher δ_t indicates a lower tradable consumption, greater borrowing needs, and stronger incentives to issue LCDs in the nonbinding states. The same logic applies to the comparison between the high- and low-income levels. If the constraint is not binding, the higher income boosts consumption, decreases borrowing, and reduces the agent's incentive to use LCDs. Since the income process is persistent, a higher income also improves the price of LCD and reduces the exchange rate risk premium. More importantly, a positive income shock greatly relaxes the financial constraint and shifts the binding region to the right.

[21] When the economy has a very low debt balance and is far away from the collateral constraint, the private agent borrows exclusively in the form of FCD. The reason is that when b_t is very low, the economy has nearly a zero probability of hitting the financial constraint in the next period. So, the cost of issuing LCD exceeds its hedging benefit. In the long run, the probability that decentralized agents choose a zero share of LCD is 15.3%.

[22] The overall debt balance is $b_t \delta_t p_t^C + b_t(1-\delta_t)$.

Figure 3: Policy Functions under Competitive Equilibrium



NOTE: This figure display decision rules in the decentralized equilibrium. We plot the policy functions for a continuum of debt balance b_t at two different levels of $y_{T,t}$ and $\delta_t \equiv b_t^C/(b_t^C + b_t^T)$. The high-$y_T$ (low-y_T) state is set to one standard deviation above (below) its mean. The high-δ (low-δ) state refers to the CP's (CE's) simulation average. The exchange rate risk premium is defined as $\mathbb{E}_t[R_{t+1}^C - R_t^T]$, where $R_{t+1}^C = p_{t+1}^C/q_t^C$ and $R_t^T = 1/q_t^T$.

4.3 Analysis of Optimal Policies

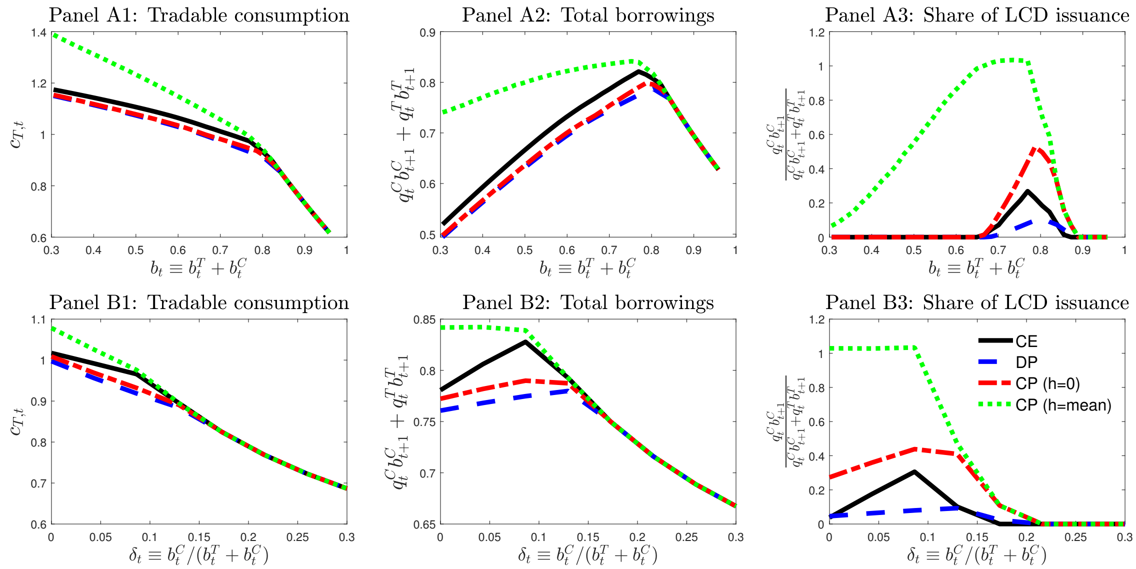
Next, we compare the decentralized equilibrium with the two social planners' allocations.

4.3.1 Comparing Policy Functions

Figure 4 compares the policy functions under the CE, DP, and CP. We notice that the two types of social planners improve welfare in different ways. First, compared to the decentralized agents, the discretionary planner internalizes how the restrained borrowing impacts the collateral constraint and real burden of LCD. Lacking commitment power, she also understands that the price of LCD is determined by the next-period consumption, which depends on her borrowing and denomination decisions today. In figure 4, we find that in states where the constraint is not binding, the DP borrows less than the decentralized agents so as to preserve liquidity and reduce the probability of hitting a financial constraint. However, in equilibrium, this lower level of borrowing reduces tradable consumption and makes the issuance of LCD less desirable. As a result, the discretionary planner borrows less than the private agents and borrows predominately in FCD.

We then consider the policy functions of a commitment planner who, unlike the discretionary planner, can promise a consumption plan in the next period and will therefore manipulate the debt payoff schedule to influence the endogenous bond price. In particular, the planner has incentives to increase consumption at certain states of nature to obtain a better bond price, which allows

Figure 4: A Comparison of Policy Functions in Three Equilibria



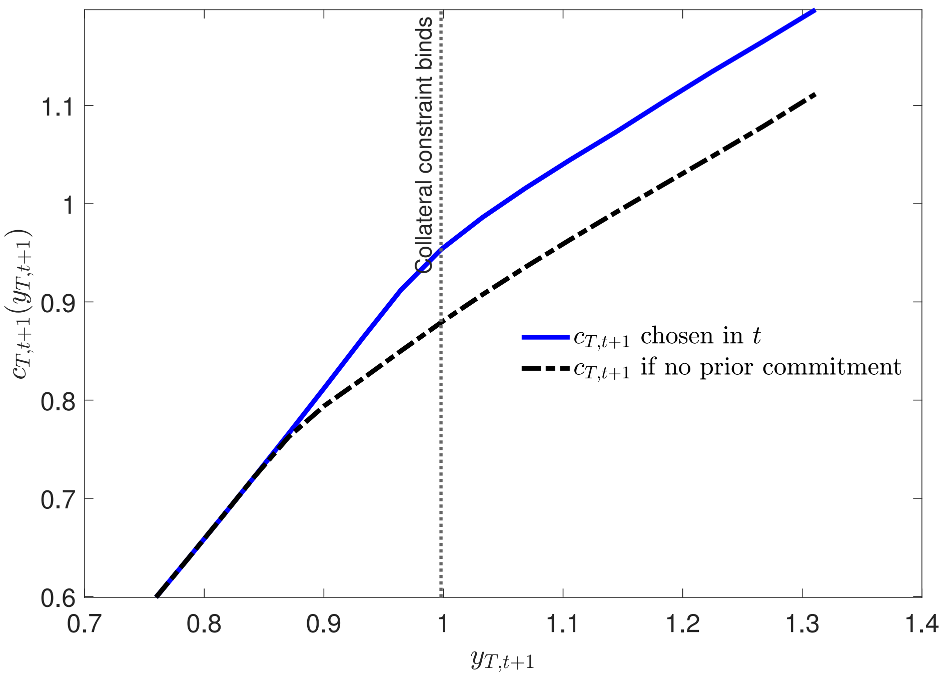
NOTE: This figure compares decision rules under the three equilibria. The tradable endowment is set to its mean. Panels A1-A3 show the decision rules for different b_t while keeping δ_t at the CP's ergodic mean. Panels B1-B3 show the decision rules for different δ_t while keeping b_t at the CP's ergodic mean. To construct the policies for the commitment planner, we set the state of prior commitment (h_t) either to zero or to its long-run mean.

her to reap the benefit of issuing LCDs at a lower cost. As shown in panels A2 and B2, the commitment planner borrows more aggressively than the discretionary planner. When the state of prior commitment equals its simulation mean (h_t = mean), the total amount of borrowing is even larger than that in the decentralized market.[23] Panels A3 and B3 show that the commitment planner denominates a larger fraction of local currency debt relative to the CE and DP.

In figure 5, we compare the time-t planner's committed consumption profile (the blue solid line) with a consumption profile chosen by a period-$t+1$ planner who discards her prior commitment (the black dashed line). The difference between these two lines captures the planner's incentive to manipulate the next-period consumption ($\{c_{T,t+1}\}_{y_{T,t+1}\in\mathcal{Y}}$) in order to improve the current utility. At the low-income realizations, the collateral constraint binds, and the consumption is uniquely determined by the collateral value. As a result, the two lines coincide. As tradable income increases and the collateral constraint becomes slack, the blue line rises above the black dashed line, implying that the commitment planner would prefer to choose a higher consumption than without prior commitment. Committing to a high consumption in the next period improves the ex ante bond

[23]On the one hand, the commitment planner has more precautionary motives, making her borrow less than the decentralized agents. On the other hand, having access to a safer debt portfolio (i.e., one that means lower debt repayments in bad times) allows her to borrow more. Whether the decentralized economy features "underborrowing" or "overborrowing" depends on the state of prior commitment h_t. Note that even if the commitment planner borrows more, this does not mean she would subsidize debts. Our simulation in section 4.5 shows that the commitment planner always imposes positive tax rates on FCD and LCD issuance, consistent with Arce et al. (2023).

Figure 5: The Next-Period Consumption Profiles w/ and w/o Prior Commitment



NOTE: This figure compares the period-t+1 consumption profiles ($c_{T,t+1}(y_{T,t+1})$) chosen by commitment planners with and without prior commitment. The blue solid line plots the period-t+1 consumption schedule that a period-t commitment planner would choose when b_t, δ_t, and the prior commitment state $h_t = \left(\lambda_{t-1}^{CP} - \mu_{t-1}^{CP}\right) b_t^C \frac{\partial p_t^C}{\partial c_{T,t}} \mathcal{M}(s_{t-1}, s_t) \frac{1}{\beta}$ are set to their ergodic means. $y_{T,t}$ is also set to its mean value. The black dashed line plots the consumption schedule if the commitment planner reneges on her previous commitment in period-t+1 (by setting h_{t+1} to 0) and rechooses a $c_{T,t+1}$ after observing the realization of $y_{T,t+1}$.

price (q_t^C) and helps the economy mitigate the consumption collapse during sudden stops using a better debt structure.[24]

4.3.2 Simulation Results

Table 2 reports long-run simulation moments in the three equilibria. First, the discretionary planner internalizes the effects of her borrowing decisions on the collapse of collateral value when a future financial crisis hits. That leads her to borrow less in the international market (31.6%) compared to the private equilibrium (33.2%). However, the incentive to deflate debt burdens ex post makes it more costly to issue LCD ex ante. In the long run, the DP only denominates 5.8% of her debts in local currency, even lower than the average share of the CE (13.6%).

Second, because the commitment planner can commit to a better consumption profile and is

[24] Figure G.5 in appendix G shows the histogram of h in the simulation of the commitment planner's problem. We find that the planner tends to promise a high h_{t+1} when the current tradable endowment ($y_{T,t}$) is low to buffer against the upcoming financial crisis.

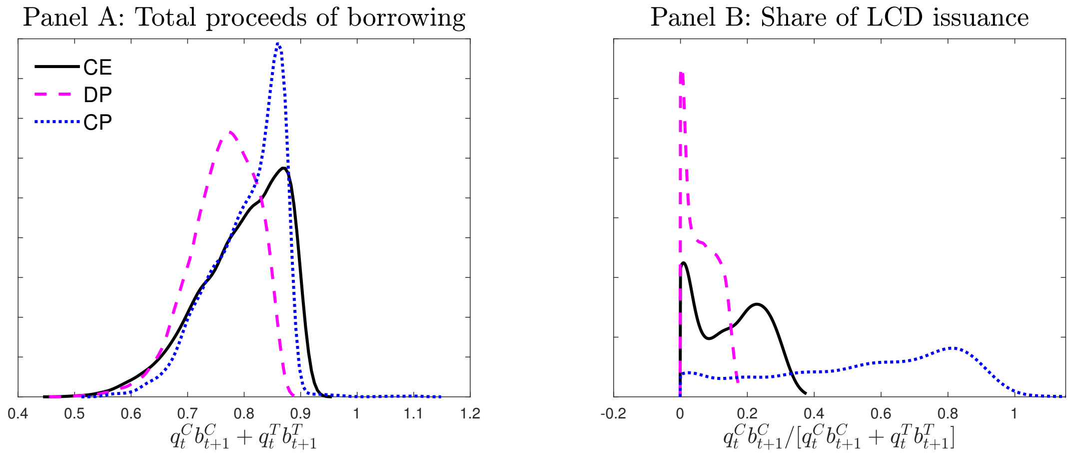
Table 2: Simulation Results

	Decentralized Equilibrium	Discretionary Planner	Commitment Planner
Avg. debt burden/y	33.2%	31.6%	33.6%
Avg. share of LCD	13.6%	5.8%	53.2%
- *Prob. of LCD=0*	15.3%	19.0%	4.5%
- *Prob. of positive reserve*	0%	0%	0.4%
Std(c_T)/Std(y_T)	1.17	1.12	1.03
Std(ca)/Std(y_T)	0.53	0.31	0.41
Corr(c_T, y_T)	0.92	0.97	0.97
Corr(ca, y_T)	-0.24	-0.32	-0.36
Std(debt burden/y_T)	5.6%	5.5%	4.4%
Std(share of LCD issuance)	10.7%	5.0%	28.9%
Corr(debt burden, y_T)	0.77	0.75	0.85
Corr(share of LCD issuance, y_T)	-0.39	-0.69	-0.63
Avg. spread on LCD	2.03%	1.77%	1.01%
- *Avg.* $std_t(p^C_{t+1})$	19.2	16.6	14.3
- *Avg.* $cov_t(\mathcal{M}_{t,t+1}, p^C_{t+1})$	-4.67	-4.02	-3.49
Std(spread)	0.24%	0.16%	1.00%
Corr(spread, y_T)	-0.26	-0.56	0.41
Prob. of crises	5.5%	1.4%	2.8%
Sev. of crises (%Δc_T)	-25.2%	-19.0%	-16.5%
Avg. tax on FCD: τ^T	–	6.51%	5.93%
Avg. tax on LCD: τ^C	–	6.19%	5.43%
Avg. tax discrimination: $\tau^T - \tau^C$	–	0.32%	0.50%
Corr($\frac{\tau^T+\tau^C}{2}$, y_T)	–	-0.79	-0.57
Corr($\tau^T - \tau^C$, y_T)	–	-0.51	-0.74
Avg. wel. gain rel. to DE	–	0.019%	0.071%

NOTE: We simulate each of the three economies for 100,000 periods and discard the first 10,000 periods for burning-in. We repeat the simulation 50 times and then take the average across simulations. A sudden stop is defined as the period when collateral constraint binds and the current account level exceeds two standard deviations above its mean. The spread on LCD is defined as $\mathbb{E}_t[R^C_{t+1} - R^T_t]$, where $R^C_{t+1} = p^C_{t+1}/q^C_t$ and $R^T_t = 1/q^T_t$. The overall debt burden is $p^C_t b^C_t + b^T_t$, while $p^C_t b^C_t/[p^C_t b^C_t + b^T_t]$ is the share of LCD in a country's liability. The share of LCD issuance is defined as $q^C_t b^C_{t+1}/[q^C_t b^C_{t+1} + q^T_t b^T_{t+1}]$. "Prob. of LCD=0" denotes the frequency of periods without any LCD issuance. "Prob. of positive reserve" denotes the frequency of periods where households borrow only in local currency while holding foreign currency assets as reserves. The average welfare gain represents the percentage of permanent consumption that households would like to sacrifice to move to the social planners' economies. Moments of tax rates are computed based on the periods when the financial constraint does not bind.

flexible in using LCD to insure against the downside risk, she denominates 53.2% of the debt in local currency, and her average indebtedness is even larger than that in the CE (33.6% vs. 33.2%). In the decentralized market, the average probability of time that agents issue no LCD is 15.3%. The probability is higher at 19% in the DP's economy but as low as 4.5% in the CP's economy. In the long run, there are even 0.4% of the periods when the CP issues debts only in local currency while holding assets in hard currency as reserves.

Figure 6: Ergodic Distributions of Borrowings and Shares of LCD



NOTE: This figure plots the ergodic distributions of total borrowings and the share of LCD issuance in the three economies.

By restricting overall debt issuance, the DP reduces the crisis probability to 1.4%, a significant decrease from the 5.5% in a decentralized economy without any policy. The lowered leverage ratio also mitigates the crisis severity. The DP's equilibrium is associated with lower volatilities of consumption and current account. On the other hand, the CP improves consumption-smoothing by issuing more debts in local currency, which is evident by examining the fluctuation of debt burden (std. and its correlation with income). Because the payoff of LCD is contingent on the realizations of real exchange rate, a larger share of LCD reduces the volatility on debt burden (4.4% in the CP vs. 5.6% in the CE).

The middle panel of table 2 shows the moments on the spread of returns between local and foreign currency debts. The spreads are reduced for both planners because financial regulations always mitigate the consumption collapse during the crises. The spreads decline because the future exchange rate is less volatile and less negatively correlated with the lenders' pricing kernel. By committing to a future consumption plan, the CP enjoys the lowest spread out of the three equilibria. The spreads are weakly countercyclical in the decentralized market and the discretionary economy but become procyclical in the model with commitment.

The CP's flexibility in choosing portfolio can be easily seen from the moments on the local currency share (std. and its correlation with income). Compared with the CE and DP, the CP has the highest standard deviation on the share of LCD issuance (28.9% vs. 10.7% and 5.0%). Also, the negative correlation indicates that the planner prefers to issue a larger fraction of LCD in economic downturns. Figure G.3 in appendix G shows the scatter plots of portfolio distributions in the three equilibria. Only the CP's equilibrium displays a large variation in the local currency share. Figure G.4 shows how the debt denomination decisions correlate with the exchange rate volatility and currency risk premium along the equilibrium paths.

Figure 6 plots the ergodic distributions of total borrowings and the share of debt denominated in local currency.[25] From panel A, we find that the discretionary planner strongly constrains the level of borrowings in the financial market. The commitment planner, on the other hand, borrows a similar level of debt as the private agents. Because the CP can hedge negative income shocks by denominating a large fraction of debt in local currency, at certain economic states, she borrows even more aggressively than the private agents.

Panel B in figure 6 compares the distributions of portfolio shares. Due to the lack of commitment, the DP has the lowest local currency share. In contrast, the CP, who enjoys a better domestic bond price by committing to future policies, issues the largest amount of debt in local currency. More importantly, we notice that the share of LCD issuance is widely dispersed in the CP's simulation. In certain states, the share even exceeds 100%, meaning that the home country holds foreign currency assets as international reserves.

4.3.3 Crisis Events

This section compares the models' performance during sudden stop crises. We begin by simulating the competitive equilibrium model for 500,000 periods and dropping the first 10,000 periods for burning-in. We then identify 1,000 sudden stop episodes from the simulated data and extract a 9-period event window for each identified sudden stop period. Next, we extract the sequences of shocks during the crisis event windows and the initial states before the crises and feed them into the social planners' equilibria.[26] Figure 7 shows the average simulation paths around a sudden stop event.

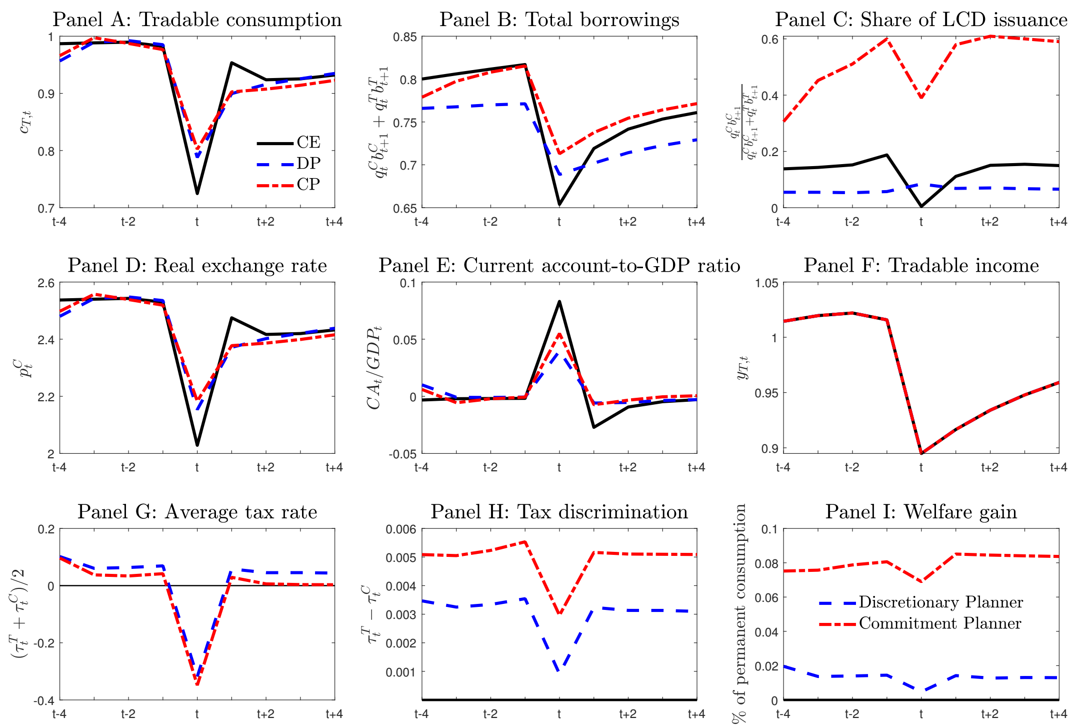
The event window under the CE (black solid lines) displays a standard sudden stop phenomenon. Financial crises are often triggered by a sudden collapse in tradable income after a sequence of positive shocks (panel F). The decline in tradable income forces agents to cut consumption, depreciating the real exchange rate. The reduced nontradable price, in turn, tightens the financial constraint and amplifies the collapse in the borrowing limit and consumption demand. Ultimately, the sudden stop event is featured by the drastic drops in consumption and borrowing, large real depreciations, and big current account reversals. We also observe from panel C that the higher income before the crisis induces agents to issue more LCDs because they have a stronger incentive to hedge the upcoming financial risk when their leverage rises.

Compared with the CE, both discretionary and commitment planners experience less severe recessions under the same sequences of shocks. The collapses in consumption, real exchange rate, and borrowings are milder in the planners' equilibria, and the current account reversals are also smaller. However, the two planners achieve financial stability through different strategies. The DP internalizes the pecuniary externality in the collateral constraint and, therefore, preserves

[25] Figure G.1 in appendix G shows the distributions of debt burden in local and foreign currencies. We find that in the DP's problem, the country's foreign currency exposure is even higher than that under the CE.

[26] Our method to conduct crisis event analysis is the same as Bianchi & Mendoza (2018).

Figure 7: Event Window Analysis



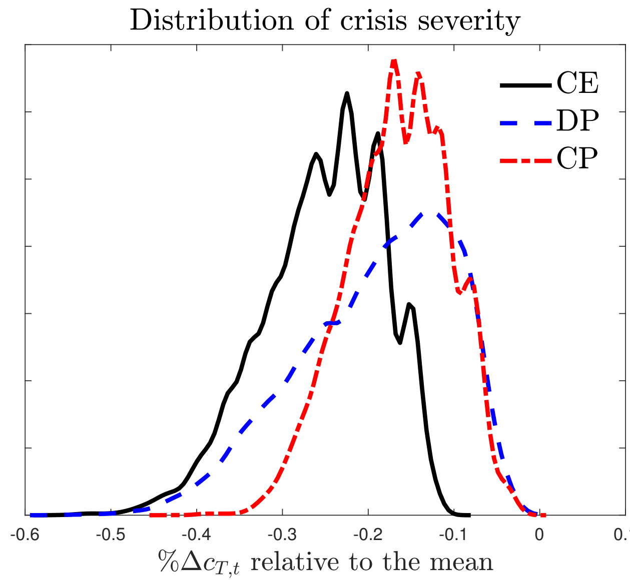
NOTE: The graph compares the sudden stop event windows in the three environments. For comparison, we first identify 1,000 sudden stop events from the simulations of the CE and extract the income process during the crises and initial states before the crises. We then feed the series of shocks and initial states into alternative economies. The graph shows the average path of simulations across the event windows. The welfare gain represents the percentage of permanent consumption that households would like to sacrifice to move to the social planners' economies. The initial h state is set to 0 at period $t-4$ in the commitment planner's simulation.

liquidity by borrowing less compared to the CE (panel B). This additional liquidity alleviates the amplification effect from the financial constraint when a negative shock hits and allows the planner to achieve a milder fluctuation.

The CP, on the other hand, tends to manipulate the debt payoff by committing to a certain consumption plan in the future. Such a manipulation allows her to reap the insurance benefits of LCD at a lower cost. Panel C shows that the CP issues the largest amount of local currency debts. The extra LCD provides her a buffer to hedge the downward risk and also reduces her need to constrain borrowings. Therefore, the planner still borrows aggressively during the boom periods preceding the financial crisis.

Figure 8 shows the distributions of crisis severity during sudden stop events in the three equilibria. The crisis severity is measured by the degree of consumption collapse (in percentage deviation from the long-run mean). Both social planners can mitigate the crisis severity by shifting the distribution to the right. By restricting the overall capital inflows, the DP enjoys a smaller consumption collapse when the same sudden stop shock hits the economy. But there is still a significant proba-

Figure 8: Distribution of Crisis Severity



NOTE: The figure shows the distributions of tradable consumption collapse during the identified sudden stop episodes in the three economies. The crisis severity is measured as the percentage deviation of c_T at the time of a sudden stop from its long-run average.

bility that the consumption drop exceeds 40% in a sudden stop, similar to what we observe under the CE. The CP obtains the mildest consumption drop during sudden stop episodes, even though the restriction on debt issuance is more lenient. Using LCD allows the economy to avoid most of the extreme consumption collapses experienced by private agents. We notice that the distribution displays a thinner left tail compared to the other two economies.

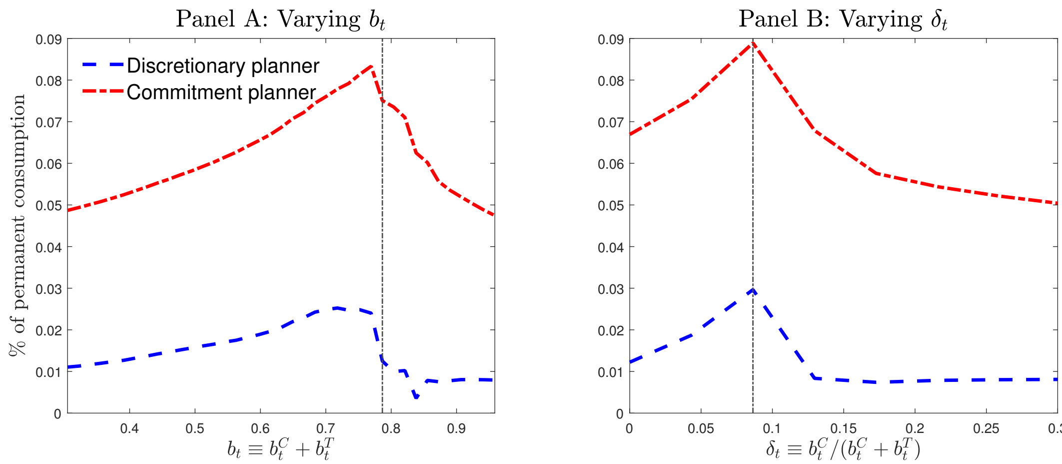
4.4 Welfare Implications

To gauge the benefits of policy intervention, we calculate state-contingent welfare gains achieved by the discretionary and commitment planners. The welfare gains are measured as the percentage of permanent consumption that households are willing to sacrifice to live in the world with either a discretionary or commitment planner. We first compute value functions in the three economies. In each economy, the household's value function is defined on the state space as follows:

$$V^i(\mathcal{S}) = \frac{c(\mathcal{S})^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{s'|s} V^i\left(b^{C'}(\mathcal{S}), b^{T'}(\mathcal{S}), s'\right), \text{ where } i = \{CE, DP, CP\} \tag{53}$$

where $\{c(\mathcal{S}),\ b^{C'}(\mathcal{S}),\ b^{T'}(\mathcal{S})\}$ denote the optimal decisions in the corresponding economy. In the CE's and the DP's problem, the state vector is $\mathcal{S} = \{b^C, b^T, s\}$. In the CP's problem, the value functions and decision rules are defined on the extended state space $\tilde{\mathcal{S}} = \{b^C, b^T, h, s\}$, where the auxiliary state h refers to the prior commitment made by the social planner in previous periods.

Figure 9: State-Contingent Welfare Gains: $\gamma^{DP}(\mathcal{S})$ and $\gamma^{CP}(\mathcal{S})$



NOTE: The figure shows the state-contingent welfare gains for the discretionary and commitment planners. The value represents the percentage of permanent consumption that households are willing to sacrifice to live in an economy with social planners. To compute the welfare gain by the commitment planner, we assume there is no prior commitment by setting $h_t = (\lambda_{t-1}^{CP} - \mu_{t-1}^{CP}) b_t^C \mathcal{M}(s_{t-1}, s_t) \frac{\partial p_t^C}{\partial c_{T,t}} \frac{1}{\beta}$ to zero. In the left panel, we keep the share of LCD ($\delta_t \equiv b_t^C/(b_t^C + b_t^T)$) at the CE's ergodic mean and vary the total debt level ($b_t \equiv b_t^C + b_t^T$). In the right panel, we hold the total debt level b_t at the CE's mean and vary the debt share δ_t. The tradable endowment is always set to its mean.

Iteration of the value functions yields the following objects: $V^{CE}(\mathcal{S})$, $V^{DP}(\mathcal{S})$, $V^{CP}(\tilde{\mathcal{S}})$.

Since the utility is in CRRA form, we can calculate the welfare gains using the following expression:

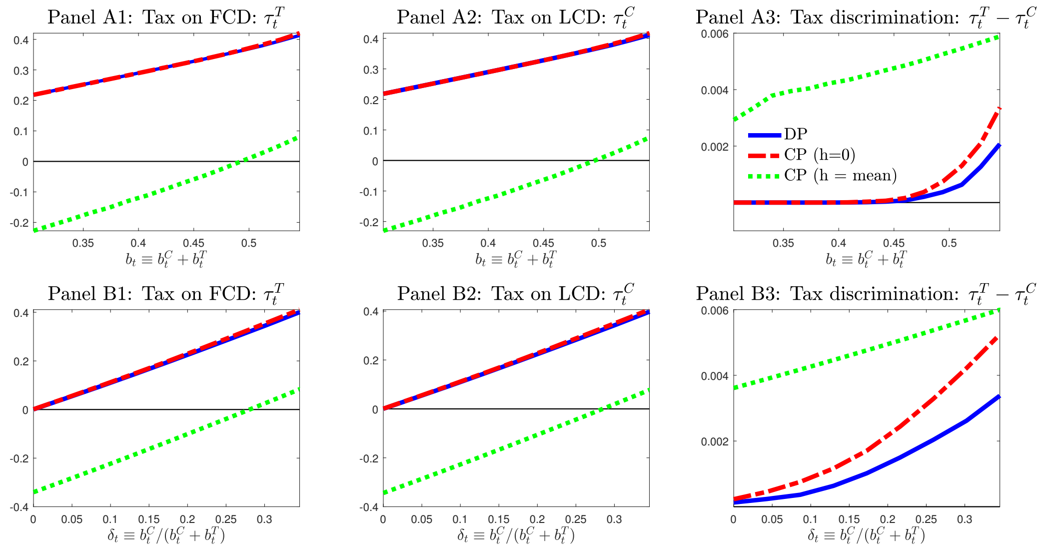
$$\left[1 + \gamma^{DP}(\mathcal{S})\right]^{1-\sigma} V^{CE}(\mathcal{S}) = V^{DP}(\mathcal{S}), \qquad \left[1 + \gamma^{CP}(\mathcal{S})\right]^{1-\sigma} V^{CE}(\mathcal{S}) = \hat{V}^{CP}(\mathcal{S}), \tag{54}$$

where $\gamma^{DP}(\mathcal{S})$ and $\gamma^{CP}(\mathcal{S})$ represent the percent of permanent consumption making the household indifferent between living in a competitive equilibrium and in the two social planners' economies. For ease of comparison, we redefine the CP's value function by removing the additional dimension in the state vector. To do so, we set the value of prior commitment to zero: $\hat{V}^{CP}(b^C, b^T, s) = V^{CP}(b^C, b^T, 0, s)$. The attenuated value function $\hat{V}^{CP}(\mathcal{S})$ is then defined on the original state space, which is comparable to the other two cases.[27]

Figure 9 displays the state-contingent welfare gains in the discretionary and commitment planners' economies. Both planners achieve positive welfare gains across the entire state space, and the gains are larger at the medium debt levels (or medium level of δ_t) where the collateral constraint is currently slack but the probability of hitting the constraint in the next period is relevant. In

[27] We also use alternative methods to calculate the state-contingent welfare gains of a commitment planner. For example, we simulate the path of the "prior commitment state" ($\{h_t\}_{t=1}^T$) under the CE and compute the CP's welfare gains based on the CE's long-run ergodic distribution over the extended state space $\{\tilde{\mathcal{S}}_t = (b_t^C, b_t^T, h_t, s_t)\}_{t=1}^T$. We find that the welfare implication of our model does not depend on the method of calculating welfare gains.

Figure 10: Tax Rate Schedules to Restore Planners' Allocations



NOTE: The figure shows the schedules of state-contingent capital control taxes that restore the social planners' allocations. The tradable endowment is set to its mean. Panels A1-A3 show the tax rates for different $b_t \equiv b_t^C + b_t^T$ while holding the portfolio share $\delta_t \equiv b_t^C/(b_t^C + b_t^T)$ at the CP's ergodic mean. Panels B1-B3 show the tax rates for different δ_t while keeping b_t at the CP's ergodic mean. For the commitment planner, we set the value of prior commitment h_t either to 0 or to the model's long-run simulation mean.

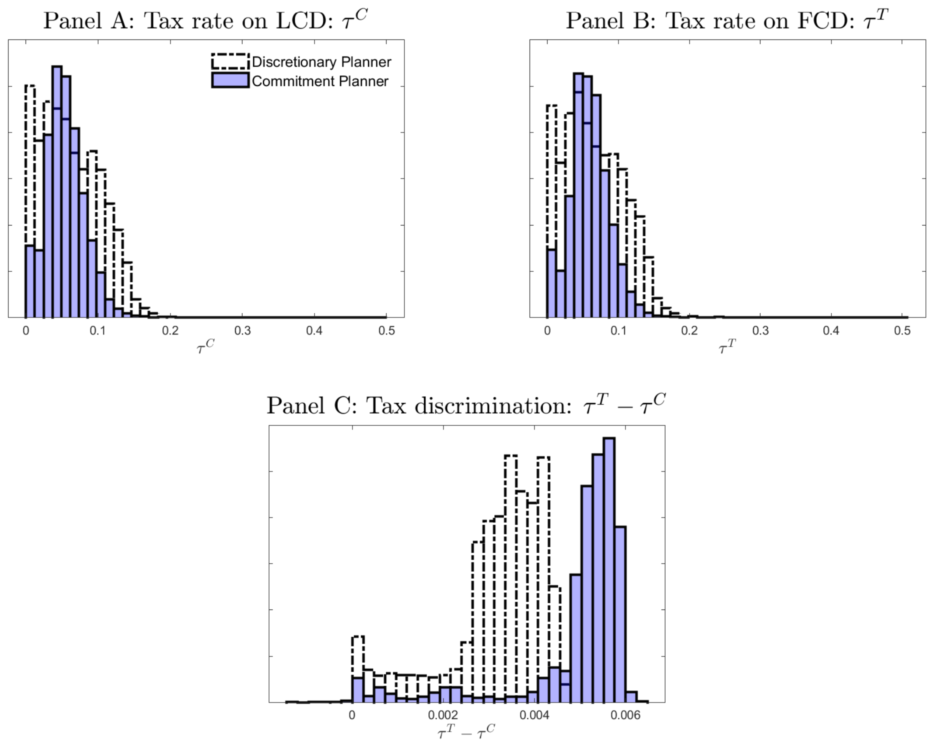
these financially fragile states, the DP's welfare gain comes from the restriction on capital inflow volumes and the associated decline in financial crisis probability. In contrast, the CP obtains the welfare benefit by managing its portfolio and denominating more debts in local currency. The LCD plays an insurance role by reducing consumption volatility and mitigating financial amplification. Making commitments also allows the economy to enjoy a lower risk premium on LCD.

Lastly, at high-b states (or high-δ) where the constraint is binding, both social planners are trapped in a financial crisis, so their welfare gains are relatively smaller. The bottom row of table 2 shows that in the long run, the DP and CP obtain the average welfare gains of 0.019% and 0.071% in terms of permanent consumption equivalence, respectively.

4.5 Capital Controls Taxes

Figure 10 shows the functions of tax policies that restore the social planners' allocations. For the commitment planner's case, we set the level of prior commitment either to 0 or to its long-run simulation mean. The figure only reports tax rates in states where the financial constraint does not bind. Overall, the DP tends to impose positive capital control taxes on local and foreign currency borrowings across the entire state space, and the difference between the two tax rates is relatively small (right panels). On average, the tax rate on FCD is higher than the tax rate on LCD by

Figure 11: Distributions of Capital Control Taxes across Simulations



NOTE: The figure shows the histograms of capital control taxes in the models' long-run simulations. We only include the periods when the financial constraint does not bind. Panels A and B show the distributions of tax rates on FCD and LCD, respectively. Panel C displays the tax discrimination that is defined as their difference. A positive number indicates that the planner tends to restrict foreign currency borrowings more heavily than local currency borrowings.

0.32%.

The commitment planner's tax schedule is more complicated. First, the optimal financial regulation highly depends on the level of prior commitment. For any nonzero value of h_t, the CP tends to choose a lower tax rate than the DP at the same economic state. Second, in certain low-b or low-δ states, the CP levies a negative tax to encourage borrowings. In states with low financial risk, the probability of hitting the financial constraint in the next period is minimal. Also, when δ_t is low, the fluctuation of tradable consumption has a minor effect on the overall debt burden. As a result, the planner's incentive to constrain borrowing or to deflate the debt is dominated by her willingness to increase consumption and preserve a better bond price. More importantly, to encourage local currency borrowings, the CP sets nonuniform taxes based on the currency denomination of capital inflows. In our simulations, the average tax on the FCD borrowings is 0.5% higher than the average tax on the LCD. As seen from panels A3 and B3, the tax discrimination is stronger when the economy is more indebted or the level of prior commitment is higher.

Figure 11 shows the ergodic distributions of capital control taxes. The bottom panel of table 2 reports the moments on tax rates.[28] Both social planners tend to impose positive capital control taxes on international borrowings, and the tax discrimination is stronger in the CP than in the DP. The discrimination of this size is large enough to incentivize agents in the CP's economy to issue a significantly greater amount of LCD. Table 2 also shows that the average taxes and tax discrimination negatively correlate with the tradable income.

It is worth to mention that the commitment planner never implements a negative tax rate along the equilibrium paths. In this environment, the use of capital control tax is governed by two forces. On the one hand, the CP has incentives to use negative taxes to induce borrowings and tilt up consumption to improve the previous-period bond price. Such an incentive is captured by the region of negative tax policies in figure 10. On the other hand, the planner also has an incentive to use positive tax rates to discourage borrowing and reduce the probability of crises. Our simulation result indicates that the second incentive dominates, so the negative tax rates never materialize along the equilibrium path.

5 Discussion: Financial Integration vs. Financial Regulation

As emerging countries are gradually integrated into the global financial market, they are equipped with a greater ability to issue LCD. This section compares our baseline model with the traditional sudden-stop models with only FCD. We consider the welfare benefits of introducing LCD into a sudden stop economy and its implications for designing macroprudential policies. Table 3 shows the long-run simulation moments, including the results of Bianchi (2011)'s constrained-efficient outcome.

First, introducing LCD delivers a sizable welfare improvement (0.065% of permanent consumption) even without any capital control policies. The magnitude of this gain is comparable to the one achieved by enforcing financial regulations in the FCD-only economy (0.055%). Compared to the FCD-only environment (CE), the ability to issue LCD reduces the consumption volatility and fluctuation of debt burden. It also reduces the average severity of financial crises (-25.2% vs. -28.3%). The introduction of LCD also changes the design of capital control regulations. As financial markets become more integrated, policymakers should pay special attention to the capital flows' currency denominations when designing capital control policies. Using two state-contingent tax rates, the policymaker under commitment can further reduce the crisis probability (2.8% vs. 5.6%) and lessen the crisis severity (-16.5% vs. -28.3%) while sustaining a relatively high level of debt (33.6% vs. 32.6%). Ultimately, the commitment planner achieves the highest welfare across different economies.

Figure 12 shows the distributions of crisis severity. We first extract the tradable endowment

[28] Figure G.2 in appendix G shows the scatter plots of capital control taxes against tradable outputs. We find that the cyclicality of capital control tax and tax discrimination is quite different in the two social planning problems.

Table 3: A Comparison of Simulation Results with Bianchi (2011)

	FCD Only CE	FCD Only SP	Decentralized Equilibrium (FCD + LCD)	Discretionary Planner (FCD + LCD)	Commitment Planner (FCD + LCD)
Avg. debt burden/y	32.6%	31.6%	33.2%	31.6%	33.6%
Std(c_T)/Std(y_T)	1.24	1.15	1.17	1.12	1.03
Std(ca)/Std(y_T)	0.53	0.35	0.53	0.31	0.41
Avg. $\sigma_t(p^C_{t+1})$	20.5	18.0	19.2	16.6	14.3
Corr(c_T, y_T)	0.91	0.96	0.92	0.97	0.97
Corr(ca, y_T)	-0.23	-0.29	-0.24	-0.32	-0.36
Std(debt burden/y_T)	6.4%	5.8%	5.6%	5.5%	4.4%
Corr(debt burden, y_T)	0.70	0.72	0.77	0.75	0.85
Prob. of crises	5.6%	2.2%	5.5%	1.4%	2.8%
Sev. of crises (%Δc_T)	-28.3%	-21.1%	-25.2%	-19.0%	-16.5%
Avg. tax rate	–	5.71%	–	6.35%	5.68%
Corr(tax rate, y_T)	–	-0.82	–	-0.79	-0.57
Avg. wel. gains rel. to - FCD only (CE)	–	0.055%	0.065%	0.073%	0.133%

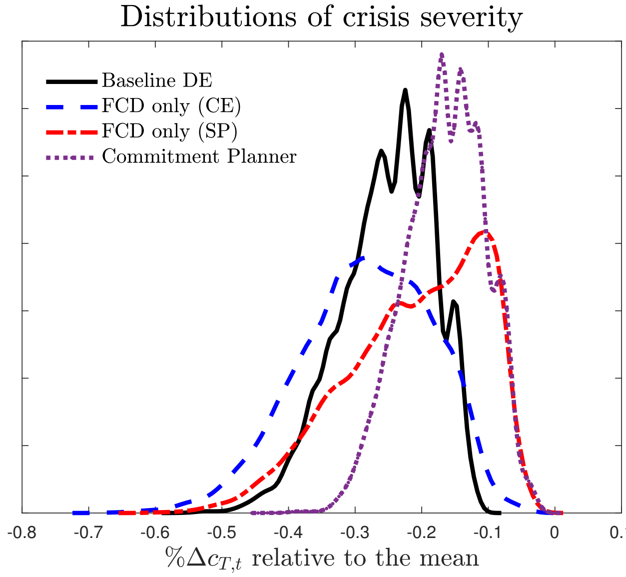
NOTE: We use the same parameters as in our baseline model to simulate the FCD-only economies. The welfare gains are the percentage of permanent consumption that the households living in the FCD-only economy (CE) would like to pay to move to an alternative environment. The average tax rates under "Discretionary Planner" and "Commitment Planner" are $(\tau^{T,DP} + \tau^{C,DP})/2$ and $(\tau^{T,CP} + \tau^{C,CP})/2$, respectively. See the notes under table 2 for details.

shocks from the simulation of the baseline CE. The shocks are then fed into alternative models, and we calculate the degree of consumption collapse during sudden stop crises. We notice that in the model without LCD, the distribution of crisis severity is very dispersed with a fat left tail. Things are different in the social planner's economy with only FCD. In this environment, the policymaker imposes a tough restriction on international borrowings so that it can reduce the crisis severity by a significant amount.

Meanwhile, due to the insurance provided by LCD, our baseline model features a less dispersed crisis severity distribution than the FCD-only economy (CE). Even though the commitment planner borrows a higher amount of debt relative to the baseline, the improvement in capital structure further alleviates the sudden stop crises. In the end, the economy under the CP has the most concentrated distribution of crisis severity in figure 12.

Figure 13 displays the sudden stop event windows in the four economies. Without LCD, an FCD-only economy (CE) is subject to a larger collapse in consumption and a greater trade balance reversal relative to our baseline model. The CP, on the other hand, enjoys the smallest consumption drop during a sudden stop due to the large share of LCD. Panel C in figure 13 shows the paths of total borrowings. We notice that the financial regulation in an FCD-only model implies the restriction on credit volumes, resulting in a smaller amount of borrowings in the social planner's economy. On the contrary, in the model with LCD, the CP borrows a similar level of debt as the decentralized market in periods before the financial crises.

Figure 12: Distribution of Crisis Severity: Models w/ and w/o LCD

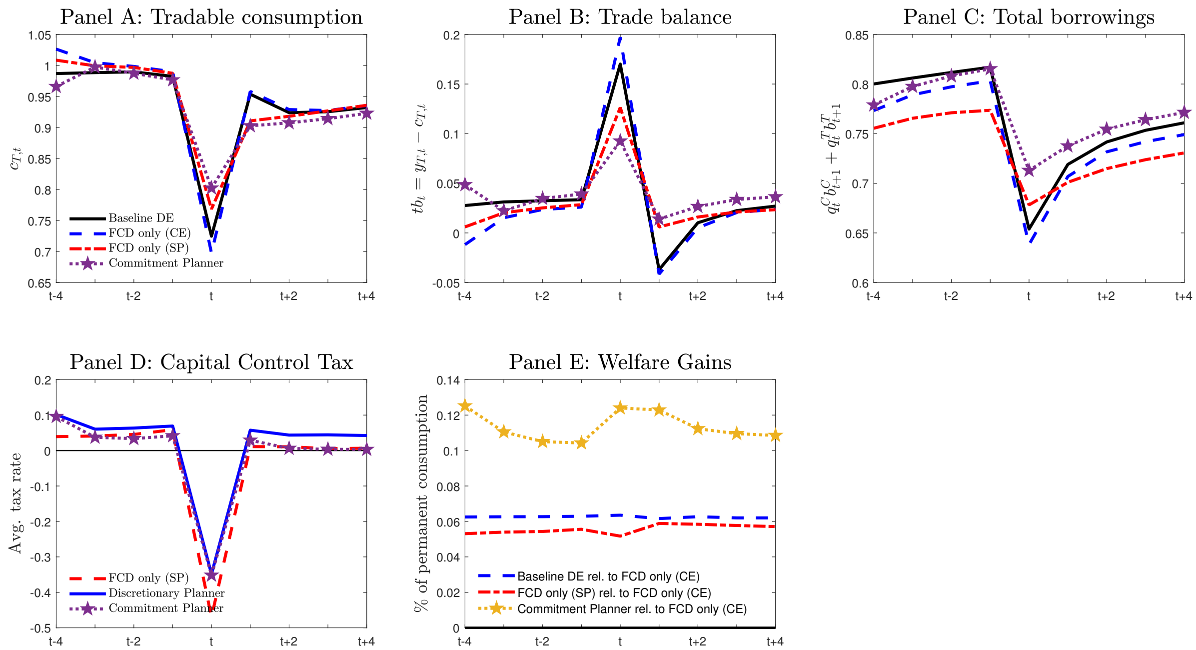


NOTE: The figure shows the distributions of crisis severity measured as the percentage of consumption collapse in a sudden stop period from its long-run mean. To construct this figure, we extract shocks from the simulation of the baseline DE and feed them into other economies starting from the DE's initial debt states before the crises. We then take the average across all event windows.

The bottom panels of figure 13 show the average capital control taxes and welfare improvement relative to an FCD-only economy (CE) around sudden stop periods. The commitment planner's allocation entails the mildest capital control relative to alternative policy environments. The magnitude of welfare gain from introducing LCD is comparable to the one achieved by capital control regulation in an FCD-only model.

It is widely accepted in the literature that prudential regulations are intended to target the crisis episode to limit consumption collapse during a financial crisis. However, the exercise here highlights the welfare benefit of introducing LCD, which is due to the progress of financial integration. This paper sheds a new perspective on the design of capital control policies as emerging economies have been gradually integrated into the global financial market over the past decades. A key takeaway from table 3 and figures 12-13 is that although these two objectives change the financial market in different directions, their welfare benefits have some overlap. For example, the use of LCD can mitigate the crisis severity when a sudden stop hits, similar to Bianchi (2011)'s social planner economy. Meanwhile, the capital control in an FCD-only economy delivers a similar welfare benefit to the one achieved by introducing LCD into a sudden stop model (0.065% vs. 0.055%). This provides a testament that financial integration could be a partial substitute for financial regulation in a dollar-debt economy.

Figure 13: Event Window: Models w/ and w/o LCD



NOTE: The figure shows the dynamics of endogenous variables and welfare gains around sudden stop episodes. To construct this figure, we extract shocks from the simulation of the baseline DE and feed them into other economies starting from the DE's initial debt states before the crises. We then take the average across all event windows. The welfare gain is measured as the percentage of permanent consumption that households living in an FCD-only economy (CE) would like to sacrifice to move to an alternative environment.

6 Conclusion

Given that the composition of capital inflows to emerging economies has changed over the past two decades, this paper introduced the debt denomination choice into a sudden stop model and investigated its implication on capital control policies. Compared to a dollar-debt economy, the presence of LCD offers risk-sharing opportunities for small open economies, even though the exchange rate risk underlying LCD entails a risk premium. It allows a country to smooth consumption, mitigates crisis severity, and delivers a welfare gain similar to the one achieved by the macroprudential policy in an FCD-only economy. In addition, the introduction of LCD adds new policy implications from pecuniary externalities and a time-inconsistency issue, thus calling for a renewed perspective on financial regulations.

Without the commitment, the Markov planner has an incentive to dilute the payoff of LCD through a real exchange rate depreciation. Such an incentive worsens the ex ante bond price and makes the issuance of LCD undesirable. In contrast, a social planner under commitment can discipline the debt-reduction motive by promising a state-contingent consumption profile in the future. The planner tends to tilt up consumption in good states of nature in order to earn a better bond price of LCD. The improvement in bond price reduces the LCD issuance cost, creates a better debt structure, and delivers a larger welfare gain than the discretionary optimal policy.

One of the key policy implications from our analysis is to use capital controls to change the composition of credit flows, in addition to restricting their aggregate volumes. Ideally, the optimal policy should result in a higher share of LCD and improve the financial stability of the indebted economies. However, such a policy goal can only be achieved with policy commitment.

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Appendix

A Data Appendix

The data on the net foreign asset (% of GDP) and the local currency share (% of total external liability) come from Bénétrix, Gautam, Juvenal, & Schmitz (2019). We calculate the net foreign asset as the difference between the total external asset and total external liability. The dataset includes 49 countries with 23 emerging and 26 advanced economies from 1990 to 2017.

Advanced countries (26) include Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, and the United States.

Emerging economies (23) include Argentina, Brazil, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, South Africa, Sri Lanka, Thailand, Tunisia, Turkey, and Uruguay.

Our data on the dynamics of sudden stop episodes come from Korinek & Mendoza (2014). The list of sudden stop episodes is shown in table A.1.

Table A.1: Sudden Stop Episodes

Low local currency share in external liability			**High local currency share in external liability**		
Country	Event Year	Local Currency Share	Country	Event Year	Local Currency Share
Morocco	1994	0.087	Denmark	1999	0.452
Finland	1995	0.112	Mexico	1995	0.453
Morocco	1995	0.113	Czech Republic	1998	0.471
Pakistan	1997	0.115	China	1992	0.473
Pakistan	2002	0.126	Australia	2008	0.481
Indonesia	1998	0.14	Tunisia	2007	0.485
Morocco	1997	0.17	Malaysia	1994	0.513
Turkey	1994	0.171	Poland	2009	0.526
Greece	1993	0.192	Canada	1996	0.538
Turkey	1999	0.217	Austria	2002	0.555
Peru	1998	0.238	Chile	1999	0.583
Turkey	2001	0.269	France	1997	0.591
Argentina	1995	0.271	Turkey	2009	0.608
Morocco	2002	0.288	Netherlands	2003	0.669
Philippines	1998	0.293	China	2005	0.689
Argentina	2002	0.3	Malaysia	1998	0.696
Thailand	1998	0.332	Ireland	2012	0.7
Korea	1998	0.341	China	2010	0.737
Hungary	1995	0.342	Czech Republic	2009	0.757
Norway	1999	0.344	Netherlands	2009	0.799
Russia	1998	0.349	United States	2009	0.804
United Kingdom	2008	0.372	Germany	2004	0.812
Italy	1993	0.378	South Africa	2009	0.812
Russia	1999	0.381	Germany	2009	0.826
Colombia	1999	0.39	Germany	2006	0.827
Brazil	2003	0.392	Belgium	2010	0.831
Sweden	2003	0.398	Greece	2012	0.832
Pakistan	2009	0.407	Spain	2009	0.853
Sweden	2006	0.415	Spain	2012	0.878
New Zealand	2009	0.428	Portugal	2012	0.919
Norway	2008	0.436	Italy	2012	0.935

Table A.2: Empirical Estimation: The Effect of Sudden Stop in the Environment with Local Currency Liabilities

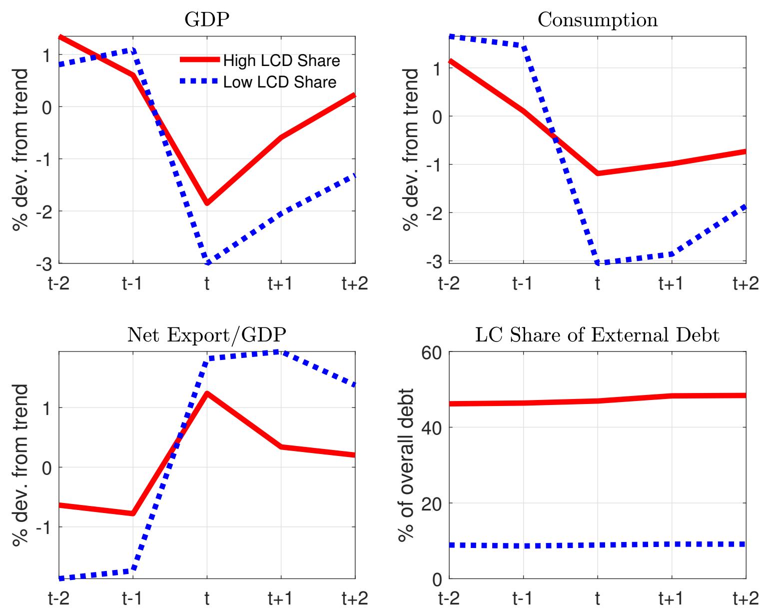
	GDP Growth				Consumption				Current Account Balance				Investment			
	(1) All Countries	(2) All Countries	(3) AE	(4) EME	(5) All Countries	(6) All Countries	(7) AE	(8) EME	(9) All Countries	(10) All Countries	(11) AE	(12) EME	(13) All Countries	(14) All Countries	(15) AE	(16) EME
Crisis	-5.57***	-6.03***	-1.61	-8.08***	-5.58***	-5.98***	-3.81	-6.57***	2.39**	2.83***	2.19	4.73***	-31.63**	-33.82**	-8.87	-43.77*
	(1.68)	(1.77)	(2.52)	(2.29)	(1.63)	(1.76)	(2.80)	(2.24)	(0.92)	(0.99)	(2.02)	(1.14)	(12.20)	(13.84)	(7.65)	(21.08)
Crisis × Local Currency Share (t-1)	5.35**	6.85**	0.65	11.38**	4.33	5.95**	3.36	6.83	-1.44	-1.22	0.38	-7.03***	35.43*	41.14**	7.77	61.68
	(2.48)	(2.64)	(3.33)	(4.23)	(2.63)	(2.75)	(3.83)	(4.27)	(1.75)	(1.99)	(3.21)	(2.45)	(18.41)	(20.31)	(10.29)	(39.32)
Local Currency Share (t-1)	0.85	5.23***	1.58	4.89*	0.92	3.02	0.18	2.01	-9.21***	-7.70**	-0.41	-13.44**	-11.95	-12.97	-1.60	-25.41
	(1.32)	(1.69)	(2.35)	(2.48)	(1.32)	(1.93)	(2.18)	(3.49)	(3.28)	(3.29)	(3.55)	(5.60)	(10.15)	(20.77)	(7.37)	(30.86)
Net Foreign Asset (t-1)		0.02*	0.00	0.02		0.03***	0.02**	0.03		0.04**	0.04	0.06		0.04	0.05*	0.01
		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.02)		(0.02)	(0.03)	(0.04)		(0.05)	(0.03)	(0.11)
log GDP per capita (t-1)		-4.87*	-8.00***	-5.35		-1.92	-1.84	-4.81		-0.76	-3.94	-0.81		-16.60**	-14.03*	-20.73
		(2.45)	(2.36)	(3.34)		(1.92)	(4.05)	(2.96)		(1.52)	(3.23)	(1.72)		(7.84)	(7.06)	(12.79)
Trade (t-1)		1.86	5.00**	1.86		2.35*	1.64	4.11*		8.43***	10.01***	7.81**		8.22	15.98**	11.56
		(1.38)	(1.88)	(1.73)		(1.38)	(1.43)	(2.17)		(2.32)	(3.41)	(3.04)		(8.34)	(5.98)	(12.24)
Credit (t-1)		-1.88**	-0.92	-2.63		-1.50*	-0.66	-2.77		-0.18	1.00	1.33		-9.57*	-2.68	-24.65
		(0.75)	(0.77)	(1.71)		(0.84)	(0.67)	(2.33)		(0.74)	(0.69)	(2.76)		(5.00)	(2.47)	(16.69)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	961	822	378	444	951	817	378	439	908	802	358	444	951	817	378	439
Adjusted R-squared	0.276	0.399	0.552	0.384	0.178	0.245	0.345	0.277	0.143	0.239	0.253	0.283	0.090	0.101	0.425	0.088

NOTE: This table estimates the following empirical relationship:

$$y_{i,t} = \alpha_i + \beta_1 SS_{i,t} + \beta_2 SS_{i,t} * LCD_{i,t-1} + \beta_3 LCD_{i,t-1} + X_{i,t-1} + \varepsilon_{i,t},$$

where $y_{i,t}$ represents the GDP growth rate, final consumption growth rate, current account balance (% GDP), and the growth rate of capital investment for country i at year t. $SS_{i,t}$ is the dummy for sudden stops episodes by Korinek & Mendoza (2014) (see table A.1). $LCD_{i,t-1}$ indicates the local currency share (% of total external liability). $X_{i,t-1}$ includes country-level control variables such as net foreign asset (% GDP), GDP per capita (in logs), trade (% GDP), and credit (% GDP). All country-level variables are from the World Development Indicator except for the local currency share, which is from Bénétrix et al. (2019). We include the country and year fixed effects in all the specifications. We first run regressions on the entire sample, then separately on advanced economies (AE) and emerging market economies (EME). The standard errors are clustered at country level. *, ** and *** indicate the significance at the 10%, 5%, and 1% level, respectively.

Figure A.1: Sudden Stop Event Window: All Countries



NOTE: The data of sudden stop episodes comes from Korinek & Mendoza (2014). The description is given in table A.1. We classify the sudden stop events into two groups based on the countries' local currency share of external liability in the pre-crisis year. The data on local currency share (% of external liability or % of external debt) come from Bénétrix et al. (2019). We classify the advanced countries and emerging and developing economies based on IMF's definition.

Figure A.2: Sudden Stop Event Window: Only EME

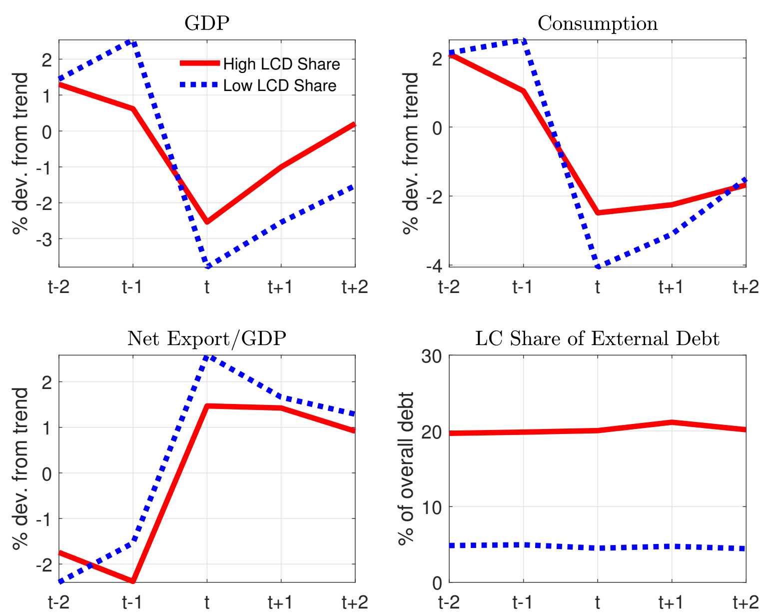
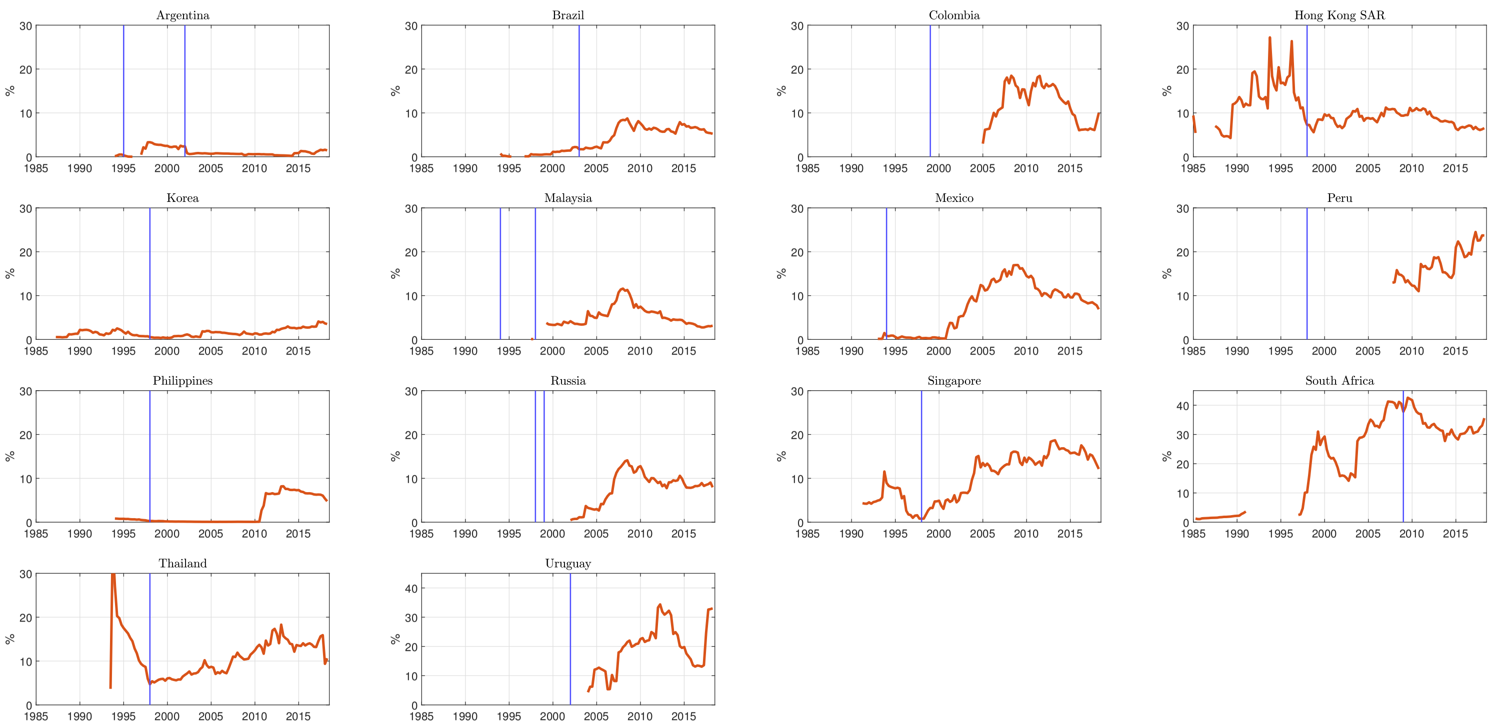


Figure A.3: Percentage of External Debt Denominated in Local Currency



NOTE: The figure plots the percentage of debt securities denominated in domestic currency for a group of sudden stop countries. The data are from the debt securities statistics by the Bank of International Settlement (BIS). We only consider the issuance in international markets. The vertical lines indicate the dates of sudden stop crises in each country.

B Proof of Propositions

B.1 Proof of Proposition 1

Proof. First, we solve for the capital control tax rates that restore the discretionary planner's allocations. In a tax-regulated economy, households' budget constraint is written as

$$c_{T,t} + p_t^N c_{N,t} + p_t^C b_t^C + b_t^T = y_{T,t} + p_t^N \overline{y}_N + q_t^C b_{t+1}^C \frac{1}{1+\tau_t^C} + q_t^T b_{t+1}^T \frac{1}{1+\tau_t^T} + T_t, \tag{B.1}$$

where T_t denotes the lump-sum transfers from the government. Taking derivatives on the bond issuance yields the first-order conditions as in equations (40)-(41). Comparing the Euler equations (40)-(41) with DP's optimality conditions (26)-(28), we can derive the following expressions of tax rates imposed on FCD and LCD respectively,

$$\tau_t^{T,DP} = \frac{\frac{\beta}{\mathcal{A}_t^{DP}} \mathbb{E}_t \left[u_T(t+1) + \mu_{t+1}\Psi_{t+1} \right] \left(\frac{1+\Xi_t}{1+\Xi_{t+1}} \right) + \mu_t(\Xi_t - \Psi_t)}{\frac{\beta}{q_t^T} \mathbb{E}_t u_T(t+1)} - 1, \tag{B.2}$$

$$\tau_t^{C,DP} = \frac{\frac{\beta}{\mathcal{B}_t^{DP}} \mathbb{E}_t \left[u_T(t+1) + \mu_{t+1}\Psi_{t+1} \right] p_{t+1}^C \left(\frac{1+\Xi_t}{1+\Xi_{t+1}} \right) + \mu_t(\Xi_t - \Psi_t)}{\frac{\beta}{q_t^C} \mathbb{E}_t u_T(t+1) p_{t+1}^C} - 1, \tag{B.3}$$

with the following auxiliary variables

$$\Psi_t = \kappa \overline{y}_N \frac{\partial p_t^N}{\partial c_{T,t}}, \qquad \Xi_t = b_t^C \frac{\partial p_t^C}{\partial c_{T,t}},$$

$$\mathcal{A}_t^{DP} = q_t^T + \frac{\partial q_t^C}{\partial b_{t+1}^T} b_{t+1}^C, \qquad \mathcal{B}_t^{DP} = q_t^C + \frac{\partial q_t^C}{\partial b_{t+1}^C} b_{t+1}^C.$$

Ψ_t captures the pecuniary externality through p^N in the collateral constraint. Ξ_t represents the effect of consumption on the fluctuations of p^C in the debt burden. The last two terms in $\mathcal{A}_t^{DP}$ and $\mathcal{B}_t^{DP}$ indicate that the domestic bond price (q^C) is elastic to agents' borrowing and portfolio decisions.

Similarly, we can back out the capital control tax rates that restore the allocations in the commitment planner's problem. Still, comparing the regulated CE's bond Euler equations (40)-(41) with CP's optimality conditions (37)-(39) yields the following state-contingent tax rates on

FCD and LCD respectively,

$$\tau_t^{T,CP} = \frac{\frac{\beta}{q_t^T}\mathbb{E}_t\left[u_T(t+1)+\mu_{t+1}\Psi_{t+1}+h_{t+1}\right]\left(\frac{1+\Xi_t}{1+\Xi_{t+1}}\right)-h_t+\mu_t(\Xi_t-\Psi_t)}{\frac{\beta}{q_t^T}\mathbb{E}_t u_T(t+1)} - 1, \tag{B.4}$$

$$\tau_t^{C,CP} = \frac{\frac{\beta}{q_t^C}\mathbb{E}_t\left[u_T(t+1)+\mu_{t+1}\Psi_{t+1}+h_{t+1}\right]p_{t+1}^C\left(\frac{1+\Xi_t}{1+\Xi_{t+1}}\right)-h_t+\mu_t(\Xi_t-\Psi_t)}{\frac{\beta}{q_t^C}\mathbb{E}_t u_T(t+1)p_{t+1}^C} - 1, \tag{B.5}$$

where the auxiliary variables are

$$\Psi_t = \kappa\bar{y}_N\frac{\partial p_t^N}{\partial c_{T,t}}, \qquad \Xi_t = b_t^C\frac{\partial p_t^C}{\partial c_{T,t}},$$
$$h_t = (\lambda_{t-1}^{CP}-\mu_{t-1}^{CP})b_t^C\frac{\partial p_t^C}{\partial c_{T,t}}\mathcal{M}(s_{t-1},s_t)\frac{1}{\beta}.$$

Similarly, Ψ_t shows the pecuniary externality through the collateral constraint, and Ξ_t indicates the effect of consumption on debt burden fluctuations. h_t is an auxiliary state that keeps track of the committed consumption plan made in the previous periods. Meanwhile, the commitment planner's current decision in period t determines h_{t+1}, which is the planner's state variable in the next period. □

B.2 Proof of Proposition 2

Proof. In the discretionary planner's second-period problem, the state-contingent consumption and interest rate are given by,

$$c_{T,2}^H = \frac{\bar{y}^T+(R^*-1)y_{T,2}^H-(R^*-1)(1-\delta_2)\bar{I}R_2^*}{\varphi_2+R^*-1+(1-\frac{1}{R^*})\delta_2\bar{I}R_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}} \equiv \mathcal{C}^H(R_2,\delta_2,\varphi_2), \tag{B.6}$$

$$c_{T,2}^L = \frac{(1+\kappa)y_{T,2}^L-(1-\delta_2)\bar{I}R_2^*}{1-(\kappa\bar{y}_N-\delta_2\bar{I}R_2)\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}} \equiv \mathcal{C}^L(R_2,\delta_2), \tag{B.7}$$

$$R_2 = \frac{1}{(1-p)\mathcal{M}_2^H\frac{1-\omega}{\omega}\frac{c_{T,2}^H}{\bar{y}_N}+p\mathcal{M}_2^L\frac{1-\omega}{\omega}\frac{c_{T,2}^L}{\bar{y}_N}} \equiv \mathcal{R}(c_{T,2}^H,c_{T,2}^L). \tag{B.8}$$

with the following auxiliary variables

$$\varphi_2 = \varphi(R_2,\delta_2) = [1+\phi(R_2,\delta_2)]^{\frac{1}{1-(-\sigma+1)\omega}}, \qquad \phi(R_2,\delta_2) = \delta_2\bar{I}R_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}. \tag{B.9}$$

$\phi(R_2,\delta_2)$ captures the balance sheet effect of consumption fluctuation. φ_2 is greater than 1 whenever $\delta_2>0$. The equation (B.6) indicates that the DP internalizes the effect of consumption fluctuation on the debt burden due to the presence of LCD. We also find that only in the high-income state consumption depends on φ_2. Because $\varphi_2 > 1$, compared with CE's equilibrium, c_2^H is less sensitive to the income shock realization y_2^H.

From equations (B.6)-(B.8), it is straightforward that the following relations hold with relevant parameter values:

$$\frac{\partial \mathcal{C}^H(R_2, \delta_2, \varphi_2)}{\partial R_2} < 0, \text{ for any fixed } \varphi_2, \qquad \frac{\partial \mathcal{C}^L(R_2, \delta_2)}{\partial R_2} < 0, \tag{B.10}$$

$$\frac{\partial \mathcal{R}(c_{T,2}^H, c_{T,2}^L)}{\partial c_{T,2}^H} < 0, \qquad \frac{\partial \mathcal{R}(c_{T,2}^H, c_{T,2}^L)}{\partial c_{T,2}^L} < 0, \tag{B.11}$$

$$\frac{\partial \varphi(R_2, \delta_2)}{\partial R_2} > 0, \qquad \frac{\partial \mathcal{C}^H(R_2, \delta_2, \varphi_2)}{\partial \varphi_2} < 0. \tag{B.12}$$

These inequalities indicate that a higher interest rate increases the debt burden and reduces consumption. The lowered consumption further increases the interest rate. Apart from that, the DP's balance sheet externality (φ_2) augments this feedback mechanism.

For each portfolio share (δ_2), the DP's second-period problem can be solved by bringing equations (B.6)-(B.7) into (B.8). The problem is reduced to a single equation with an unknown interest rate,

$$R_2^{DP} = \mathcal{R}\Big(\mathcal{C}^H\left(R_2^{DP}, \delta_2, \varphi(R_2^{DP}, \delta_2)\right),\ \mathcal{C}^L(R_2^{DP}, \delta_2)\Big). \tag{B.13}$$

The root of this equation is the DP's equilibrium interest rate in the second period. Similarly, by setting $\varphi(R_2^{DP}, \delta_2) = 1$, the solution of the CE's second-period problem is given by

$$R_2^{CE} = \mathcal{R}\Big(\mathcal{C}^H\left(R_2^{CE}, \delta_2, 1\right),\ \mathcal{C}^L(R_2^{CE}, \delta_2)\Big). \tag{B.14}$$

Now, we use the above properties in (B.10)-(B.12) to prove that for each $\delta_2 > 0$, the solution of equation (B.13) is larger than the solution of equation (B.14). First, we define $\tilde{R}_2(\varphi_2)$ as the solution to the following auxiliary problem,

$$\tilde{R}_2 = \mathcal{R}(\mathcal{C}^H(\tilde{R}_2, \delta_2, \varphi_2), \mathcal{C}^L(\tilde{R}_2, \delta_2)). \tag{B.15}$$

for any value of $\varphi_2 > 1$. Then, in order to show $R_2^{DP} > R_2^{CE}$, it suffices to show that for any $\varphi_2 > 1$, we have $\frac{\partial \tilde{R}_2}{\partial \varphi_2} > 0$.

Taking derivatives with respect to φ_2 in equation (B.15) yields the following,

$$\frac{\partial \tilde{R}_2}{\partial \varphi_2} = \frac{\partial \mathcal{R}}{\partial c_{T,2}^H}\left[\frac{\partial \mathcal{C}^H}{\partial R_2}\frac{\partial \tilde{R}_2}{\partial \varphi_2} + \frac{\partial \mathcal{C}^H}{\partial \varphi_2}\right] + \frac{\partial \mathcal{R}}{\partial c_{T,2}^L}\frac{\partial \mathcal{C}^L}{\partial R_2}\frac{\partial \tilde{R}_2}{\partial \varphi_2}.$$

Then,

$$\frac{\partial \tilde{R}_2}{\partial \varphi_2}\left[1-\frac{\partial \mathcal{R}}{\partial c^H_{T,2}}\frac{\partial \mathcal{C}^H}{\partial R_2}-\frac{\partial \mathcal{R}}{\partial c^L_{T,2}}\frac{\partial \mathcal{C}^L}{\partial R_2}\right]=\frac{\partial \mathcal{R}}{\partial c^H_{T,2}}\frac{\partial \mathcal{C}^H}{\partial \varphi_2}. \tag{B.16}$$

The fact that DP problem has a unique solution requires the regularity condition: $\frac{\partial \mathcal{R}}{\partial c^H_{T,2}}\frac{\partial \mathcal{C}^H}{\partial R_2}+\frac{\partial \mathcal{R}}{\partial c^L_{T,2}}\frac{\partial \mathcal{C}^L}{\partial R_2}<1$ at the equilibrium interest rate. The condition states that the sensitivity of consumption to the interest rate is not strong enough to generate two equilibrium interest rates. Explicitly, the condition can be expressed as the following,

$$\left(R_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}\right)^2\delta_2\bar{I}\Bigg[(1-p)\mathcal{M}^H_2 c^H_{T,2}\left(1-\frac{1}{R^*}\right)\frac{1}{\varphi_2+R^*-1+(1-\frac{1}{R^*})\delta_2\bar{I}R_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}$$
$$+p\mathcal{M}^L_2 c^L_{T,2}\frac{1}{1-(\kappa\bar{y}_N-\delta_2\bar{I}R_2)\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}\Bigg]<1.$$

A sufficient condition for this to hold is that the local currency share (δ_2) is not too large.

Besides, from the functional forms, we know that $\frac{\partial \mathcal{R}}{\partial c^H_{T,2}}\frac{\partial \mathcal{C}^H}{\partial \varphi_2}>0$. Hence, equation (B.16) implies that $\frac{\partial \tilde{R}_2}{\partial \varphi_2}>0$ for $\varphi_2>1$. The result indicates that a stronger balance sheet effect (larger φ_2) can increase the equilibrium interest rate on LCD in the problem of DP.

On the other hand, a higher interest rate further augments the balance sheet externality in DP's problem because $\frac{\partial \varphi(R_2,\delta_2)}{\partial R_2}>0$. This feedback effect further pushes φ_2 above 1. Therefore, for each $\delta_2>0$, the DP's equilibrium interest rate (the solution to equation B.13) must be strictly higher than the CE's equilibrium interest rate (the solution to equation B.14). Ultimately, we have $R_2^{DP}>R_2^{CE}$ for each $\delta_2>0$.

Based on the form of consumption functions $\mathcal{C}^H(R_2,\delta_2,\varphi_2)$, $\mathcal{C}^L(R_2,\delta_2)$ in equations (B.6) and (B.7), the higher equilibrium interest rate ($R_2^{DP}>R_2^{CE}$) and the associated stronger balance sheet externality ($\varphi_2>1$) for the DP imply the lower consumption in both states compared to the allocations of CE; that is, $c^{H,DP}_{T,2}<c^{H,CE}_{T,2}$ and $c^{L,DP}_{T,2}<c^{L,CE}_{T,2}$.

Thus, we have proved that for any $\delta_2>0$, DP has obtained lower consumption and a higher domestic interest rate than under the CE. □

B.3 Proof of Proposition 3

Proof. First, we prove that CP can achieve the same second-period allocations as in the CE problem at a given δ_2. We denote the CP's solution as $\{\tilde{c}^L_{T,2},\ \tilde{c}^H_{T,2},\ \tilde{c}^L_{T,3},\ \tilde{c}^H_{T,3},\ \tilde{R}_2,\ \tilde{\lambda}^L_2,\ \tilde{\lambda}^H_2,\ \tilde{\mu}^L_2,\ \tilde{\delta}_2\}$. The necessary condition is that the solution must satisfy the following first-order conditions,

$$\tilde{c}^L_{T,2}: \quad \tilde{c}^L_{T,2} = \frac{(1+\kappa)y^L_{T,2} - (1-\tilde{\delta}_2)\bar{I}R^*_2}{1-(\kappa\bar{y}_N - \tilde{\delta}_2\bar{I}\tilde{R}_2)\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}, \tag{B.17}$$

$$\begin{aligned}\tilde{\lambda}^L_2: \quad & pu_T(\tilde{c}^L_{T,2}) + p\tilde{\mu}^L_2\kappa\frac{1-\omega}{\omega} - p\tilde{\lambda}^L_2\left(1+\tilde{\delta}_2\bar{I}\tilde{R}_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}\right) \\ & - \left[p\tilde{\lambda}^L_2\tilde{c}^L_{T,2} + (1-p)\tilde{\lambda}^H_2\tilde{c}^H_{T,2}\right]\tilde{\delta}_2\bar{I}\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}\frac{\partial\mathcal{R}(c^H_{T,2},c^L_{T,2})}{\partial c^L_{T,2}} = 0,\end{aligned} \tag{B.18}$$

$$\tilde{\mu}^L_2: \quad \tilde{\lambda}^L_2 - \tilde{\mu}^L_2 = \tilde{\lambda}^L_3 = u_T(\tilde{c}^L_{T,3}), \tag{B.19}$$

$$\begin{aligned}\tilde{\lambda}^H_2: \quad & (1-p)u_T(\tilde{c}^H_{T,2}) - (1-p)\tilde{\lambda}^H_2\left(1+\tilde{\delta}_2\bar{I}\tilde{R}_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}\right) \\ & - \left[(1-p)\tilde{\lambda}^H_2\tilde{c}^H_{T,2} + p\tilde{\lambda}^L_2\tilde{c}^L_{T,2}\right]\tilde{\delta}_2\bar{I}\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}\frac{\partial\mathcal{R}(c^H_{T,2},c^L_{T,2})}{\partial c^H_{T,2}} = 0,\end{aligned} \tag{B.20}$$

$$\tilde{c}^H_{T,2}: \quad \tilde{\lambda}^H_2 = u_T(\tilde{c}^H_{T,3}), \tag{B.21}$$

$$\tilde{R}_2: \quad \tilde{R}_2 = \mathcal{R}(\tilde{c}^H_{T,2}, \tilde{c}^L_{T,2}) = \frac{1}{\mathbb{E}\mathcal{M}_2\tilde{p}^N_2}, \tag{B.22}$$

$$\tilde{\delta}_2: \quad \frac{R^*_2}{\tilde{R}_2}\mathbb{E}\tilde{\lambda}_2 = \mathbb{E}\left[\tilde{\lambda}_2\tilde{p}^N_2\right], \tag{B.23}$$

together with the budget constraint, collateral constraint, and the definition of exchange rate; that is $\tilde{p}^{N,H}_2 = \frac{1-\omega}{\omega}\frac{\tilde{c}^H_{T,2}}{\bar{y}_N}$, $\tilde{p}^{N,L}_2 = \frac{1-\omega}{\omega}\frac{\tilde{c}^L_{T,2}}{\bar{y}_N}$.

The derivatives to the domestic interest rate are expressed as

$$\frac{\partial\mathcal{R}(c^H_{T,2},c^L_{T,2})}{\partial c^L_{T,2}} = -\frac{p\mathcal{M}^L_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}{\left((1-p)\mathcal{M}^H_2\frac{1-\omega}{\omega}\frac{c^H_{T,2}}{\bar{y}_N} + p\mathcal{M}^L_2\frac{1-\omega}{\omega}\frac{c^L_{T,2}}{\bar{y}_N}\right)^2}, \tag{B.24}$$

$$\frac{\partial\mathcal{R}(c^H_{T,2},c^L_{T,2})}{\partial c^H_{T,2}} = -\frac{(1-p)\mathcal{M}^H_2\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}}{\left((1-p)\mathcal{M}^H_2\frac{1-\omega}{\omega}\frac{c^H_{T,2}}{\bar{y}_N} + p\mathcal{M}^L_2\frac{1-\omega}{\omega}\frac{c^L_{T,2}}{\bar{y}_N}\right)^2}. \tag{B.25}$$

Taking the expressions of (B.24) and (B.25) into equations (B.18) and (B.20), and under the assumption that $\tilde{\lambda}^H_2 = \theta^*\mathcal{M}^H_2$ and $\tilde{\lambda}^L_2 = \theta^*\mathcal{M}^L_2$, we have the following:

$$u_T(\tilde{c}^L_{T,2}) + \tilde{\mu}^L_2\kappa\frac{1-\omega}{\omega} - \tilde{\lambda}^L_2 = 0, \tag{B.26}$$

$$u_T(\tilde{c}^H_{T,2}) - \tilde{\lambda}^H_2 = 0. \tag{B.27}$$

Therefore, at any portfolio choice $\tilde{\delta}_2$, the CP's second-period solution $\{\tilde{\lambda}^L_2, \tilde{\lambda}^H_2, \tilde{\mu}_2, \tilde{c}^L_{T,2}, \tilde{c}^H_{T,2}, \tilde{R}_2\}$ can be characterized by the equation system (B.17), (B.19), (B.21), (B.22), (B.26), (B.27). Since equations (B.17), (B.21), (B.22), (B.27) take the same form as in the CE's equilibrium, the two equilibria must have the same consumption and interest rate. Specifically, at CP's optimal portfolio share $\tilde{\delta}_2$, we must have

$$\tilde{c}^H_{T,2} = c^H_{T,2}(\tilde{\delta}_2), \quad \tilde{c}^L_{T,2} = c^L_{T,2}(\tilde{\delta}_2), \quad \tilde{R}_2 = R_2(\tilde{\delta}_2), \quad \tilde{\lambda}^H_2 = \lambda^H_2(\tilde{\delta}_2).$$

Because agents can smooth consumption since the second period, the above result also implies $\tilde{c}^H_{T,3} = c^H_{T,3}(\tilde{\delta}_2), \quad \tilde{c}^L_{T,3} = c^L_{T,3}(\tilde{\delta}_2)$. Next, the Lagrange multipliers at the low-income state $\{\tilde{\lambda}^L_2, \tilde{\mu}^L_2\}$ can be backed out using equations (B.19) and (B.26). Compared to their expressions under the CE, we have the following

$$\tilde{\mu}^L_2(\tilde{\delta}_2) = \frac{u_T(\tilde{c}^L_{T,2}(\tilde{\delta}_2)) - u_T(\tilde{c}^L_{T,3}(\tilde{\delta}_2))}{1 - \kappa\frac{1-\omega}{\omega}} > u_T(c^L_{T,2}(\tilde{\delta}_2)) - u_T(c^L_{T,3}(\tilde{\delta}_2)) = \mu^L_2(\tilde{\delta}_2),$$

and

$$\tilde{\lambda}^L_2(\tilde{\delta}_2) = u_T(\tilde{c}^L_{T,2}(\tilde{\delta}_2)) + \tilde{\mu}^L_2(\tilde{\delta}_2)\kappa\frac{1-\omega}{\omega} > u_T(\tilde{c}^L_{T,2}(\tilde{\delta}_2)) = \lambda^L_2(\tilde{\delta}_2(\tilde{\delta}_2)).$$

It implies that even though CP's second-period solutions coincide with the ones under the CE, the CP has a higher valuation of wealth in the low-income state where financial constraint binds.

Next, we prove that under specified conditions, CP would choose a higher level of LCD relative to CE. Since $\tilde{\delta}_2$ is CP's optimal portfolio, by definition, equations (B.22)-(B.23) must hold at $\tilde{\delta}_2$. The Euler equation on the portfolio choice can be rewritten as

$$\frac{(1-p)\tilde{\lambda}^H_2\frac{1-\omega}{\omega}\frac{\tilde{c}^H_{T,2}}{\bar{y}_N} + p\tilde{\lambda}^L_2\frac{1-\omega}{\omega}\frac{\tilde{c}^L_{T,2}}{\bar{y}_N}}{(1-p)\tilde{\lambda}^H_2 + p\tilde{\lambda}^L_2} = \frac{\mathcal{M}^H_2\frac{1-\omega}{\omega}\frac{\tilde{c}^H_{T,2}}{\bar{y}_N} + p\mathcal{M}^L_2\frac{1-\omega}{\omega}\frac{\tilde{c}^L_{T,2}}{\bar{y}_N}}{(1-p)\mathcal{M}^H_2 + p\mathcal{M}^L_2}. \tag{B.28}$$

Since the financial constraint only binds in the low-income state, it must be the case that $\tilde{c}^H_{T,2} > \tilde{c}^L_{T,2}$.

Moreover, from the above analysis, we know that CP's second-period consumption schedules are the same as the ones under CE at $\tilde{\delta}_2$, but the CP's marginal valuation of wealth at the low-income state is higher than CE's; that is $\tilde{\lambda}^H_2 = \lambda^H_2(\tilde{\delta}_2)$ and $\tilde{\lambda}^L_2 > \lambda^L_2(\tilde{\delta}_2)$. Hence, based on equation (B.28), we can see following

$$\frac{(1-p)\lambda^H_2(\tilde{\delta}_2)\frac{1-\omega}{\omega}\frac{c^H_{T,2}(\tilde{\delta}_2)}{\bar{y}_N} + p\lambda^L_2(\tilde{\delta}_2)\frac{1-\omega}{\omega}\frac{c^L_{T,2}(\tilde{\delta}_2)}{\bar{y}_N}}{(1-p)\lambda^H_2(\tilde{\delta}_2) + p\lambda^L_2(\tilde{\delta}_2)} > \frac{\mathcal{M}^H_2\frac{1-\omega}{\omega}\frac{c^H_{T,2}(\tilde{\delta}_2)}{\bar{y}_N} + p\mathcal{M}^L_2\frac{1-\omega}{\omega}\frac{c^L_{T,2}(\tilde{\delta}_2)}{\bar{y}_N}}{(1-p)\mathcal{M}^H_2 + p\mathcal{M}^L_2}. \tag{B.29}$$

This inequality indicates that CE's first-period Euler equation satisfies: $\frac{\mathbb{E}[\lambda_2(\delta_2)p^N_2(\delta_2)]}{\mathbb{E}\lambda_2(\delta_2)} - \frac{R^*}{R_2(\delta_2)} > 0$ at CP's optimal portfolio $\tilde{\delta}_2$.

Because the function $\frac{\mathbb{E}[\lambda_2(\delta_2)p^N_2(\delta_2)]}{\mathbb{E}\lambda_2(\delta_2)} - \frac{R^*}{R_2(\delta_2)}$ is monotonically increasing in δ_2, and there exists an unique solution to the equation $\frac{\mathbb{E}[\lambda_2(\delta_2)p^N_2(\delta_2)]}{\mathbb{E}\lambda_2(\delta_2)} - \frac{R^*}{R_2(\delta_2)} = 0$ (given by δ^{CE}_2), we must have that $\delta^{CE}_2 < \tilde{\delta}_2$. That is to say, the CP tends to issue a larger fraction of debt in local currency than the agents under competitive equilibrium.

Figure D.1 (right panel) in appendix D.5 compares the equilibrium consumption schedules under CP and CE. We can see that LCD allows domestic households to share risk in the international market. When δ_2 increases, the consumption dispersion between high- and low-income states becomes smaller. The smaller consumption dispersion makes repaying LCD less risky from foreign lenders' perspective. The monotonicity condition implies that as the consumption risk declines, domestic agents' certainty equivalence on p_2^N fluctuation increases faster than the lenders' certainty equivalence. In other words, the proposition requires that the domestic agent is more sensitive to the exchange rate risk than the foreign lenders.

□

C Solution Methods

Since the model has occasionally-binding constraints, we use a global solution method with time iteration on Euler equations. More specifically, a set of decision rules is defined and approximated by policy functions on the discretized state space. For each iteration and at each grid point, we find the solution to a system of equations, and the decision rules are updated accordingly. We iterate on these policies until the difference between successive iterations is small enough.

To make the numerical solution more tractable and stable, we use a one-to-one mapping to reshape the state space in our model $\{(b^C \in \mathbb{R}, b^T \in \mathbb{R}) | b^C \geq 0\}$ into a new state space $\{(b \in \mathbb{R}, \delta \in \mathbb{R}) | \delta \geq 0\}$, where $b_t \equiv b_t^C + b_t^T$ is the total debt and $\delta_t \equiv b_t^C/(b_t^C + b_t^T)$ is the fraction of debt in local currency. Under this definition, $1 - \delta$ is the fraction of debt in foreign currency. Under the condition that the economy is a net borrower $b_t^C + b_t^T > 0$, such a transformation is a one-to-one mapping from the old to the new state space. After redefining the state space, all three problems (private agents' problem and social planners' problem) in this paper can be re-formulated on the new state space. The choice variables thus become $\{c_T, b', \delta'\}$ in the transformed problem, instead of $\{c_T, b^{C'}, b^{T'}\}$ in the original problem.

Similarly, we can rewrite the household's budget constraint, the collateral constraint, and the non-negativity constraint on LCD issuance based on the new state space,

$$c_{T,t} + p_t^N c_{N,t} + p_t^C b_t \delta_t + b_t(1 - \delta_t) = y_{T,t} + p_t^N \overline{y}_N + q_t^C b_{t+1}\delta_{t+1} + q_t^T b_{t+1}(1 - \delta_{t+1}), \tag{C.1}$$

$$q_t^C b_{t+1}\delta_{t+1} + q_t^T b_{t+1}(1 - \delta_{t+1}) \leq \kappa \left(y_{T,t} + p_t^N \overline{y}_N \right), \tag{C.2}$$

$$\delta_{t+1} \geq 0. \tag{C.3}$$

We follow Tauchen (1986) and discretize the tradable income process into 17 points with ± 2.5 standard deviations around its unconditional mean. We use an equidistant b grid, which has 39 points and is spaced over [0.3, 0.96]. The δ grid has 45 points and is spaced over [0, 1.9]. The h grid (state of prior commitment) has 27 points and is spaced over [0, 0.65]. To solve for the next-period policy values, we use linear interpolation to find the values of b, δ, and h that are not on the grids.

For the competitive equilibrium and discretionary planner's problem, the vector of transformed states is $\mathbf{S} = (b, \delta, y_T)$. For the commitment planner's problem, we need an additional state variable h to capture the commitment made in the previous periods. The extended state vector is $\tilde{\mathbf{S}} = (b, \delta, h, y_T)$. In all three environments, there are two occasionally binding constraints: (1) a collateral constraint and (2) a non-negativity constraint on LCD issuance: $\delta_{t+1} \geq 0$.

To deal with the occasionally binding constraints, we follow the method discussed in Zangwill & Garcia (1981) to transform the inequality constraints into equalities by adding an auxiliary variable. This method allows us to solve a system of equations with a Newton-type solver. Specifically, denote μ_t as the Lagrange multiplier associated with the collateral constraint. We define an auxiliary variable μ_t^* such that $\mu_t = \max\{\mu_t^*, 0\}^3$.[29] Then, the complementarity slackness conditions on the collateral constraint

$$
\begin{aligned}
&\mu_t \geq 0, \\
&\kappa(y_{T,t} + p_t^N \overline{y}_N) - \left[q_t^C b_{t+1}\delta_{t+1} + q_t^T b_{t+1}(1-\delta_{t+1})\right] \geq 0, \\
&\mu_t \bigg(\kappa(y_{T,t} + p_t^N \overline{y}_N) - \left(q_t^C b_{t+1}\delta_{t+1} + q_t^T b_{t+1}(1-\delta_{t+1})\right) \bigg) = 0,
\end{aligned}
$$

can be replaced by a single equation:

$$
\max\{-\mu_t^*, 0\}^3 = \kappa(y_{T,t} + p_t^N \overline{y}_N) - \left[q_t^C b_{t+1}\delta_{t+1} + q_t^T b_{t+1}(1-\delta_{t+1})\right]. \tag{C.4}
$$

Note that $\mu_t^* > 0$ if the collateral constraint binds, and $\mu_t^* < 0$ if the constraint is slack. So, the complementarity slackness conditions are indeed satisfied. We can replace the collateral constraints in all three problems with (C.4) and solve for $\mu_t^* \in \mathbb{R}$ instead of $\mu_t \geq 0$. After solving for μ_t^*, we can easily back out $\mu_t = \max\{\mu_t^*, 0\}^3 \geq 0$. The complementarity slackness conditions on the non-negativity constraint $\delta_{t+1} \geq 0$ are transformed in a similar way. Let η_t be the Lagrange multiplier on $\delta_{t+1} \geq 0$. We create an auxiliary variable η_t^* such that $\eta_t = \max\{\eta_t^*, 0\}^3$ and the following condition holds,

$$
\max\{-\eta_t^*, 0\}^3 = \delta_{t+1}. \tag{C.5}
$$

As a result, $\eta_t^* < 0$ if $\delta_{t+1} > 0$, while $\eta_t^* > 0$ if $\delta_{t+1} = 0$. So, we can replace the non-negativity constraint with (C.5) and solve for η_t^* instead of η_t and δ_{t+1}.

Next, we describe how we solved each equilibrium in detail.

[29] There is no consensus in the literature on what order of exponent to use. Zangwill & Garcia (1981) mention that people can use any order of any positive integer in the transformation. Benigno et al. (2013) and Benigno et al. (2016) use the square. Stepanchuk & Tsyrennikov (2015) mention that they solved their model using the exponent order of $\{2, 4, 6, \cdots\}$. Liu (2022), on the other hand, uses a cubic exponent. We use a cubic exponent since it works well for our numerical solutions.

C.1 Algorithm for Competitive Equilibrium

Under the set of constraints (C.1)-(C.3), the optimality conditions in the competitive equilibrium become

$$\lambda_t = u_T(t), \tag{C.6}$$

$$\lambda_t q_t^T = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t q_t^T, \tag{C.7}$$

$$\lambda_t q_t^C + \frac{\eta_t}{b_{t+1}} = \beta \mathbb{E}_t \lambda_{t+1} p_{t+1}^C + \mu_t q_t^C, \tag{C.8}$$

where λ_t, μ_t, and η_t are the Lagrange multipliers on equations (C.1), (C.2), and (C.3) respectively. The algorithm to solve the decentralized equilibrium is detailed as follows.

1. Make an initial guess on the set of decision rules over the state space $\mathbf{S}_t = (b_t, \delta_t, y_{T,t})$, which are written as $\mathbb{C}_0 = \Big\{ b'^0(\mathbf{S}_t),\ \delta'^0(\mathbf{S}_t),\ c_T^0(\mathbf{S}_t),\ p^{N,0}(\mathbf{S}_t),\ p^{C,0}(\mathbf{S}_t),\ q^{C,0}(\mathbf{S}_t),\ q^{T,0}(\mathbf{S}_t),\ \lambda^0(\mathbf{S}_t),$ $\eta^{*,0}(\mathbf{S}_t),\ \mu^{*,0}(\mathbf{S}_t) \Big\}$.

2. Set $j = 0$.

3. For a given j and decision rules $\mathbb{C}_j$, we use a Newton-type solver to solve for a new set of decision rules that satisfy the system of equations (7), (8), (11), (13), (14), (16), (17), (C.4), (C.5), (C.7), (C.8). The new set of decision rules are given by: $\mathbb{C}_{j+1} = \Big\{ b'^{j+1}(\mathbf{S}_t),\ \delta'^{j+1}(\mathbf{S}_t),$ $c_T^{j+1}(\mathbf{S}_t),\ p^{N,j+1}(\mathbf{S}_t),\ p^{C,j+1}(\mathbf{S}_t),\ q^{C,j+1}(\mathbf{S}_t),\ q^{T,j+1}(\mathbf{S}_t),\ \lambda^{j+1}(\mathbf{S}_t),\ \eta^{*,j+1}(\mathbf{S}_t),\ \mu^{*,j+1}(\mathbf{S}_t) \Big\}$.
   Note that, as described above, we replace the complementarity slackness conditions with equations (C.4)-(C.5) when solving for the new decision rules.

4. Compute the distance between the new policy functions and the old ones. Stop if $|\mathbb{C}_j - \mathbb{C}_{j+1}| < 1e-6$ for all the states. Otherwise, go back to step 3 with $j = j + 1$.

C.2 Algorithm for Discretionary Planner

Taking derivatives with respect to $c_{T,t}$, b_{t+1}, and δ_{t+1}, we have the following optimality conditions for the discretionary planner's problem,

$$\lambda_t^{DP}\left(1 + b_t \delta_t \frac{\partial p_t^C}{\partial c_{T,t}}\right) = u_T(t) + \mu_t^{DP} \kappa \overline{y}_N \frac{\partial p_t^N}{\partial c_{T,t}}, \tag{C.9}$$

$$\left(\lambda_t^{DP} - \mu_t^{DP}\right)\left[q_t^T + \frac{\partial q_t^C}{\partial b_{t+1}} b_{t+1}\delta_{t+1} - \frac{\partial q_t^C}{\partial \delta_{t+1}} {\delta_{t+1}}^2\right] = \beta \mathbb{E}_t \lambda_{t+1}^{DP}, \tag{C.10}$$

$$\left(\lambda_t^{DP} - \mu_t^{DP}\right)\left[q_t^C + \frac{\partial q_t^C}{\partial b_{t+1}} b_{t+1}\delta_{t+1} + \frac{\partial q_t^C}{\partial \delta_{t+1}} \delta_{t+1}(1-\delta_{t+1})\right] + \frac{\eta_t^{DP}}{b_{t+1}} = \beta \mathbb{E}_t \lambda_{t+1}^{DP} p_{t+1}^C, \tag{C.11}$$

where λ_t^{DP}, μ_t^{DP}, and η_t^{DP} are the corresponding Lagrange multipliers.

The main difference between the solutions of DP and CE is that we need to differentiate the local currency bond price with respect to planner's choices, which correspond to the terms $\partial q_t^C / \partial b_{t+1}$ and $\partial q_t^C / \partial \delta_{t+1}$ in equations (C.10)-(C.11). To compute DP's equilibrium, we use a nested fixed point algorithm similar to Bianchi & Mendoza (2018). Given the two derivative functions ($\mathcal{D}_1(b_{t+1}, \delta_{t+1}, y_{T,t}) \equiv \partial q_t^C / \partial b_{t+1}$ and $\mathcal{D}_2(b_{t+1}, \delta_{t+1}, y_{T,t}) \equiv \partial q_t^C / \partial \delta_{t+1}$), we solve for the discretionary planner's decisions using time iteration on Euler equations as an inner loop. In the outer loop, we update these two derivatives using the updated policy functions. The algorithm is detailed as follows.

1. Generate grid points on the choice space $\hat{\mathbf{S}} = (b', \delta', y_T)$. We use the same grid points as the state space points on $\mathbf{S}$. We use linear interpolation for the values of b' and δ' that are not on the state space grid.

2. Make an initial guess on the derivative functions $\mathcal{D}_1$ and $\mathcal{D}_2$ on the discretized choice space $\hat{\mathbf{S}}$. Our initial guess is that $\mathcal{D}_1^{guess}(\hat{\mathbf{S}}) = \mathcal{D}_2^{guess}(\hat{\mathbf{S}}) = 0$ for all (b', δ', y_T).

3. Given $\mathcal{D}_1^{guess}(\hat{\mathbf{S}})$ and $\mathcal{D}_2^{guess}(\hat{\mathbf{S}})$, we solve for the discretionary planner's policy functions $\mathbb{C}^{DP} = \left\{ b'(\mathbf{S}),\ \delta'(\mathbf{S}),\ c_T(\mathbf{S}),\ p^N(\mathbf{S}),\ p^C(\mathbf{S}),\ q^C(\mathbf{S}),\ q^T(\mathbf{S}),\ \lambda(\mathbf{S}),\ \eta^*(\mathbf{S}),\ \mu^*(\mathbf{S}) \right\}$ using the Euler equation iteration method. Specifically, for each iteration and at each grid point, we jointly solve a system of equations (19), (22), (23), (24), (25), (C.4), (C.5), (C.9), (C.10), (C.11). We iterate the policy functions until the distance between successive iterations is smaller than 1e-8.

4. Based on the updated policy functions, we calculate the endogenous bond price $q^C(\hat{\mathbf{S}})$ at each point of (b', δ', y_T). Then, we calculate the implied the derivative functions $\mathcal{D}_1^{implied}(\hat{\mathbf{S}})$ and $\mathcal{D}_2^{implied}(\hat{\mathbf{S}})$ at each (b', δ', y_T) using the numerical differentiation.

5. In the outer loop, if the maximum difference between the guessed and implied derivative functions of $\mathcal{D}_1$ and $\mathcal{D}_2$ is smaller than 1e-6 for all grid points on (b', δ', y_T), stop. Otherwise, update the guess using a convex combination. That is

$$\mathcal{D}_1^{guess} = (1-x)\mathcal{D}_1^{guess} + x\mathcal{D}_1^{implied},$$
$$\mathcal{D}_2^{guess} = (1-x)\mathcal{D}_2^{guess} + x\mathcal{D}_2^{implied},$$

where $x \in (0, 1)$. Then, go back to step 2.

C.3 Algorithm for Commitment Planner

For commitment planner, the optimality conditions with respect to $c_{T,t}$, b_{t+1}, and δ_{t+1} are written as

$$\lambda_t^{CP}\left(1+\frac{\partial p_t^C}{\partial c_{T,t}}b_t\delta_t\right)=u_T(t)+\mu_t^{CP}\kappa\bar{y}_N\frac{\partial p_t^N}{\partial c_{T,t}}+(\lambda_{t-1}^{CP}-\mu_{t-1}^{CP})b_t\delta_t\mathcal{M}(s_{t-1},s_t)\frac{\partial p_t^C}{\partial c_{T,t}}\frac{1}{\beta}, \tag{C.12}$$

$$\left(\lambda_t^{CP}-\mu_t^{CP}\right)q_t^T=\beta\mathbb{E}_t\lambda_{t+1}^{CP}, \tag{C.13}$$

$$\left(\lambda_t^{CP}-\mu_t^{CP}\right)q_t^C+\frac{\eta_t^{CP}}{b_{t+1}}=\beta\mathbb{E}_t\lambda_{t+1}^{CP}p_{t+1}^C. \tag{C.14}$$

The last term in (C.12) captures the fact that the period t planner internalizes the effects of its consumption $c_{T,t}$ on the period $t-1$ bond price. Then, we define an auxiliary state variable h_t that summarizes the commitment made in the period periods,

$$h_t\equiv(\lambda_{t-1}-\mu_{t-1})b_t\delta_t\mathcal{M}(y_{T,t-1},y_{T,t})\frac{\partial p_t^C}{\partial c_{T,t}}\frac{1}{\beta}.$$

Then, the social marginal value of wealth can be expressed as

$$\lambda_t^{CP}=\frac{u_T(t)+\mu_t^{CP}\Psi_t+h_t}{1+\Xi_t},$$

where $\Psi_t=\kappa y_{N,t}\frac{\partial p_t^N}{\partial c_{T,t}}$ and $\Xi_t=\frac{\partial p_t^C}{\partial c_{T,t}}b_t\delta_t$ represent the externalities on the collateral and budget constraints.

The commitment planner's decision in period t also influences the auxiliary h_{t+1} for the next-period problem, which is given by

$$h_{t+1}=(\lambda_t-\mu_t)b_{t+1}\delta_{t+1}\mathcal{M}(y_{T,t},y_{T,t+1})\frac{\partial p_{t+1}^C}{\partial c_{T,t+1}}\frac{1}{\beta}. \tag{C.15}$$

As shown in equation (C.15), h_{t+1} depends on the realization of period $t+1$ tradable income $y_{T,t+1}$. Taking into the CP's policy functions, h_{t+1} is an implicit function of b_t, δ_t, h_t, $y_{T,t}$, and $y_{T,t+1}$. When we solve for CP's decisions, we need the law of motion of "h"; that is written as

$$h'=\mathbf{H}(b,\delta,h,y_T,y_T'). \tag{C.16}$$

Denote $\tilde{\mathbf{S}}=(b,\delta,h,y_T)$ as the extended state vector for CP's problem. The algorithm is detailed as follows.

1. Make an initial guess on policy functions $\mathbb{C}_0^{CP}=\Big\{c_T^0(\tilde{\mathbf{S}}),\ b'^0(\tilde{\mathcal{S}}),\ \delta'^0(\tilde{\mathbf{S}}),\ q^{T,0}(\tilde{\mathbf{S}}),\ q^{C,0}(\tilde{\mathcal{S}}),$ $\lambda^0(\tilde{\mathbf{S}}),\ \eta^{*,0}(\tilde{\mathbf{S}}),\ \mu^{*,0}(\tilde{\mathbf{S}}),\ p^{N,0}(\tilde{\mathbf{S}}),\ p^{C,0}(\tilde{\mathbf{S}}),\ h'^0(\tilde{\mathbf{S}},y_T')\Big\}$. We use the CE's policy functions as the initial guess, and assume $h'^0(\tilde{\mathcal{S}},y_T')=0$ for all states.

2. Set $j = 0$.

3. For a given j and policy functions functions $\mathbb{C}_j^{CP}$, we use the non-linear solver to solve for $\mathbb{C}^{CP,j+1} = \Big\{ c_T^{j+1}(\tilde{\mathbf{S}}),\ b'^{j+1}(\tilde{\mathbf{S}}),\ \delta'^{j+1}(\tilde{\mathbf{S}}),\ q^{T,j+1}(\tilde{\mathbf{S}}),\ q^{C,j+1}(\tilde{\mathbf{S}}),\ \lambda^{j+1}(\tilde{\mathbf{S}}),\ \eta^{*j+1}(\tilde{\mathbf{S}}),\ \mu^{*j+1}(\tilde{\mathbf{S}}),$ $p^{N,j+1}(\tilde{\mathbf{S}}),\ p^{C,j+1}(\tilde{\mathbf{S}}) \Big\}$ that satisfy equilibrium conditions: (30), (33), (34), (35), (36), (C.4), (C.5), (C.12), (C.13), (C.14). Simultaneously, we obtain the function of auxiliary state $h'^{j+1}(\tilde{\mathbf{S}}, y'_T)$ from equation (C.15).

   One step that deserves a discussion is that, while searching for the b' and δ' that satisfy the equilibrium conditions, we need to know h' to form an expectation on the next-period variables. For example, we need to know the next-period tradable consumption $c_T(b', \delta', h', y'_T)$ to calculate the expected marginal utility in the Euler equations. For each $(b', \delta', y'_T, b, \delta, h, y_T)$, we use the following steps to find the corresponding h':

   (a) We first make a guess of h', which we refer to as h'_{guess}.

   (b) By plugging h'_{guess} into the right hand side of equation (C.15) and using the j-step guessed policy functions $c_T^j(b', \delta', h'_{guess}, y'_T)$ to calculate the term $\partial p^{C'}/\partial c'_T$, we can calculate an implied value of h', which we denote as $h'_{implied}$.

   (c) If $|h'_{implied} - h'_{guess}| < 1e{-}10$, we have found the h' that corresponds to $(b', \delta', y'_T, b, \delta, h, y_T)$. Otherwise, we revise our guess h'_{guess} and calculate again.

4. After finding the policy functions at the $\mathbb{C}_{j+1}^{CP}$ in the $j+1$ step, check whether the distance between successive iterations is small enough. If $|\mathbb{C}_j^{CP} - \mathbb{C}_{j+1}^{CP}| < 1e - 6$, we have found an equilibrium. Otherwise, make $j = j + 1$ and repeat from step 3.

D Details for the Simplified Model

D.1 First-Order Conditions for the CE

We describe the problem in a backward way. Since the second period, the private agents make consumption-borrowing decisions: $\{c_{T,t}, c_{N,t}, b_{t+1}\}_{t=2}^{\infty}$ to maximize their life-time utilities, with the constraints given by equations (42)-(44). The second-period values are

$$V_2^{CE}(\delta_2, y_{T,2}) = \max_{b_{t+1}, t \geq 2} \left\{ \sum_{t=2}^{\infty} \beta^{t-2} u(c_{T,t}, \bar{y}_N) \right\}, \qquad \text{s.t.} \quad \text{equations (42)-(44).} \tag{D.1}$$

The Euler equations are

$$\lambda_t = u_T(c_{T,t}), \quad t \geq 2 \tag{D.2}$$

$$\lambda_2 - \mu_2 = \lambda_3, \tag{D.3}$$

$$\lambda_t = \lambda_{t+1}, \quad t \geq 3 \tag{D.4}$$

$$p_t^N = \frac{1-\omega}{\omega}\frac{c_{T,t}}{\bar{y}_N}, \quad t \geq 2, \tag{D.5}$$

where λ_t is the private sector's marginal value of wealth. μ_2 is the Lagrange multiplier on the second-period financial constraint, which only binds in low state; that is $\mu_2^H = 0$ and $\mu_2^L > 0$. The solution is characterized by equations (D.2)-(D.5) together with the resource and collateral constraints:

$$c_{T,2} + (1-\delta_2)\bar{I}R_2^* + \delta_2\bar{I}R_2p_2^N = \frac{1}{R^*}b_3 + y_{T,2}, \tag{D.6}$$

$$\frac{1}{R^*}b_3 \leq \kappa(y_{T,2} + p_2^N\bar{y}_N), \tag{D.7}$$

$$c_{T,t} + b_t = \frac{1}{R^*}b_{t+1} + y_{T,t}, \quad t \geq 3. \tag{D.8}$$

In the first period, the agent chooses the fraction of debt denominated in local currency to maximize her life-time utility:

$$V_1^{CE} = \max_{\delta_2}\left[(1-p)V_2^{CE}(\delta_2, y_{T,2}^H) + pV_2^{CE}(\delta_2, y_{T,2}^L)\right], \tag{D.9}$$

The Euler equation with respect to δ_2 is

$$\frac{R_2^*}{R_2}\mathbb{E}\lambda_2 = \mathbb{E}\left[\lambda_2 p_2^N\right]. \tag{D.10}$$

The bond-pricing equations from foreign lenders' problem are

$$\frac{1}{R_2} = \mathbb{E}[\mathcal{M}_2 p_2^N], \qquad \frac{1}{R_2^*} = \mathbb{E}\mathcal{M}_2. \tag{D.11}$$

The equilibrium solution consists of $\{c_{T,2}^{H,CE}, c_{T,2}^{L,CE}, b_3^{H,CE}, b_3^{L,CE}, \mu_2^{L,CE}, \lambda_2^{H,CE}, \lambda_2^{L,CE}, \delta_2^{CE}, R_2^{CE}\}$.

D.2 Discretionary Planner's Problem (DP)

The social planner internalizes the effect of her portfolio decision on the exchange rate fluctuations but cannot commit to future policies. In the first period, the DP decides on the proportion of LCD,

$$V_1^{DP} = \max_{\delta_2}\left[(1-p)V_2^{DP}(\delta_2, y_{T,2}^H) + pV_2^{DP}(\delta_2, y_{T,2}^L)\right], \tag{D.12}$$

where $V_2(\delta_2, y_{T,2})$ denotes the value function in the second period. Meanwhile, the interest rate on LCD (R_2) is contracted in ex ante between the borrower and foreign lenders.

In the second period, the social planner solves the following problem while taking R_2 and δ_2 as given,

$$\begin{aligned} V_2^{DP}(\delta_2, y_{T,2}) = & \max_{b_3} \left\{ u(c_{T,2}, \bar{y}_N) + \beta V_3(b_3) \right\}, \\ \text{s.t.} \quad \lambda_2^{DP}: \quad & c_{T,2} + (1-\delta_2)\bar{I}R_2^* + \delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{c_{T,2}}{\bar{y}_N} = \frac{1}{R^*}b_3 + y_{T,2}, \\ \mu_2^{DP}: \quad & \frac{1}{R^*}b_3 \le \kappa\left(y_{T,2} + \frac{1-\omega}{\omega}c_{T,2}\right), \end{aligned} \tag{D.13}$$

where we use the expression of p_2^N in the resource and the financial constraints. The continuing value in the third period can be easily derived as $V_3(b_3) = \frac{1}{1-\beta}u(c_{T,3}(b_3), \bar{y}_N)$, where the consumption is $c_{T,3}(b_3) = \bar{y}_T - \left(1 - \frac{1}{R^*}\right) b_3$.

The model can be solved recursively from the second period. Taking derivatives with respect to $c_{T,2}$, $c_{T,3}$, and b_3 yields the following optimality conditions,

$$\lambda_2^{DP} = \frac{u_T(c_{T,2}) + \mu_2^{DP}\kappa\frac{1-\omega}{\omega}}{1+\phi(R_2,\delta_2)}, \tag{D.14}$$

$$\frac{u_T(c_{T,2}) + \mu_2^{DP}\kappa\frac{1-\omega}{\omega}}{1+\phi(R_2,\delta_2)} - \mu_2^{DP} = u_T(c_{T,3}). \tag{D.15}$$

The term $\phi(R_2, \delta_2) = \delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}$ indicates that the planner internalizes the balance sheet effect of changing consumption on the debt burden.

In the first period, taking derivative with respect to δ_2 yields the following portfolio Euler equation,

$$\frac{R_2^*}{R_2}\mathbb{E}\lambda_2^{DP} = \mathbb{E}\left[\lambda_2^{DP} p_2^N\right]\left(1 + \frac{\delta_2}{R_2}\frac{\partial R_2(\delta_2)}{\partial \delta_2}\right). \tag{D.16}$$

The DP's equilibrium solution consists of $\{c_{T,2}^{H,DP}$, $c_{T,2}^{L,DP}$, $b_3^{H,DP}$, $b_3^{L,DP}$, $\mu_2^{L,DP}$, $\lambda_2^{H,DP}$, $\lambda_2^{L,DP}$, δ_2^{DP}, $R_2^{DP}\}$.

D.3 Commitment Planner's Problem (CP)

In the first period, the social planner under commitment makes a portfolio decision (δ_2), and at the same time, commit to consumption profiles in the second period ($c_{T,2}^H$, $c_{T,2}^L$). In the second period, the government chooses certain borrowing levels (b_3^H, b_3^L) that satisfy the budget constraint and collateral constraint taking the pre-committed consumption as given. The CP's global optimization problem is

$$V_1^{CP} = \max_{\delta_2, c_{T,2}^H, c_{T,2}^L, b_3^H, b_3^L} \left\{ (1-p)\left[u(c_{T,2}^H, \bar{y}_N) + \beta V_3(b_3^H)\right] + p\left[u(c_{T,2}^L, \bar{y}_N) + \beta V_3(b_3^L)\right] \right\}, \tag{D.17}$$

$$\begin{aligned}
\text{s.t.} \quad \lambda_2^{H,CP}: &\quad c_{T,2}^H + (1-\delta_2)\bar{I}R_2^* + \delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{c_{T,2}^H}{\bar{y}_N} = \frac{1}{R^*} b_3^H + y_{T,2}^H, \\
\lambda_2^{L,CP}: &\quad c_{T,2}^L + (1-\delta_2)\bar{I}R_2^* + \delta_2 \bar{I} R_2 \frac{1-\omega}{\omega}\frac{c_{T,2}^L}{\bar{y}_N} = \frac{1}{R^*} b_3^L + y_{T,2}^L, \\
\mu_2^{CP}: &\quad \frac{1}{R^*} b_3^L = \kappa \left(y_2^L + \frac{1-\omega}{\omega} c_{T,2}^L \right), \\
&\quad R_2 = \frac{1}{(1-p)\mathcal{M}_2^H \frac{1-\omega}{\omega}\frac{c_{T,2}^H}{\bar{y}_N} + p\mathcal{M}_2^L \frac{1-\omega}{\omega}\frac{c_{T,2}^L}{\bar{y}_N}}.
\end{aligned}$$

The first-order conditions of this problem is given by equations (B.17)-(B.22) in the proof of proposition 3 in appendix B. The CP's equilibrium solution consists of $\{c_{T,2}^{L,CP}$, $c_{T,2}^{H,CP}$, $\lambda_2^{L,CP}$, $\lambda_2^{H,CP}$, $\mu_2^{L,CP}$, R_2^{CP}, $\delta_2^{CP}\}$.

Among the optimality conditions, the equation that determines the portfolio choice is

$$\frac{R_2^*}{R_2}\mathbb{E}\lambda_2^{CP} = \mathbb{E}\left[\lambda_2^{CP} p_2^N\right]. \tag{D.18}$$

D.4 Capital Control Measures

The allocations of social planners can be decentralized by imposing a pair of tax rates on the decentralized economy. Suppose in the second period, there is a portfolio tax on the FCD repayment (τ_1); there is also a borrowing tax on the new debt issuance in a high-income state (τ_2^H). In the tax-regulated economy, the budget constraint becomes

$$c_{T,2} + p_2^N c_{N,2} + (1+\tau_1)(1-\delta_2)\bar{I}R_2^* + \delta_2 \bar{I} R_2 p_2^N = \frac{1}{R^*} b_3 \frac{1}{1+\tau_2^H} + y_{T,2} + p_2^N \bar{y}_N.$$

The following proposition characterizes the form of tax policies to restore the social planners' allocations.

Proposition 4 (Capital Control Measures).

a. The discretionary planner's allocations can be restored by imposing a tax to adjust the period-1 *portfolio share and a tax to constrain the period-2 debt issuance in the high-income state:*

$$\tau_1^{DP} = \frac{p\mu_2^{DP}\kappa\frac{1-\omega}{\omega}\left(1 - \frac{R_2^{DP}p_2^{N,L,DP}}{R_2^*}\right) - \left[1+\phi(R_2^{DP},\delta_2^{DP})\right]\frac{\delta_2^{DP}}{R_2^*}\frac{\partial R_2^{DP}}{\partial \delta_2}\mathbb{E}[\lambda_2^{DP}p_2^{N,DP}]}{\mathbb{E}u_T^{DP}(2)}, \tag{D.19}$$

$$\tau_2^{H,DP} = \phi(R_2^{DP},\delta_2^{DP}) \equiv \delta_2^{DP}\bar{I}R_2^{DP}\frac{1-\omega}{\omega}\frac{1}{\bar{y}_N}, \tag{D.20}$$

where all variables are evaluated at the DP's optimal decision; that is δ_2^{DP}.
b. Given that the conditions in proposition 3 hold, the commitment planner's allocations can be implemented by imposing a tax to adjust the period-1 portfolio share:

$$\tau_1^{CP} = \frac{p\mu_2^{CP}\kappa\frac{1-\omega}{\omega}\left(1 - \frac{R_2^{CP}p_2^{N,L,CP}}{R_2^*}\right)}{\mathbb{E}u_T^{CP}(2)}. \tag{D.21}$$

where all variables are evaluated at the CP's optimal decision; that is δ_2^{CP}.

Proof. **a.** We construct the capital control measures to restore the DP's allocations. With a portfolio tax on the share of FCD (τ_1) and a borrowing tax on further debt issuance (τ_2^H), the households' budget constraint in the second period writes as

$$c_{T,2} + p_2^N c_{N,2} + (1+\tau_1)(1-\delta_2)\bar{I}R_2^* + \delta_2\bar{I}R_2p_2^N = \frac{1}{R^*}b_3\frac{1}{1+\tau_2^H} + y_{T,2} + p_2^N\bar{y}_N. \tag{D.22}$$

Taking derivatives with respect to δ_2 and b_3 yields the following first-order conditions in the tax-regulated equilibrium,

$$\mathbb{E}[\lambda_2 p_2^N] - \frac{R_2^*}{R_2}\mathbb{E}\lambda_2 - \tau_1\frac{R_2^*}{R_2}\mathbb{E}\lambda_2 = 0, \tag{D.23}$$

$$u_T(c_{T,2}^H)\frac{1}{1+\tau_2^H} = u_T(c_{T,3}^H). \tag{D.24}$$

Comparing equation (D.24) with the DP's bond Euler equation (D.15) when $\mu_2^{H,DP} = 0$, we can derive the formula of period-2 capital control tax rate as

$$\tau_2^{H,DP} = \phi(R_2^{DP},\delta_2^{DP}).$$

Comparing equation (D.23) with the DP's portfolio Euler equation (51), we can back out the formula of portfolio tax rate as

$$\tau_1^{DP} = \frac{p\mu_2^{DP}\kappa\frac{1-\omega}{\omega}\left(1 - \frac{R_2^{DP}p_2^{N,L,DP}}{R_2^*}\right) - \left[1+\phi(R_2^{DP},\delta_2^{DP})\right]\frac{\delta_2^{DP}}{R_2^*}\frac{\partial R_2^{DP}}{\partial \delta_2}\mathbb{E}[\lambda_2^{DP}p_2^{N,DP}]}{\mathbb{E}u_T^{DP}(2)}.$$

With both the ex ante and ex post policy measures: $\{\tau_1^{DP}, \tau_2^{H,DP}\}$, the tax-regulated equilibrium shares the same Euler equations (47), (48), (D.23), (D.24) as the ones under DP's problem. Therefore, the two equilibria deliver the same allocations.

b. We show that implementing a single portfolio tax in period 1 allows the government to restore CP's equilibrium allocations. With the portfolio tax τ_1 on the FCD repayment, the regulated CE has the Euler equation (D.23). Comparing this equation with the CP's portfolio Euler equation (52), we can back out the formula of tax rate as

$$\tau_1^{CP} = \frac{p\mu_2^{CP}\kappa\frac{1-\omega}{\omega}\left(1 - \frac{R_2^{CP}p_2^{N,L,CP}}{R_2^*}\right)}{\mathbb{E}u_T^{CP}(2)}.$$

Under the assumptions in proposition 3, in the second period, the tax-regulated CE has the same first-order conditions (47), (48), (D.3) as in the problem of CP (B.17), (B.21), (B.22). As a result, the two problems generate the same equilibrium allocations: $c_{T,2}^{H*}$, $c_{T,2}^{L*}$, R_2^*, and δ_2^*.

Therefore, the government can obtain CP's allocations by simply tilting portfolio toward LCD using a portfolio tax τ_1^{CP} in the first period. □

From proposition 4, we notice that DP's allocations can be achieved by combining an ex ante macroprudential tax to adjust the portfolio share and an ex post borrowing tax to eliminate the debt-reduction incentive. In contrast, CP's allocations can be restored by simply using an ex ante tax to correct the externality on debt denominations. The expression of portfolio tax in equation (D.21) is easy to interpret. The tax consists of three parts: the crisis probability, the crisis severity, and a term that indicates the benefit of using LCD in crisis. However, for the discretionary policy, the portfolio tax is augmented by the bond price effect of increasing local currency share, as indicated by the term with $\partial R_2(\delta_2)/\partial\delta_2$.

D.5 Numerical Illustration

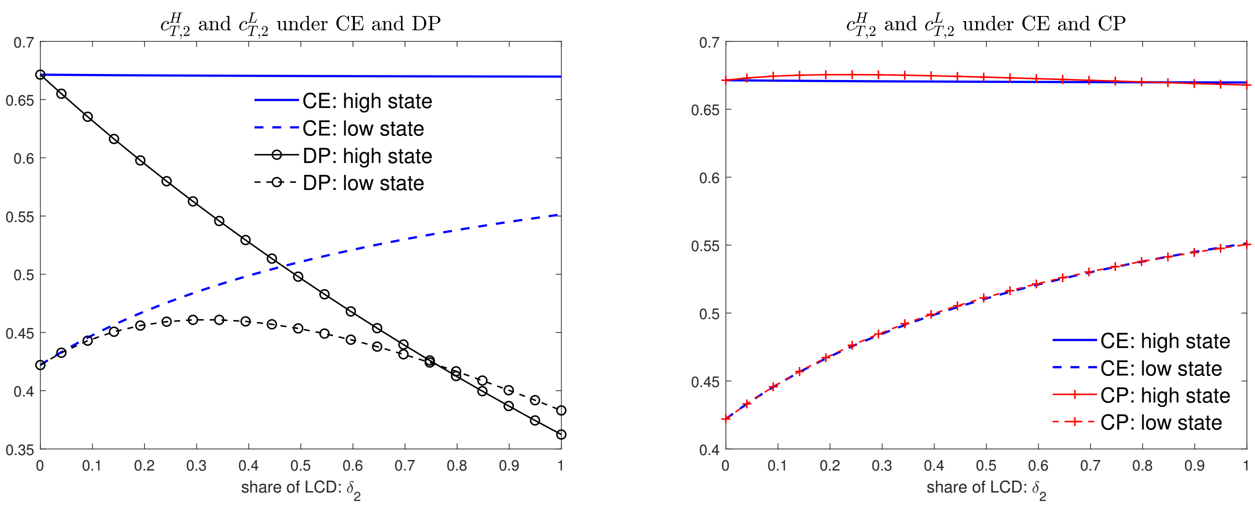
Table D.1: Parameter Values in the Numerical Example

$\sigma = 2$	$\beta = 0.94$	$\omega = 0.4$	$\bar{y}_T = 0.7$	$y_{T,2}^H = 1.05 * \bar{y}_T$	$\mathcal{M}_2^H \sim g^H = 1.05^{-5}$
$\kappa = 0.3$	$p = 0.2$	$\gamma = 5$	$\bar{y}_N = 1$	$y_{T,2}^L = 0.85 * \bar{y}_T$	$\mathcal{M}_2^L \sim g^L = 0.95^{-5}$

Table D.2: Numerical Results for the Simple Model

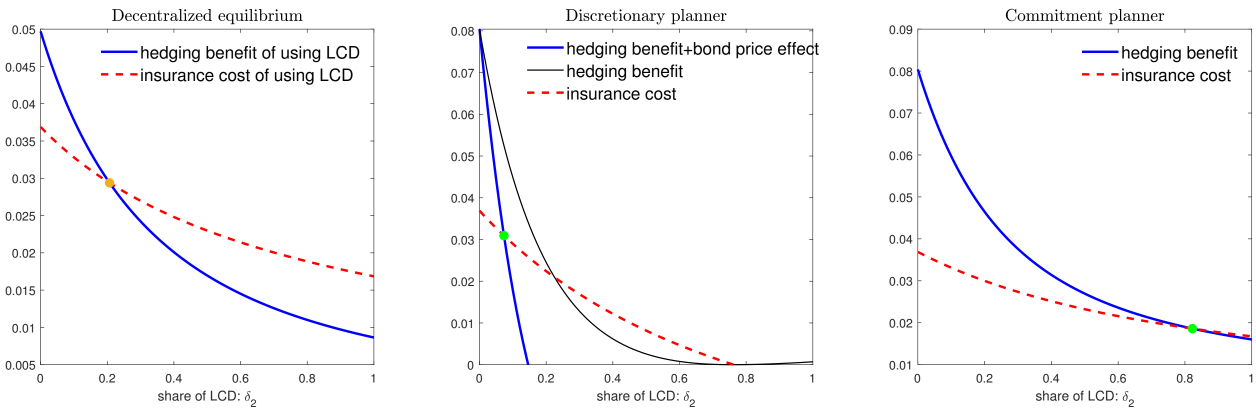
	Decentralized Equilibrium	Discretionary Planner	Commitment Planner
δ_2^*	0.21	0.07	0.82
$c_{T,2}^H$	0.67	0.64	0.67
$c_{T,2}^L$	0.47	0.44	0.54
Exchange rate premium (ρ)	2.94%	3.09%	1.86%
Welfare gain (%)	–	-0.01%	0.007%
ex post tax τ_2^H	–	6.8%	0.0%
ex ante tax τ_1	–	-0.2%	0.8%

Figure D.1: State-Contingent Consumption Profiles in the Second Period



Note: In the left figure, we compare the second-period consumption profiles in the CE and in the DP's problem for a continuum of portfolio states (δ_2) and at two shock realizations ($y_{T,2}^H, y_{T,2}^L$). In the right panel, we compare the consumption in the CE with the pre-committed consumption functions in the CP's problem.

Figure D.2: The Portfolio Decision: The Relative Hedging Benefit and Insurance Cost of Raising Local Currency Shares



NOTE: This graph shows the values at the left and right sides of portfolio Euler equations (49), (51) and (52) in the three problems. For the CE and DP, the second-period problem is solved conditional on a given state of δ_2. For the CP's problem, at an arbitrary $\delta_2 \in [0,1]$, we solve the remaining equations in the system (B.17)-(B.21) that jointly determine the values of other variables: $c^H_{T,2}(\delta_2)$, $c^L_{T,2}(\delta_2)$, $\lambda^H_2(\delta_2)$, $\lambda^L_2(\delta_2)$, $\mu^L_2(\delta_2)$, $R_2(\delta_2)$.

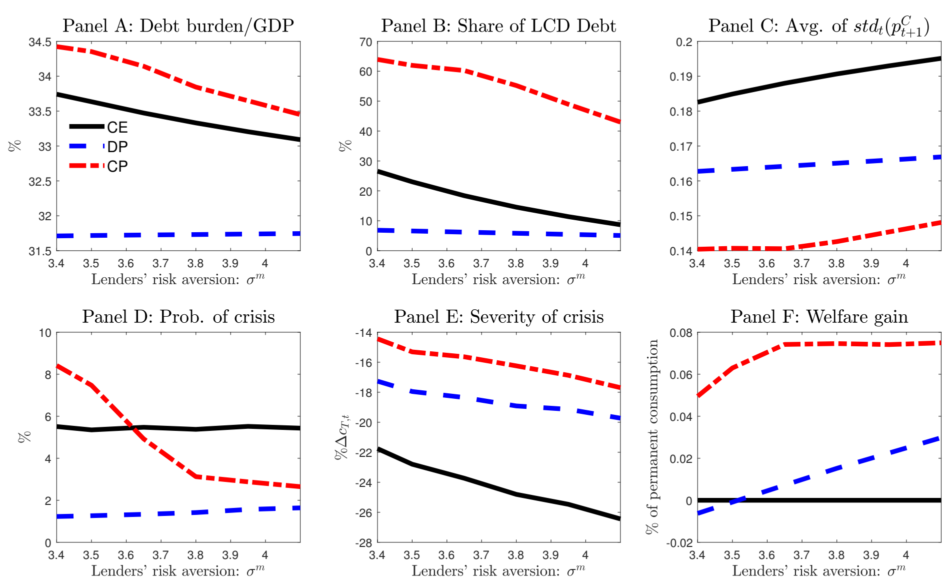
E Comparative Statics

This section provides comparative statics. We solve the model for different values of lenders' risk aversion (σ^m), income volatility (σ_ϵ), and leverage ratio (κ). We also solve a model with a different intertemporal elasticity of substitution ($\sigma = 2$).

Lenders' Risk Aversion. Figure E.1 shows how the simulation results vary with lenders' risk aversion σ^m. As the lenders become more risk-averse (a higher σ^m), they charge a higher premium on the LCD as the payoff is uncertain. As a result, issuing LCDs becomes more costly. We find from figure E.1 that in all three economies, the average share of LCD decreases as σ^m increases. As the liability structure deteriorates, the conditional volatility of the exchange rate ($std_t(p^C_{t+1})$) also rises up.

The economy under the discretionary policy has the lowest debt among the three equilibria. Since the commitment planner can obtain a favorable LCD bond price by committing to future consumption, she chooses the highest LCD share and borrows the most aggressively. With this insurance benefit, the commitment planner enjoys the lowest consumption volatility and crisis severity. The relationship on crisis probability is ambiguous. The reason is that even though the commitment planner improves the country's liability structure, the economy is more indebted than the competitive equilibrium. Panel F shows that the commitment planner always achieves the highest welfare among the three equilibria. The discretionary policy's welfare gain turns negative when σ^m is low enough.

Figure E.1: Comparative Statics: Lenders' Risk Aversion σ^m

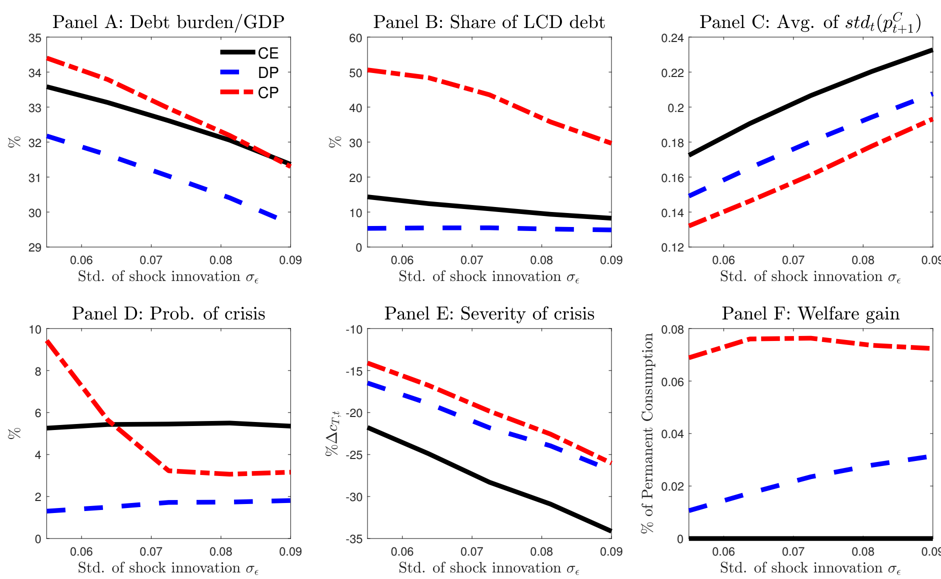


NOTE: This figure shows the comparative statics with respect to lenders' risk aversion σ^m.

Income Volatility. Figure E.2 shows how the simulation results vary with the income volatility σ_ϵ. As the tradable income becomes more volatile, issuing LCDs becomes more costly. So, in the decentralized economy, the average LCD share decreases as σ_ϵ goes up. The higher income volatility increases the conditional volatility of the exchange rate $(std_t(p^C_{t+1}))$. Panel A shows that a higher σ_ϵ induces stronger precautionary savings, leading to a lower debt balance. The same effect applies to the social planners' equilibria: the average LCD share decreases in σ_ϵ, while the conditional standard deviation and the crisis severity all increase in σ_ϵ.

Figure E.2 also shows that the relationship between the three equilibria is robust to the variation of σ_ϵ. The discretionary planner has the lowest debt balance, LCD share, and crisis probability. In contrast, the commitment planner can actively manage its portfolio by changing the share of LCD. In equilibrium, she enjoys the highest level of LCD on average and achieves the highest welfare gain. As σ_ϵ increases, her welfare gain barely changes. In contrast, the discretionary planner's welfare gain increases in σ_ϵ.

Figure E.2: Comparative Statics: Income Volatility σ_ϵ

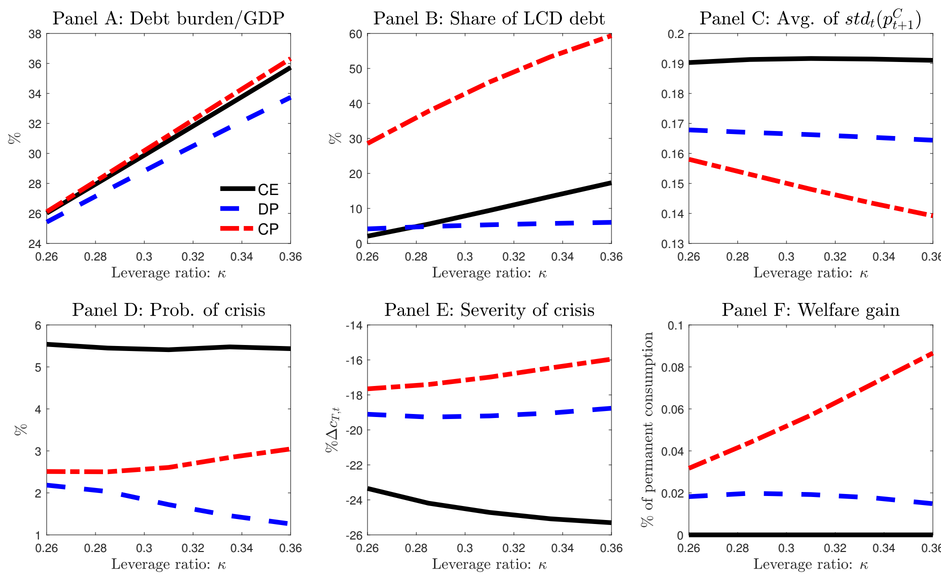


NOTE: This figure shows the comparative statics with respect to the standard deviation of tradable endowment shock σ_ϵ.

Leverage Ratio. Figure E.3 shows how the simulation results vary with the collateral rate κ. Since agents are impatient, a higher κ invites a greater amount of borrowings in all three economies. As the financial risk becomes more relevant, the relative benefit of using LCD is also larger. Therefore, we find the average share of LCD increases in κ, as shown in panel B. A higher κ increases a country's borrowing opportunity and, at the same time, increases its indebtedness. These two forces go against each other. We notice that under the decentralized equilibrium, the conditional volatility of exchange rate, probability and severity of crises only marginally change as κ varies.

The optimal policy under commitment always leads to smaller exchange rate volatility, milder recessions during sudden stops, and less frequent crises relative to the private equilibrium. Moreover, under commitment, the crisis probability and severity slightly decrease as κ rises. So, a higher κ generates a larger welfare gain. On the other hand, the discretionary planner always borrows the least amount of debt and denominates the smallest share of LCD. Even though the crisis probability is reduced to the minimum out of the three equilibria, the welfare gain of discretionary policy is relatively small.

Figure E.3: Comparative Statics: Leverage Ratio κ



NOTE: This figure shows the comparative statics with respect to the leverage ratio κ.

Intertemporal Elasticity of Substitution (IES). Next, we consider whether our results still hold in an environment with a higher intertemporal elasticity of substitution ($\sigma = 2$). Table E.1 compares the long-run simulation moments in the three environments. Figure E.4 illustrates the effect of policies around sudden stop episodes. We find that all our baseline conclusions apply to the new parameter setting. Due to the debt-deflation incentive, the discretionary planner denominates fewer debts in local currency than the private agents (5.5% vs. 12.5%), and capital controls are primarily used to restrict borrowing volumes (32.1% vs. 33.5%). On the other hand, the commitment planner enjoys greater borrowing opportunities above the decentralized market (34.2%) while denominating a larger fraction of debt in local currency (58.7%).

The middle panel of table E.1 shows that both policy environments result in smaller consumption standard deviation and conditional volatility of exchange rate. Also, both planners can reduce the frequency of crises and generate milder consumption drops during sudden stops. The commitment planner achieves the highest welfare benefit compared to alternative environments.

Table E.1: Simulation Results in a Model with $\sigma = 2$

	FCD Only CE	FCD Only SP	Decentralized Equilibrium (FCD + LCD)	Discretionary Planner (FCD + LCD)	Commitment Planner (FCD + LCD)
Avg. debt burden/y	33.1%	32.2%	33.5%	32.1%	34.2%
Avg. share of LCD	–	–	12.5%	5.5%	58.7%
Std(debt burden/y_T)	6.6%	5.7%	5.7%	5.3%	3.8%
Std(share of LCD issuance)	–	–	8.9%	4.0%	25.5%
Corr(debt burden, y_T)	0.73	0.76	0.79	0.78	0.90
Corr(share of LCD issuance, y_T)	–	–	-0.23	-0.63	-0.44
Avg. $\sigma_t(p^C_{t+1})$	22.5	19.4	20.8	17.7	14.9
Std(c_T)/Std(y_T)	1.30	1.18	1.22	1.14	1.02
Std(ca)/Std(y_T)	0.64	0.42	0.62	0.37	0.48
Prob. of crises	6.2%	2.3%	5.5%	1.4%	2.3%
Sev. of crises (%Δc_T)	-29.7%	-21.5%	-25.7%	-19.0%	-15.6%
Avg. tax rate $\frac{(\tau^T+\tau^C)}{2}$	–	5.94%	–	6.64%	6.26%
Avg. tax discrimination ($\tau^T - \tau^C$)	–	–	–	0.32%	0.48%
Avg. wel. gain rel. to - Baseline DE	–	–	–	0.023%	0.080%
Avg. wel. gain rel. to - FCD only (CE)	–	0.059%	0.065%	0.078%	0.138%

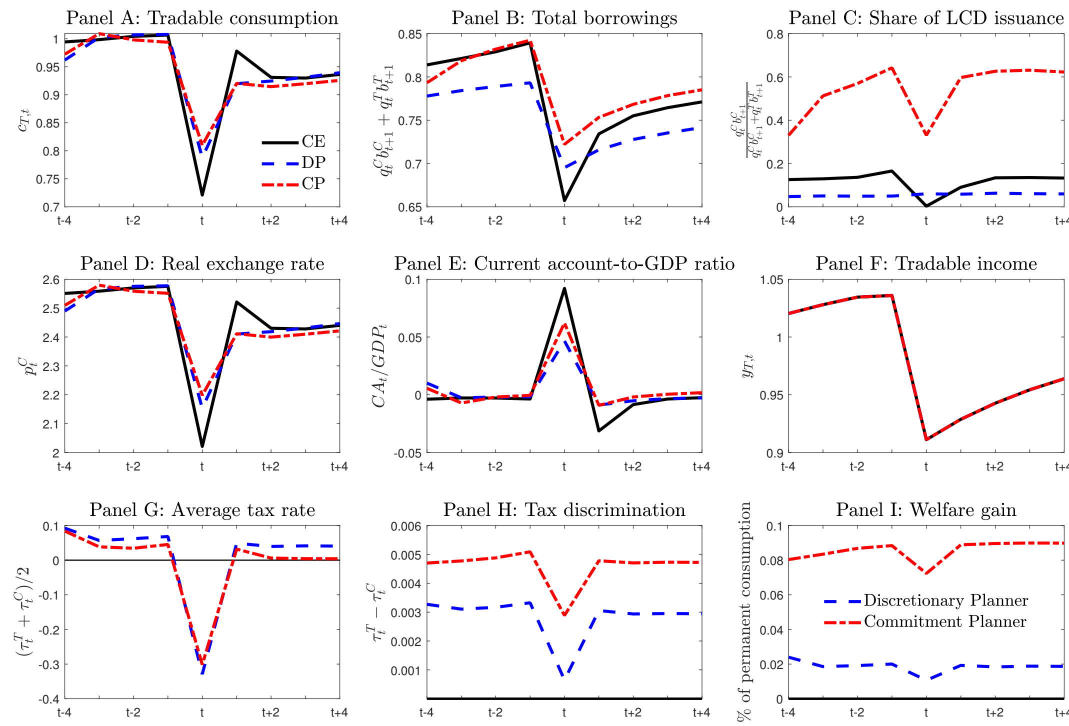
NOTE: We recalibrate the model with $\sigma = 2$ to target data moments, which leads to $\beta = 0.86$, $\sigma^m = 3.25$, $\kappa = 0.334$. Other parameters are the same as in our calibration calibration.

Figure E.4 shows the average paths of a typical sudden stop event in the three environments. Both the discretionary and commitment policies result in milder recessions, but the commitment planner can borrow more aggressively. This is due to the difference in debt denominations. Panel C shows that under commitment, the LCD share quickly rises up in periods preceding the crisis, leading to a smoother exchange rate fluctuation (panel D) and a milder current account reversal (panel E) during the financial turmoil. The tax rates implied by the two social planners are also different. The capital control tax enforced by the commitment planner is looser on average (panel G) and features larger discrimination based on currency denomination (panel H).

F A Model with Issuance Cost of LCD

In our baseline model, a country's external portfolio is uniquely determined by lenders' risk aversion parameter σ^m. We ask the following question: does the social planners' mechanism depend on the lenders' risk aversion? In this section, we assume international lenders are all risk neutral: $\sigma^m = 0$. To pin down households' portfolio decision, we further assume a quadratic cost on the issuance of

Figure E.4: Event Windows in Models with $\sigma = 2$



NOTE: The figure shows event window analysis in a recalibrated model with $\sigma = 2$. For comparison, we first identify 1,000 sudden stop events from the simulations of the CE and extract the income process during the crises and initial states before the crises. We then feed the series of shocks and initial states into alternative economies. The graph shows the average path of simulations across the event windows. The welfare gain represents the percentage of permanent consumption households would like to sacrifice for moving to social planners' economies. The initial h state is set to 0 at period $t-4$ for the commitment planner.

LCD. Given everything else the same, the households' budget constraint is written as

$$c_{T,t} + p_t^N c_{N,t} + p_t^C b_t^C + b_t^T + \frac{\phi}{2}(b_{t+1}^C)^2 = y_{T,t} + p_t^N \overline{y}_N + q_t^C b_{t+1}^C + q_t^T b_{t+1}^T. \tag{F.1}$$

The quadratic cost in the budget constraint represents some institutional distortions (i.e., less disciplined monetary policy, incomplete financial integration, etc.) that might prohibit the use of LCD in the international market. A reduction in ϕ captures the disappearance of "original sin" that arises from long-run factors such as improved institutional quality, increasing financial integration, or more disciplined central banks.

In the private equilibrium, the Euler equations on bond issuances are given by

$$q_t^T(\lambda_t - \mu_t) = \beta \mathbb{E}_t \lambda_{t+1}, \tag{F.2}$$

$$q_t^C(\lambda_t - \mu_t) - \lambda_t \phi b_{t+1}^C = \beta \mathbb{E}_t \lambda_{t+1} p_{t+1}^C, \tag{F.3}$$

where λ_t and μ_t are still Lagrange multipliers on the budget and collateral constraints. The additional term with ϕ in equation (F.3) indicates that agents incorporate the debt issuance cost when choosing debt denominations. The problems of social planners can be defined correspondingly.

Results. Table F.1 shows the simulation results of the three equilibria in an environment with risk-neutral lenders. We use the debt issuance cost parameter ϕ to target the average share of LCD in the Mexican economy. We find that our main channel in the baseline model still holds in this new environment. Due to pecuniary externalities in the collateral and budget constraint, the private agents tend to over-borrow relative to social planners' equilibria and denominate too much debt in foreign currency. The discretionary planner, however, denominates a smaller fraction of debt in local currency since the domestic bond price is distressed by her debt-deflation incentive. Only the social planner under commitment can increase the share of LCD in her liability structure.

However, compared to the benchmark model where the portfolio is pinned down by lenders' risk aversion, in this new environment, the commitment planner's ability to improve debt structure is heavily restricted. It is because the issuance cost of LCD takes a quadratic form. As we notice from table F.1, the average share of LCD under the CP is only slightly higher than that in the private equilibrium (15.6% vs. 13.6%), and her average borrowing is lower than the private agents' (32.7% vs. 33.2%). In the end, the welfare gain achieved by the commitment planner is smaller relative to the benchmark environment (0.035% vs. 0.071%).

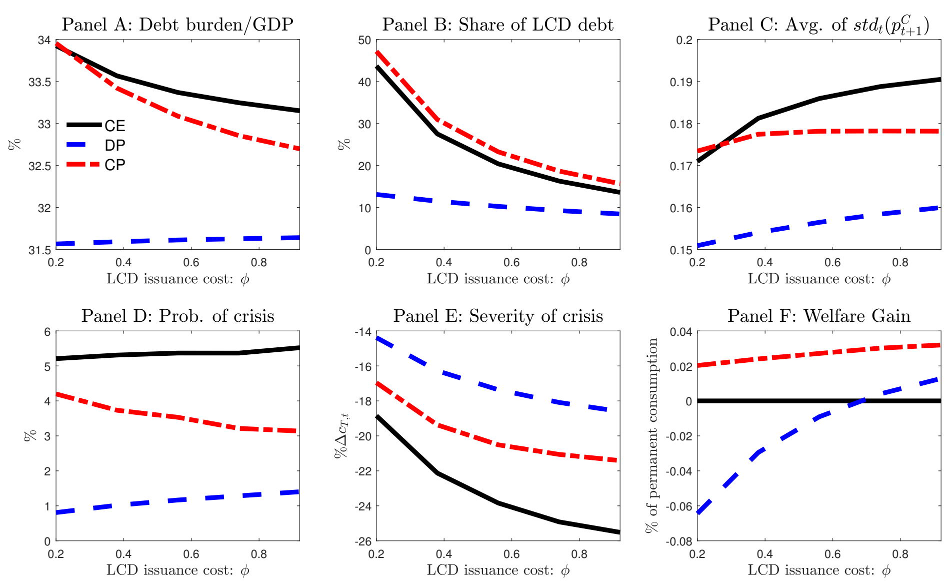
Figure F.1 displays simulation moments for a continuum of $\phi \in [0.2, 0.92]$. Apparently, the lower issuance cost leads to the higher share of LCD in simulations and the milder severity of crisis. The average welfare gain also decreases as ϕ becomes smaller, and a small enough ϕ can even produce a welfare loss for the discretionary planner.

Table F.1: Simulation Results in a Model with Risk-Neutral Lenders and Issuance Cost of LCD

	Decentralized Equilibrium	Discretionary Planner	Commitment Planner
Avg. debt burden/y	33.2%	31.6%	32.7%
Avg. share of LCD	13.6%	8.4%	15.6%
Std(c_T)/Std(y_T)	1.15	1.09	1.10
Std(ca)/Std(y_T)	0.53	0.29	0.42
Std(debt burden/y_T)	5.4%	5.3%	4.9%
Std(share of LCD issuance)	3.8%	3.0%	4.1%
Avg. $\sigma_t(p^C_{t+1})$	19.1	16.0	17.8
Prob. of crises	5.5%	1.4%	3.1%
Sev. of crises (%Δc_T)	-25.6%	-18.6%	-21.4%
Avg. tax on FCD: τ^T	–	6.59%	3.42%
Avg. tax on LCD: τ^C	–	6.79%	2.90%
Avg. tax discrimination: $\tau^T - \tau^C$	–	-0.20%	0.52%
Corr($\frac{\tau^T+\tau^C}{2}$, y_T)	–	-0.84	-0.70
Corr($\tau^T - \tau^C$, y_T)	–	-0.79	-0.82
Avg. wel. gain rel. to DE	–	0.012%	0.035%

NOTE: The table shows the simulation results in an environment with risk-neutral lenders ($\sigma^m = 0$) and issuance cost of LCD: $\phi = 0.92$. For comparison, other parameters are set to the same as in our baseline calibration.

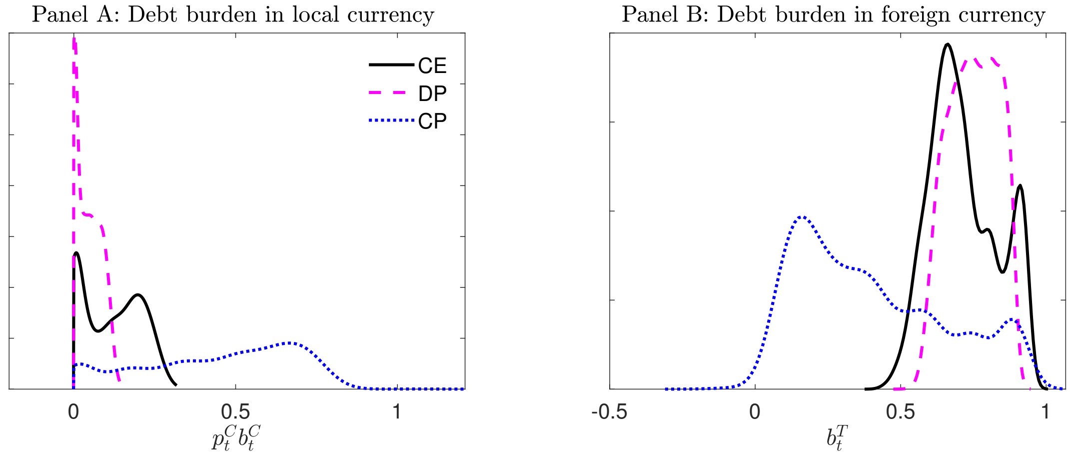
Figure F.1: Models with Risk-Neutral Lenders and Issuance Cost of LCD



NOTE: This figure shows the long-run simulation results in a model with risk-neutral lenders ($\sigma^m = 0$) and a continuum of LCD issuance cost: $\phi \in [0.2, 0.92]$. For comparison, other parameters are set to the same as in our baseline calibration.

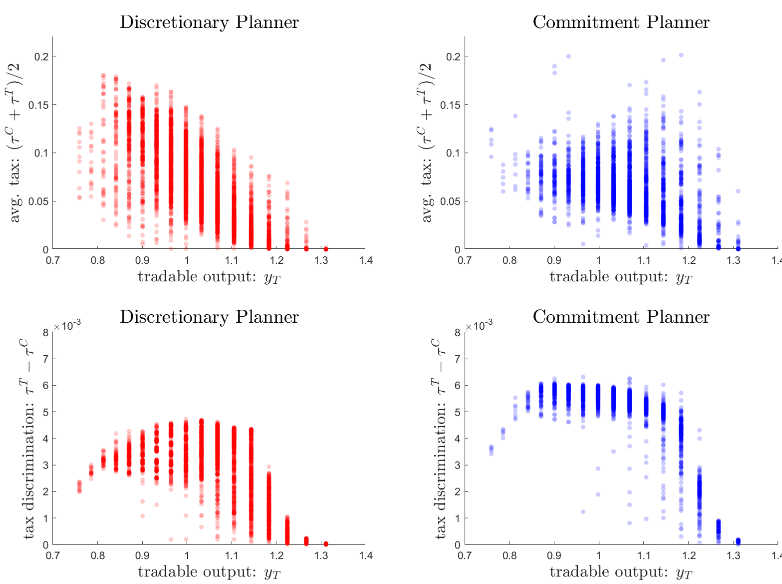
G Additional Figures

Figure G.1: Ergodic Distributions of Debt Burdens in Local and Foreign Currencies



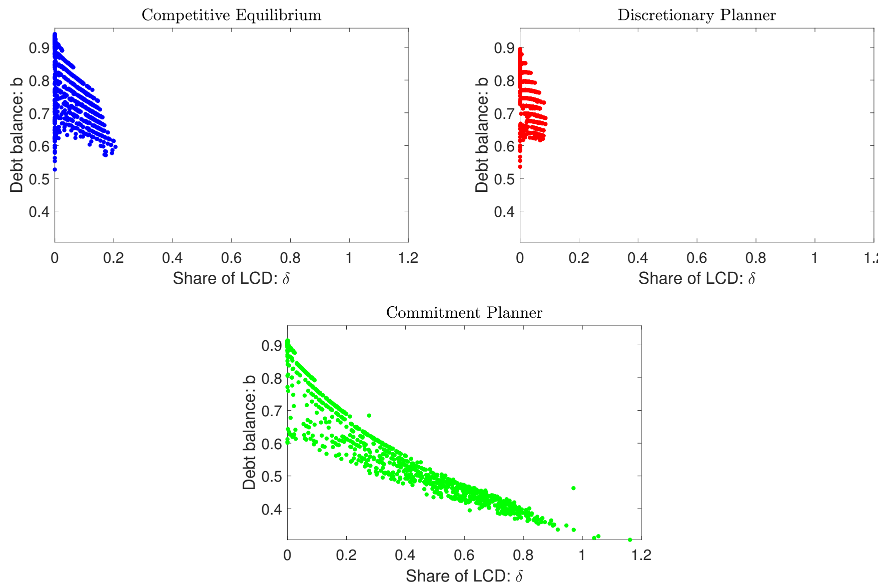
NOTE: This figure plots the ergodic distributions of debt burdens that are denominated in local ($\{p_t^C b_t^C\}_{t=1}^T$) and foreign currencies ($\{b_t^T\}_{t=1}^T$) in the three equilibria.

Figure G.2: Cyclicality of Tax Rates and Tax Discrimination



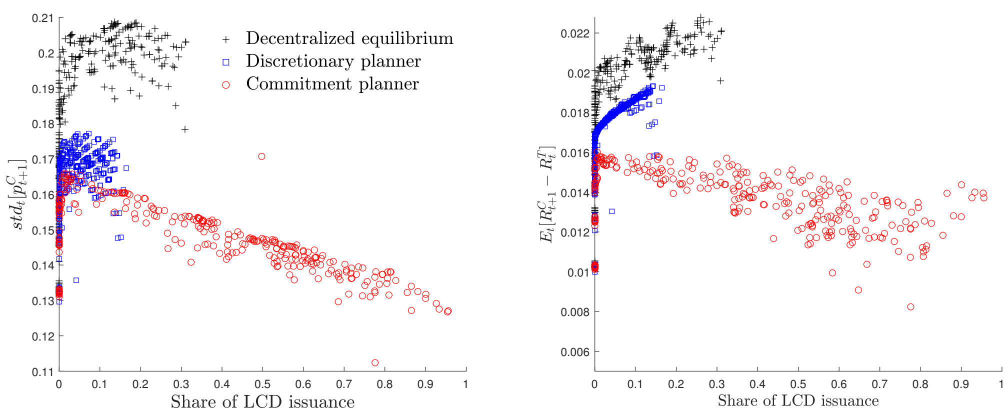
NOTE: The figure plots the average tax rates and tax discrimination against tradable endowment in the social planners' economy. The average tax rate is $\frac{\tau_t^T + \tau_t^C}{2}$. The tax discrimination is $\tau_t^T - \tau_t^C$. We only include the periods in the simulation when the collateral constraint is slack.

Figure G.3: Scatters of Portfolio Distributions in the Three Economies



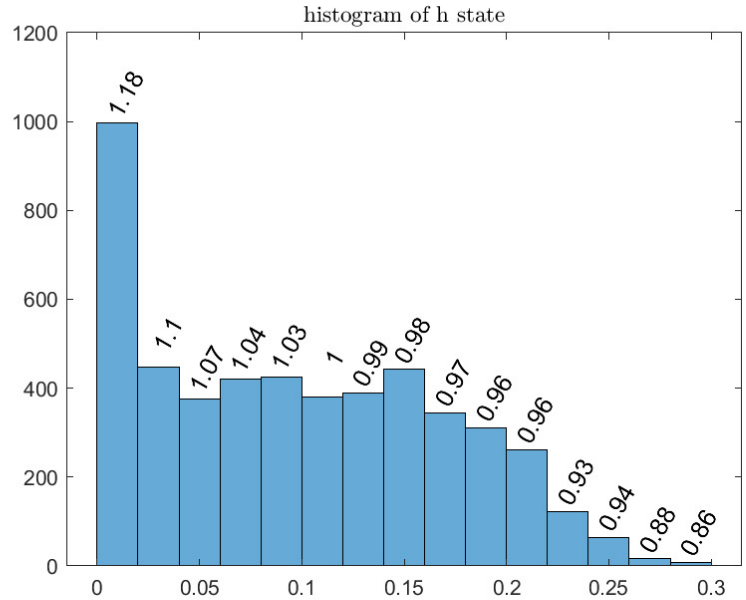
NOTE: This figure shows the scatters of $\{b_t, \delta_t\}_{t=1}^{T}$ over long-run simulations in the three economies, where $b_t = b_t^C + b_t^T$ and $\delta_t = b_t^C/(b_t^C + b_t^T)$.

Figure G.4: Local Currency Debt as a Hedging Device



NOTE: This figure shows the scatters of LCD borrowing share $(q_t^C b_{t+1}^C/(q_t^C b_{t+1}^C + q_t^T b_{t+1}^T))$ against the conditional standard deviation of exchange rate $(std_t(p_{t+1}^C))$ or exchange rate risk premium $(E(R_{t+1}^C - R_t^T))$ in all the three economies.

Figure G.5: Histogram of h State



NOTE: This figure shows the histogram of the next-period commitment state $\{h_{t+1}\}_{t=1}^{T}$ in the CP's simulation. We only include the periods when financial constraint is not binding: $\mu_t = 0$. The numbers on the top of each bin indicate the average tradable endowment in the current period ($y_{T,t}$) given that the next-period h belongs to a specific bin: $h_{t+1} \in bin_j$ for $j = 1, ..., 15$.