Online Supplement to

‘Self-Regulation Versus Government Regulation: An Externality View

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S.1 Relative Welfare Function Analysis

In this section, we analyze the desirability of self-regulation and government regulation by defining a relative welfare function as follows.

$$\Delta^{i/G} \equiv E[W(X^i;\mathcal{F}) - W(X^G;\mathcal{F})]$$

where we consider two cases with $i = \{S, CE\}$ respectively. The former measures the relative welfare between self-regulation and government regulation while the latter measures the relative welfare between competitive equilibrium and government regulation . Given these two functions, $\Delta^{S/CE} = \Delta^{S/G} - \Delta^{CE/G}$ measures the relative welfare between self-regulation and competitive equilibrium.[S1]

To get an analytical solution, I follow Weitzman (1974) and Laffont (1977) to impose information structure in the model as below and apply a second-order approximation around $x = X^G$ and $X = X^G$ as follows.

$$\begin{aligned}
u(x;\phi) &\approx u\left(X^G;\phi\right) + [\bar{u'} + \phi](x - X^G) + \frac{1}{2}\bar{u''}(x - X^G)^2 \\
U(X;\Phi) &\approx U\left(X^G;\Phi\right) + [\bar{U'} + \Phi](X - X^G) + \frac{1}{2}\bar{U''}(X - X^G)^2 \\
c(x;\theta) &\approx c\left(X^G;\theta\right) + [\bar{c'} + \theta](x - X^G) + \frac{1}{2}\bar{c''}(x - X^G)^2 \\
C(X;\Theta) &\approx C\left(X^G;\Theta\right) + [\bar{C'} + \Theta](X - X^G) + \frac{1}{2}\bar{C''}(X - X^G)^2
\end{aligned}$$

[S1] I use $\Delta^{S/G}$ and $\Delta^{CE/G}$ to derive $\Delta^{S/CE}$ because it simplifies analysis. As will be shown later, all the functions are approximated around $X = X^G$.

where the parameters have zero mean.

Under this approximation, the first-order and second-order derivatives are given as follows.

$$u'(x;\phi) = \bar{u'} + \phi + \bar{u''}(x - X^G)$$
$$U'(X;\Phi) = \bar{U'} + \Phi + \bar{U''}(X - X^G)$$
$$c'(x;\theta) = \bar{c'} + \theta + \bar{c''}(x - X^G)$$
$$C'(X;\Theta) = \bar{C'} + \Theta + \bar{C''}(X - X^G)$$
$$u''(x;\phi) = \bar{u''}$$
$$U''(X;\Phi) = \bar{U''}$$
$$c''(x;\theta) = \bar{c''}$$
$$C'(X;\Theta) = \bar{C''}$$

Using the approximation for the optimality condition of government regulation, the following relationship holds.[S2]

$$\begin{aligned} 0 &= E[u'(X^G;\phi) - U'(X^G;\Phi) - c'(X^G;\theta) - C'(X^G;\Theta)] \\ &\approx E[\bar{u'} + \phi - \bar{U'} - \Phi - \bar{c'} - \theta - \bar{C'} - \Theta] \\ &= \bar{u'} - \bar{U'} - \bar{c'} - \bar{C'} \end{aligned}$$

Similarly, using the allocations X^S for self-regulation and X^{CE} for competitive

[S2]The objective function for a benevolent government is

$$\max_{X^G} E\left[u(X;\phi) - U(X;\Phi) - c(X;\theta) - C(X;\Theta)\right]$$

The optimality condition is

$$E[u'(X^G;\phi)] = E[U'(X^G;\Phi) + c'(X^G;\theta) + C'(X^G;\Theta)] \tag{4}$$

equilibrium, the following relationship holds.[S3]

$$
\begin{aligned}
0 &= u'(X^S;\phi)+u''(X^S;\phi)X^S-c'(X^S;\theta)-C'(X^S;\Theta)\\
&\approx \bar{u}'+\phi+\bar{u}''X^S-\bar{c}'-\theta-\bar{C}'-\Theta+(\bar{u}''-\bar{c}''-\bar{C}'')(X^S-X^G)\\
&= \bar{U}'+\phi-\theta-\Theta+\bar{u}''X^S+(\bar{u}''-\bar{c}''-\bar{C}'')(X^S-X^G)
\end{aligned}
$$

and

$$
\begin{aligned}
0 &= u'(X^{CE};\phi)-c'(X^{CE};\theta)\\
&\approx \bar{u}'-\bar{c}'+\phi-\theta+(\bar{u}''-\bar{c}'')(X^{CE}-X^G)
\end{aligned}
$$

The difference between X^G and X^S (X^{CE}) can thus be written as

$$
X^S-X^G=\frac{\bar{u}''X^G+\bar{U}'+\phi-\theta-\Theta}{\bar{c}''+\bar{C}''-2\bar{u}''}\equiv\frac{\bar{u}''X^G+\bar{U}'+\phi-\theta-\Theta}{\bar{W}''_S}
$$

end

$$
X^{CE}-X^G=\frac{\bar{u}'-\bar{c}'+\phi-\theta}{\bar{c}''-\bar{u}''}\equiv\frac{\bar{u}'-\bar{c}'+\phi-\theta}{\bar{W}''_{CE}}
$$

where $\bar{W}''_S=\bar{c}''+\bar{C}''-2\bar{u}''>0$ and $\bar{W}''_{CE}=\bar{c}''-\bar{u}''>0$.

The welfare function is given by

$$
W(X;\mathcal{F})=W(X^G;\mathcal{F})+(\phi-\Phi-\theta-\Theta)(x-X^G)-\frac{1}{2}\bar{W}''(X-X^G)^2
$$

[S3]The objective function for an SRO is

$$
\begin{aligned}
\max_{X^S}\quad & p(X^S;\phi)X^S-c(X^S;\theta)-C(X^S;\Theta)\\
\text{s.t.}\quad & p(X^S;\phi)=u'(X^S;\phi)
\end{aligned}
$$

The optimality condition is

$$
u'(X^S;\phi)+u''(X^S;\phi)X^S=c'(X^S;\theta)+C'(X^S;\Theta) \tag{5}
$$

Equivalently, it can be written as

$$
u'(X^S;\phi)\left(1-\frac{1}{E_d(X^S;\phi)}\right)=c'(X^S;\theta)+C'(X^S;\Theta)
$$

where $E_d(X^S;\phi)$ is the price elasticity of demand at the point $X=X^S$.

where $\bar{W}'' = -\bar{u}'' + \bar{U}'' + \bar{c}'' + \bar{C}'' > 0$.

The relative welfare benefit of self-regulation over government regulation can be approximated as

$$\begin{aligned}\Delta^{S/G} &= E\left[(\phi - \Phi - \theta - \Theta)(X^S - X^G) - \frac{1}{2}\bar{W}''(X^S - X^G)^2\right] \\ &= \frac{\underbrace{E[\phi - \theta - \Theta]^2(\bar{W}_S'' - \bar{W}''/2)}_{\text{Information Advantage}} - \underbrace{\bar{W}''/2\left(\bar{u}''X^G + \bar{U}'\right)^2}_{\text{Externality}} - \underbrace{\bar{W}_S'' E[\Phi(\phi - \theta - \Theta)]}_{\text{Information Correlation}}}{(\bar{W}_S'')^2}\end{aligned}$$

The relative welfare benefit of government regulation over competitive equilibrium can be approximated as

$$\begin{aligned}\Delta^{CE/G} &= E\left[(\phi - \Phi - \theta - \Theta)(X^{CE} - X^G) - \frac{1}{2}\bar{W}''(X^{CE} - X^G)^2\right] \\ &= \frac{\underbrace{E[\phi - \theta]^2(\bar{W}_{CE}'' - \bar{W}''/2)}_{\text{Information Advantage}} - \underbrace{\bar{W}''/2\left(\bar{U}' + \bar{C}'\right)^2}_{\text{Externality}} - \underbrace{\bar{W}_{CE}'' E[(\Phi + \Theta)(\phi - \theta)]}_{\text{Information Correlation}}}{(\bar{W}_{CE}'')^2}\end{aligned}$$

One can see that the disadvantage of government regulation is from the asymmetric information captured by the term in the first bracket. The advantage, however, comes from the fact that the government internalizes the externalities (distortions) in the economy, captured by the term in the second bracket. The last bracket is information correlation, which vanishes if there is no correlation in the information set. It comes from the fact that the government can infer the unknown parameters from its prior knowledge about the correlation structure.

Note that superior information does not justify self-regulation (competitive equilibrium) over government regulation automatically because private agents might use those information in a way that makes the existing distortion even worse. Such an effect is captured by the term $\bar{W}_S'' - \bar{W}''/2$ in the first bracket of $\Delta^{S/G}$ and the term $\bar{W}_{CE}'' - \bar{W}''/2$ in the first bracket of $\Delta^{CE/G}$. In the case where $\bar{W}_S'' < \bar{W}''/2$ ($\bar{W}_{CE}'' < \bar{W}''/2$), self-regulation (competitive equilibrium) is likely to be inferior to government regulation.

S.2 Proofs

S.2.1 Proof of Proposition 1

Proof. If government has perfect information about $\mathcal{F}$, it can choose X^{FB} defined by the optimality condition (2). Furthermore, $X^{FB} < X^{CE}$.

To implement X^{FB}, government can regulate either consumers or producers. To regulate the consumers, government can use a Pigovian tax τ on individual consumers and rebate them by a lump-sum transfer T. For the individual consumer j, his objective function is thus

$$\max_{y_j} u(y_j;\phi) - (p+\tau)y_j - U(X;\Psi) + T$$

The optimality condition is

$$p+\tau = u'(y_j;\phi)$$

The optimality condition for producers is unaffected by the policy. Therefore, in equilibrium, the following relationship holds.

$$\tau = u'(X;\phi) - c'(X;\theta)$$

To implement the first best allocation, one can choose $\tau = U'(X^{FB};\Phi) + C'(X^{FB};\Theta)$ and $T = \tau X^{FB}$. Furthermore, one can simply put a quantity restriction $y^j \leq X^{FB}$ on the individual consumer and implement the first best allocation. The reason is that $X^{FB} < X^{CE}$ in equilibrium.

By a similar argument, one can easily show that the first best allocation X^{FB} can be implemented by a tax τ_0^* and a lump-sum transfer T_0^* on an individual producer. For individual producer i, his objective function is thus

$$\max_{x_i}(p+\tau_0^*)x_i - c(x_i;\theta) - C(X;\Theta) + T_0^*$$

The optimality condition is thus

$$p + \tau_0^* = c'(x_i;\theta)$$

The optimality condition for consumers is unaffected by the policy. Therefore, in equilibrium, the following relation holds.

$$\tau_0^* = c'(X;\theta) - u'(X;\phi)$$

By monotonicity of $c' - u'$, choosing $\tau_0^* = -U'(X^{FB};\Phi) - C'(X^{FB};\Theta)$ can implement X^{FB} in the decentralized economy. Also $T_0^* = -\tau_0^* X^{FB}$ is implied by government's budget constraint. Similarly, one can also put a production restriction $x^i \leq X^{FB}$ to implement X^{FB} because $X^{CE} > X^{FB}$ in equilibrium.

Now, we consider a case where the government allows the producers to form a industrial SRO and regulates the SRO instead. The SRO thus faces the following maximization problem.

$$\max_X (u'(X;\phi) + \tau_1^*)X - c(X;\theta) - C(X;\Theta) + T_1^*$$

The optimality condition is thus

$$u'(X;\phi) + \tau_1^* + u''(X;\phi)X = c'(X;\theta) + C'(X;\Theta)$$

Hence, one can choose $\tau_1^* = -u''(X^{FB};\phi)X^{FB} - U'(X^{FB};\Phi)$ and $T_1^* = -\tau_1^* X^{FB}$ to implement X^{FB}.

Interestingly, if $\tau_1^* = -u''(X^{FB};\phi)X^{FB} - U'(X^{FB};\Phi) > 0$, it implies that $X^S < X^{FB} < X^{CE}$. In other words, government needs to subsidize an SRO to implement the first best allocation. It turns out that there exists a specific number of monopolistic competitive SROs such that the first best allocation X^{FB} can be implemented. To see this point, first assume that there exists N SROs in the market for self-regulation and each has a market share of $\frac{1}{N}$. For each of them, the maximization problem is as follows.

$$\begin{array}{ll}\max\limits_{X_i} & P\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\phi\right)X_i - c(X_i;\theta) - C\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\Theta\right)\\ \text{s.t.} & \qquad P\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\phi\right) = u'\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\phi\right)\end{array}$$

The optimality condition is

$$\frac{1}{N}u''\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\phi\right)X_i + u'\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\phi\right) = c'(X_i;\theta)+\frac{1}{N}C'\left(\frac{X_i}{N}+\sum_{j\neq i}\frac{X_j}{N};\Theta\right)$$

By symmetry, it implies

$$\frac{1}{N}u''(X^N;\phi)X^N + u'(X^N;\phi) = c'(X^N;\theta) + \frac{C'(X^N;\Theta)}{N}$$

Realize that if $N = 1$, there is only one SRO in the market and $X^1 = X^S$; if $N = \infty$, there is a continuum of agents in the market and $X^\infty = X^{CE}$. Moreover, X^N is an increasing function of N. Therefore, if $X^S < X^{FB} < X^{CE}$, by continuity there exists N^* such that $X^{N^{FB}} = X^{FB}$. □

S.2.2 Proof of Proposition 2

Proof. Suppose government announces $\tau(X;\phi)$ to an SRO and rebates it by $T = -\tau(X;\phi)X$. The objective function for the SRO is

$$\begin{array}{ll}\max_X & [P(X;\phi)+\tau(X;\phi)]X - c(X;\theta) - C(X;\theta) + T\\ \text{s.t.} & P(X;\phi) = u'(X;\phi)\end{array}$$

Notice that by choosing $\tau(X;\phi) = -u'(X;\phi) + \frac{u(X;\phi)-E[U(X;\Phi)]}{X}$, the SRO chooses the second best allocation as in (3) □

S.2.3 Proof of Proposition 3

Proof. By choosing the price menu as $P(X) = E[u'(X;\phi) - U'(X;\Phi)]$, the government can implement $\bar{\bar{W}}$. To implement, government buys goods from an SRO according to such price menu and sells to the consumer. The difference between selling and buying is transferred to the SRO. □

S.3 Derivation of Value Function

In period 1, define the state variable as $m = \tilde{e} - d_1$ and $M = m$ in equilibrium. The value function can be written as

$$\begin{aligned} V(m;M) = \quad & \max_{d_2,\kappa} u(c_1) + c_2 \\ \text{s.t.} \quad & c_1 = m + d_2 + (1-\kappa)p, \\ & c_2 = \kappa y - d_2 \\ & d_2 \le \phi p \cdots (\lambda) \end{aligned}$$

The FOCs are

$$\begin{aligned} u'(c_1) &= 1 + \lambda \\ u'(c_1)p &= y \end{aligned}$$

In equilibrium, since the asset is held only by bankers, $\kappa = 1$ and $C_1 = M + D_2$, where the capital letters denote the aggregate level of variables. There are two states in period 1. Define c^* such that $u'(c^*) = 1$ and $\hat{M}$ such that $\hat{M} = c^* - \phi$. Then if $M \ge \hat{M}$, the economy is in the unconstrained state and $c^1 = c^*, d_2 = c^* - m, p = 1$; if $M < \hat{M}$, the economy is in the constrained state and $c_1 = m + \phi \frac{y}{u'(c_1)}$, $p = \frac{y}{u'(c_1)} \equiv p(M)$. Therefore,

$$V(m;M) = \begin{cases} u(c^*) + y + m - c^* & \text{if } M \ge \hat{M} \\ u(m + \phi p(M)) + y - \phi p(M) & \text{if } M < \hat{M} \end{cases}$$