Too Big to Fail and Optimal Regulation *

Chang Ma†
Fudan University

Xuan-Hai Nguyen‡
Chinese University of Hong Kong

March 2021

Abstract

This paper analyzes the optimal regulation for "Too Big to Fail" (TBTF) in a simple model. As government cannot credibly commit no bail-out during crises, banks have an incentive to become excessively large. In this case, no single policy can fully eliminate the inefficiencies from TBTF. The optimal regulation for the first-best allocation features a capital requirement and issuance of Contingent Convertible Bonds (CoCos) where the capital requirement addresses the moral hazard issue from government bailouts and CoCos improve the risk-sharing. Moreover, a combination of the capital requirement and size regulation can implement a second-best allocation where the government has to bail out the banking sector but the social cost of bail-out is internalized by the banks. In this case, the capital requirement forces banks to internalize the bailout cost while the size regulation directly discourages banks to become large.

Keywords: Too Big To Fail, Bailout, Optimal Regulation, CoCos
JEL Classification: G21, G28

* We wish to thank Olivier Jeanne, Anton Korinek, Laurence Ball, Jon Faust, Dang Du, Jiakai Chen (discussant), Zhen Zhou (discussant) and participants at the 2013 JHU Macro-Finance Discussion Group, the JHU Macro Lunch Seminar, the 2016 Vietnam International Conference in Finance, and the 28th Asian FA Annual Meeting for very helpful comments. Zhiyang Luo provided superb research assistance. Chang Ma gratefully acknowledges financial support from National Natural Science Foundation of China (Grant No. 72003043) and was also sponsored by Shanghai Pujiang Program. All remaining errors are ours. Declarations of interest: none.

† Corresponding author at Fanhai International School of Finance (FISF), Fudan University, Shanghai, 200433. **Email:** changma@fudan.edu.cn.

‡ Chinese University of Hong Kong, Department of Economics, Shatin, Hong Kong. **Email:** nxhaivn@gmail.com.

1 Introduction

During the 2008-09 global financial crisis, government interventions in failing financial institutions, such as American International Group (AIG), fueled a resurgence of interest in the notion of "Too Big to Fail" (TBTF). It refers to the idea that some financial institutions are so large/important/complex that their failures are costlier to society than the required expenses to save them.[1] As a result, policymakers will have to bail out those big financial institutions using taxpayers' money when they are in trouble. However, this ex post intervention creates an ex ante moral hazard problem (see Farhi and Tirole 2012; Strahan 2013; Keister 2016; Jeanne and Korinek 2020). Financial institutions have an incentive to become *large* enough to take advantage of the government's bail-out either through taking excessive risk or increasing the size of their balance sheet — an important feature of TBTF. To address this issue, policymakers have proposed several regulatory tools, such as capital requirements, a direct cap/tax on the size (balance sheet), the issuance of contingent convertible bonds (CoCos), etc. However, different tools have different impacts on the balance sheet of banks. Whether those regulations are effective is still under debate even with the enactment of the Dodd-Frank Act.[2] For example, the famous symposiums held at the Minneapolis Fed, under the leadership of President Neel Kashkari, has reignited the debate on the TBTF regulations.[3]

Several regulatory tools have been proposed. For example, Simon Johnson from MIT have suggested to limit bank size or even impose a division for the largest

[1] TBTF began to attract public attention in the 1970s when the Federal Deposit Insurance Corporation (FDIC) repeatedly bailed out institutions that it deemed "essential" to the community. In 1991, it received an official status in the financial industry when Congress passed the FDIC Improvement Act, authorizing the FDIC to grant special treatments to a number of large banks. Although Congress attempted to abolish the notion of TBTF with the passage of the Dodd-Frank Act in 2010, the Act created a new category — Systemically Important Financial Institutions (SIFIs) — which not only included all previously TBTF institutions but also many new ones.

[2] See the debate on whether Dodd-Frank is sufficient to address TBTF at https://www.nytimes.com/roomfordebate/2016/04/14/has-dodd-frank-eliminated-the-dangers-in-the-banking-system/the-financial-system-remains-too-fragile-too-distorted-too-dangerous.

[3] See the proposal "*The Minneapolis Plan to End Too Big to Fail*" issued in Dec 2017 at https://www.minneapolisfed.org/~/media/files/publications/studies/endingtbtf/the-minneapolis-plan/the-minneapolis-plan-to-end-too-big-to-fail-final.pdf.

financial institutions (see Johnson and Kwak 2011). Anat Admati from Stanford, however, proposed a higher capital requirement for big banks, i.e. asking large financial institutions to provide more capital as a buffer to absorb negative shocks (see Admati and Hellwig 2014). Meanwhile, the idea of encouraging banks to issue CoCos has gained popularity (see Flannery 2016). But how effective are those policies in addressing the TBTF? What is the optimal regulation? In addition to more empirical works, one also need a unified theoretical framework to analyze these questions.

In this paper, we provide a simple model to analyze the TBTF and its optimal regulation. Our first objective is to explain the emergence of TBTF absence from any regulatory tools discussed above. The fundamental reason for TBTF is due to the government's bail-out incentive during crises, consistent with the conventional wisdom in the literature such as Strahan (2013). Moreover, the incentive is stronger when the targeted financial institutions are deemed large enough such that their failure will create huge externalities to the society (see Lorenc and Zhang (2020) for empirical evidence). In that case, the government almost has no option but to use the taxpayers' money for bail-out even though the fiscal transfer incurs a dead-weight loss to the society. In the model, we assume a costly bankruptcy process to capture the negative externalities from bank failures to the society.[4] We then impose conditions such that the bankruptcy cost is higher than the fiscal transfer from taxpayers to the financial sector. Therefore, the government always finds it optimal to bail out the banking sector during crises. However, such a policy creates an ex ante moral hazard problem for the banks — those firms tend to become excessively large compared to a world without government bail-out. This is the notion of TBTF due to the government's inability to commit no bail-out. It happens because the government's bail-out lowers the funding cost for the banks, consistent with the empirical evidence such as in Iyer, Lærkholm Jensen, Johannesen, and Sheridan (2019). However, the TBTF equilibrium is inefficient because the banks fail to

[4]For example, with the failure of Lehman Brothers, the resolution, which happened well after the financial crisis had passed, recovered less than 30 cents on the dollar for creditors, at a cost of more than $9 billion in administrative and other expenses (see Fleming and Sarkar 2014). Estimating the bankruptcy cost is a non-trivial task. Altman (1984) finds that the cost is on average 11-17% of the firm values up to three years prior to bankruptcy.

internalize the social cost of bailing them out during crises.

To address the inefficiency from TBTF, we consider a range of possible policies, including a minimum Tier-1 or Tier-2 capital requirement, a direct cap/tax on size, and issuance of CoCos. The capital requirement is a policy that encourages banks to raise more capital which can be used to absorb losses during crises.[5] The size regulation is a policy that directly restrict banks' balance sheet.[6] It could be in the form of a cap on the maximum amount of balance sheet, or a tax schedule increasing with the balance sheet.[7] Contingent convertible bonds, however, are financial securities that convert bond holders into equity holders under circumstances. By doing so, one can ease the financial conditions for the banks and thus avoid the costly bankruptcy process. We evaluate the effectiveness of those three policy tools one by one in our model and find that none of them is sufficient to address the inefficiency from TBTF. Specifically, capital requirement forces the banks to put more stake into their investment yet fails to address the TBTF — the overall size of balance sheet is still excessive and the government's bail-out is unavoidable during crises. The size regulation, however, can reduce the excessive size taken by the banking sector. Yet, it cannot eliminate the government bail-out in equilibrium. Moreover, the cost of bail-out is not internalized by the banking sector. Different from those two regulations, the issuance of CoCos is able to resolve the TBTF inefficiency if the banks are forced to use them. However, without any regulatory requirement, the banks will never issue CoCos but use risky debt to become TBTF.

Our second result features the optimal regulation. To facilitate analysis, we define two welfare benchmarks. One is a first-best benchmark where there is no inefficiency in the allocation. The other is a second-best allocation where the government bail-out is needed but the social cost for the bail-out is internalized by the banks. Clearly, the social welfare in the first-best allocation is higher than the second-best because it avoids the dead-weight loss from fiscal transfer needed for

[5] See Haubrich (2020) for the recent development of capital requirement in the U.S..

[6] The Dodd-Frank Act actually allows the Federal Reserve to regulate systemically important institutions who are typically large.

[7] This tax can be thought of a Pigouvian tax that aims at correcting externalities in the banking sector. Such an approach is also widely proposed as a way to regulate cross-border capital flows (see Jeanne and Korinek 2018; Ma 2020a; Ma and Wei 2020; Rebucci and Ma 2020; Liu, Ma, and Shen 2021).

the government bail-out.

We then show that the first-best allocation can be implemented by the issuance of CoCos and the capital requirement. The intuition is as follows. CoCos provide a superior risk-sharing for the banking sector because the security converts the debt holder into an equity holder during crises and thus effectively conduct a transfer from the investors to the banking sector. It is beneficial because it avoids the costly bankruptcy process and does not require government bail-out. Moreover, it improves the key market friction between the banking sector and depositors, i.e. a limited risk-sharing — the banks need to pay back depositors promised payments irrelevant of the states of world; Otherwise, they need to declare bankruptcy. However, CoCos alone cannot implement the first-best allocation since the banker always finds it optimal to take advantage of the bail-out and thus issue defaultable debts, a typical moral hazard issue. Therefore, a capital requirement is necessary to rule out such a possibility. Intuitively, the existence of capital requirement helps to address the moral hazard issue incurred by the government bail-out. As a result, when it is used together with the issuance of CoCos, the first-best allocation can be achieved, consistent with Aiyar, Calomiris, and Wieladek (2015).

Similarly, the second-best allocation can be implemented by a combination of capital requirement and size regulation. Intuitively, the size regulation is able to reduce the banks' balance sheet to an optimal level. However, the government bail-out is unavoidable in equilibrium. Moreover, banks do not internalize such a bail-out cost if only size regulation is imposed. To correct this inefficiency, a capital requirement is needed to force the banker to put more stake into the investment. Therefore, in equilibrium, it is necessary to combine both the size regulation and capital requirement to reach the second-best allocation. This is consistent with the move from the Fed governor Jerome Powell who intends to make big banks "fully internalize the risk" they pose to the economy.[8]

Our model captures several layers of the issues surrounding the TBTF debate. First, quite literally, banks can become too big to fail. This is because bank failure is typically costly to creditors and depositors, as well as disruptive to the local

[8]See details at https://www.wsj.com/articles/feds-tarullo-warns-banks-of-significant-increase-in-capital-in-future-stress-tests-1464870270.

and even national economies (see Bernanke 1983; Chabot 2011; Bernanke 2013). Moreover, the larger the bank, the more costly and disruptive its failure will be (see White and Yorulmazer 2014; McAndrews, Morgan, Santos, and Yorulmazer 2014). Therefore, when a large bank finds itself on the brink of collapse, the government is inclined to intervene in the form of recapitalization by using public funds (i.e., a bail-out).[9] Second, knowing that the government will intervene, banks have a strong incentive to become TBTF. Naturally, a bank that has received either the implicit or explicit status of TBTF will face less scrutiny from the market and will be able to raise more and cheaper debts (see Strahan 2013; Santos 2014; Jacewitz and Pogach 2018). Furthermore, TBTF banks will be more willing to gamble with their investments (see Afonso, Santos, and Traina 2014; Gropp, Gründl, and Guettler 2014; Dávila and Walther 2020). Third, on anticipating such intervention and banks' behavior, authorities have tried to regulate those banks that are (or may become) TBTF. For instance, under the authority of the FDIC Improvement Act, banks that received the TBTF status (implicitly or explicitly) were subject to a broader scope of regulation and supervision. However, TBTF banks continue to get larger in good times and require ever more public assistance in bad times (see Strahan 2013).

As is known to all, regulating TBTF is not a simple task. First, it is difficult to identify and measure the TBTF problem because financial markets have grown not only in size but also in complexity (see Stern and Feldman 2004). Furthermore, the benefit that TBTF institutions receive is mostly at the margin, which can vary greatly across firms of different sizes with different portfolio compositions and performance histories (see Ennis and Malek 2005). Even considering the negative impacts of TBTF as given, the optimal regulation remains debatable.[10] For

[9] Whether the bail-out can generate some positive effects is an elusive question. See an empirical estimate on the credit market and real effects from one of the largest borrower bailouts in India by Giné and Kanz (2018).

[10] Furthermore, another fundamental concern regarding TBTF is the use of public funds to assist open banks. Strahan (2013) provides an excellent survey of the issue, arguing that TBTF is partly due to — and always reinforced by — the government's commitment to assist large financial institutions in distress. The justification for an ex post intervention can be traced back to Bagehot (1873), who explained the need and presented the principles for lending of last resort, and to Diamond and Dybvig (1983), whose model provides the rationale for policy actions that prevent widespread contagion of liquidity shocks. A formal argument for bail-out can also be found in the representation hypothesis by Dewatripont and Tirole (1994), who argue that depositors are too small, and thus

instance, Johnson and Kwak (2011) argued for a straightforward cap on size and called for division of the largest financial institutions in the United States. Others, however, strongly resisted the idea for fear of inhibiting innovation and economies of scales.[11][12]

In a simple model like ours, one can understand why regulating TBTF is a difficult task. According to the model, no single policy regulation can resolve the inefficiency with TBTF. One needs the capital requirement to let the banks put more stake in the risky investment and thus address the moral hazard problem generated by the government bail-out. However, it cannot prevent banks to become excessively (and inefficiently) large. Size regulation, used alone, also cannot fully resolve the inefficiency even though it can effectively limit the size of balance sheet for the banks. One need the capital requirement and the size regulation together to let the banks fully internalize the bail-out cost and implement the second-best allocation. However, there still exists inefficiency from the bail-out because of the dead-weight loss associated with fiscal transfer for bailing out. To completely eliminate such a distortion and implement the first-best allocation, one will need to introduce CoCos to avoid the costly bankruptcy and meanwhile use the capital requirement to address the moral hazard issue generated by government bail-out.[13]

It is noteworthy that, in this paper, we aim to highlight the discussion about TBTF in terms of bank size, and thus choose to simplify our main model with respect to systemic risk and its associated regulations. As Afonso, Santos, and Traina

need protection. Based on these reasons, governments have repeatedly provided bailouts to failing institutions throughout history, and they seemingly use more of the taxpayers' money each time. To address this issue, a growing body of literature has advocated for "bail-in" regulation, thereby shifting the burden of saving failing banks from taxpayers to holders of high-yielding bonds. In particular, Sommer (2014) and Flannery (2014) support the use of convertible debts at the largest financial institutions as a counter-measure to the moral hazard of TBTF.

[11] Krugman, Paul. *Financial Reform 101*. April 1, 2010. http://www.nytimes.com/2010/04/02/opinion/02krugman.html. Retrieved April 20, 2015.

[12] Indeed, a number of studies have found evidence of economies of scale in banking (see Hughes and Mester 1998; Feng and Serletis 2010; Wheelock and Wilson 2012; Kovner, Vickery, and Zhou 2014).

[13] One caveat in using CoCos as a way to address the TBTF lies in the complexity of the securities. Empirical evidence finds that CoCo issuance can reduce the costs of debt for banks. However, there are many heterogeneity. See Avdjiev, Bogdanova, Bolton, Jiang, and Kartasheva (2020) for further details.

(2014), Cetorelli, McAndrews, and Traina (2014), and Laeven, Ratnovski, and Tong (2014) argue, banks tend to become larger, riskier, and more complex simultaneously. Their complexity can generate systemic risk; in other words, the failure of one institution can lead to a wave of asset fire sales and credit flow disruptions in the financial system, such as the case of Lehman Brothers. For this reason, researchers and policymakers have spent a great deal of effort understanding, measuring, and mitigating systemic risk (see Acharya, Cooley, Richardson, and Walter 2010a and Tobias and Brunnermeier 2016). However, bank size regulation remains a crucial aspect of TBTF. For example, Brewer and Jagtiani (2013) examined the data on bank mergers in the United States between 1991 and 2004, finding that banks were willing to pay additional premiums in acquisitions that expanded their size into the TBTF regime. As size continues to play an important role in bankers' business decisions and policymakers' responses, the pros and cons of bank size require more attention.

The paper proceeds as follows. Section 2 reviews the relevant literature. Section 3 describes the model and examines the market outcome in a laissez-faire environment. Section 4 provides evaluate a range of policy responses to address the issue of TBTF. Section 5 concludes.

2 Literature Review

Our paper contributes to several strands of the literature. First, our paper belongs to the large literature on the emergence of TBTF and its relationship with government bailout policies. For example, Freixas (1999) provides a game-theoretic model of bank failure under the crucial assumption that the cost of bank liquidation increases with bank size. Therefore, if the regulator cannot commit to an ex ante policy, the model results in the regulator's pure strategy of bailing out sufficiently large banks. In this case, banks anticipate the policy and structure their liabilities to maximize the value of a bail-out. Similarly, Goodhart and Huang (1999) analyze TBTF from the perspective of the lender of last resort (LOLR). Given that the cost of bank failure rises (with respect to bank size) faster than the cost of bank rescue, the authors find that a sufficiently large bank will always receive liquidity injections.

The ex post bailouts not only induce banks to become excessively large but also change the behavior of smaller banks. For example, Acharya and Yorulmazer (2007) demonstrate a herding behavior by small banks in a model with ex post bail-out. Intuitively, when the number of bank failures is small, the regulator will let other banks acquire the failed banks. However, when the number is large, the social cost is sufficiently high that an ex post bailout is optimal. Anticipating the regulator's ex post decision, small banks tend to correlate their risk of failure to increase their bailout subsidy. In this case, a TBTF bank will differentiate itself from the small banks because its bailout subsidy does not increase with the herd. Similarly, Dávila and Walther (2020) also find that the presence of large banks exacerbates the risk-taking behavior of small banks and can lead to higher bailout costs. Using a model with a continuum of small banks and a definitive number of large banks, the authors show that by internalizing their size, large banks take on more risk to increase their chances of receiving bailouts. Even though small banks cannot directly influence the equilibrium bailout probability via their individual leverage decisions, they handle more risk in the presence of large banks, and hence, in aggregate, increase the probability of bailouts. Size, as Dávila and Walther (2020) concludes, does matter.

Other than the theoretical work, the empirical findings also suggest that TBTF exists. For example, Hett and Schmidt (2017) estimate the changes in market discipline in reaction to the failure of Lehman Brothers and the initiation of the Dodd-Frank Act. They find that market discipline is weaker for the government-sponsored enterprises and systemically important banks than for investment banks. Similarly, Iyer, Lærkholm Jensen, Johannesen, and Sheridan (2019) find that the TBTF banks can retain and attract uninsured deposits in a crisis at the expense of other banks even those banks differentially lower their interest rates. Using international sample of banks from 104 countries, Cubillas, Fernández, and González (2017) confirm the predominance of the too-big-to-fail hypothesis by studying the sensitivity of the cost of deposits to bank risk.

With respect to this strand of literature, our paper provides a consistent rationale for the emergence of TBTF that is due to the government's inability to commit no bail-out during crises. Differently, we focus on the different sets of ex ante

regulations to address the TBTF inefficiency. In particular, we focus on the relative scope of capital requirement, bank size regulation and issuance of CoCos. We then demonstrate the implementation of first- and second-best allocation to resolve the inefficiencies with TBTF.

Second, our paper belongs to the literature on the banking regulation. A general discussion of banking regulation can be found in classical textbook such as Dewatripont and Tirole (1994) and Freixas and Rochet (1997). Employing similar modeling techniques as Acharya and Yorulmazer (2007), Acharya (2009) finds that banks tend to invest in the same industry, increase the correlation of their returns and failures, and thereby extract greater bailout subsidies. The paper then proposes a capital requirement that considers the banks' joint risk, and shows that such a policy can alleviate systemic risk. Similarly, Farhi and Tirole (2012) examine a model in which banks tend to hold little liquidity and take on too much correlated risk. Therefore, in their model, it is optimal for the regulator to impose an ex ante liquidity requirement to eliminate ex post bailout equilibriums. Furthermore, Calomiris and Herring (2013), Chen, Glasserman, Nouri, and Pelger (2013), and Sommer (2014) propose the use of convertible debts to ensure bank solvency. Directly tackling the banks' incentive to take risk, Acharya, Pedersen, Philippon, and Richardson (2010b) argue that the optimal policy to regulate systemic risk is a "Pigouvian tax", which is based on the banks' expected losses in a systemic crisis and varies with the banks' size, leverage, risk, and correlation with the rest of the financial sector. In a similar vein, Ma (2020b) analyzes the optimal regulation in a general framework with externalities both within and across sectors and suggests to use government regulation, either in the form of quantity or price regulation, as a way to regulate any distortions generated from the banking sector.

Our paper differs from this body of literature in considering a range of ex ante regulations. First, we consider the effectiveness of individual regulation one by one in addressing the TBTF. We then focus on the combinations of those regulations that can reach the first- and second-best allocations.

Last, our paper belongs to the literature that investigates the relationship between ex ante and ex post regulations in the financial sector. For example, Jeanne and Korinek (2020) analyze the relative scope of ex ante versus ex post interven-

tion in addressing financial instability and externalities in a three-period model with collateral borrowing constraints. Their analysis suggests that a combination of both regulations is needed in the optimal regulatory design. Similarly, both Benigno, Chen, Otrok, Rebucci, and Young (2013) and Ma (2020a) provide a quantitative analysis for the welfare implication of optimal policy intervention. In particular, the Pigouvian tax in their analysis is not just used in an ex ante fashion but also in ex post.

Similar to those analysis, our paper also focuses on the ex ante intervention to address inefficiency. Differently, we view the bailout, i.e. the ex post intervention, as the source of inefficiency for TBTF and focus on the optimal ex ante policies to minimize the distortions generated from the ex post intervention. In particular, we show that the first-best allocation can be implemented by using a combination of CoCos and capital requirement, both of which belongs to the ex ante intervention. Moreover, no ex post intervention is needed in the first-best allocation.

3 The Model

Our model features a two-period economy, $t = 1, 2$, with a continuum of depositors of mass 1 and a representative entrepreneurial banker, all of whom are risk-neutral.

Depositors are assumed to be deep pocket and they (collectively) receive an arbitrarily large endowment, w, at $t = 1$. Their utilities are given as follows:

$$U^d = E_1\left[c_1^d + c_2^d\right],$$

where c_t^d is their time-t consumption.

Banker receives an endowment, e_0, at $t = 1$, and has utility function as follows:

$$U^b = E_1\left[c_1^b + c_2^b\right],$$

where c_t^b is her time-t consumption. Moreover, $c_t^b \geq 0$, which captures the idea that equity issuance is costly and the risk-sharing between depositors and the banker is limited (see Brunnermeier and Sannikov 2014).

Technology Only the banker has access to an investment technology. At date 1, the banker chooses investment scale $I \geq 0$. At date 2 the investment yields either $\rho(I)I$ in good state with probability α or $\underline{\rho}I$ in bad state with probability $1-\alpha$. Here $\underline{\rho} < 1$ captures a loss in bad state.[14]

Possibility of Bankruptcy The banker can use either internal equity E or issue debt D with a promised return $R = 1 + r$ to finance investment. Bankruptcy occurs whenever the promised repayment is not satisfied. In the case of bankruptcy, limited liability requres a liquidation of bank's asset. However, there is a deadweight loss in this process and we assume that only a fraction $\lambda \in [0,1)$ of investment can be recovered.

Issuance of Risky Debt There are three options for the banker to conduct investment: (1) use internal equity, (2) issue risk-free debt, or (3) issue risky debt. In the case of option (1) and (2), there is no bankruptcy cost and thus no role for government intervention. In the real world, neither of them is realistic. Hence, we impose the following assumption such that the banker finds it optimal to issue risky debt and the bankruptcy occurs in the bad state.

Assumption 1. *(Issuance of Risky Debt)*

Let $I_0 = \frac{e_0}{1-\underline{\rho}}$, where I_0 is the maximum amount of investment the banker can have when issuing only risk-free deposits. The following relations hold,

$$\alpha\Big[\rho(I_0)I_0\Big]' + (1-\alpha)\underline{\rho} > 1$$

and

$$I_0 \geq \alpha\rho(I_0)I_0 + (1-\alpha)\underline{\rho}I_0.$$

Assumption 1 implies that the banker finds it optimal to expand its investment scale beyond the maximum amount of investment funded only by internal equity and risk-free debt. The internal equity and risk-free debt are perfect substitute to the banker since their costs are the same.

[14] The usual concavity assumption on technology applies here, i.e. $\rho(I)I$ is strictly increasing and concave.

3.1 Laissez Faire

We only consider a Laissez Faire with risky debt due to Assumption 1. The maximization problem is given as follows, where the banker chooses internal equity E, investment size I and promised return r to maximize her utility function.[15]

$$\begin{aligned}
\Pi^{LF} &= \max_{E,I,r} E_1\left[c_1^b + c_2^b\right] = e_0 - E + \alpha\Big(\rho(I)I - (1+r)(I-E)\Big) \\
&\text{s.t.} \quad e_0 \geq E \qquad (1)\\
&\qquad \rho(I)I \geq (1+r)(I-E) \qquad (2)\\
&\qquad \alpha(1+r)(I-E) + (1-\alpha)\underline{\rho}\lambda I = I - E. \qquad (3)
\end{aligned}$$

where E, I, r are the internal equity, investment scale and promised return of risky debt $D = I - E$ for the banker. Condition (1) and (2) are imposed by the limited risk-sharing between the banker and depositors, i.e. $c_t^b \geq 0$ for $t = 1, 2$. Condition (3) is the break-even condition for depositors. Substituting condition (3) into the utility function, we get the following problem.

$$\Pi^{LF} = \max_I e_0 + \alpha\rho(I)I + (1-\alpha)\underline{\rho}\lambda I - I$$

The first order condition implies that

$$\alpha\left[\rho\left(I^{LF}\right)I^{LF}\right]' + (1-\alpha)\underline{\rho}\lambda = 1$$

Notice the marginal benefit of the project at the bad state is $\underline{\rho}\lambda$ rather than $\underline{\rho}$ due to the bankruptcy cost.[16] The cost is beared by the banker since the depositors only care about the expected return.

Note There is no difference between the internal equity and risky debt since

[15] Assumption 1 implies that banks would like to issue risky debt since she finds it optimal to increase the size I above the maximum amount of investment that she can have by issuing only the risk-free debt.

[16] One might realize that the marginal benefit under Laissez Faire is smaller than the marginal benefit of the project since $\lambda < 1$. Hence we need to impose an additional assumption such that the banker wants to take the risky debt as opposed to the risk-free debt. The assumption is that $\Pi^{LF} > e_0 + \alpha\rho(I_0)I_0 + (1-\alpha)\underline{\rho}I_0 - I_0$, which is the utility of issuing the maximum amount of risk-free debt.

their marginal costs are the same. The introduction of risky debt only reduces the marginal benefit of the project in the bad state due to the bankruptcy cost. To illustrate this point, we introduce the first-best benchmark.

3.2 The First-best

We define the first-best investment I^* that solves the following problem.

$$\Pi^{FB} = \max_I e_0 + w + \alpha\rho(I)I + (1-\alpha)\underline{\rho}I - I$$

The optimality condition implies that

$$\alpha\left[\rho(I^*)I^*\right]' + (1-\alpha)\underline{\rho} = 1$$

The difference between the Laissez Faire and the First-best lies in the marginal benefit of investment. Due to the bankrupcty cost in the bad state, the marginal benefit is lower in the Laissez Faire, corresponding to the inefficiently lower investment than the first-best. A government, in this case, might have an incentive to bail out the banker so as to save the cost of bankruptcy. As noted later, we argue that this motive creates a moral hazard problem and leads to the issue of TBTF.

3.3 Government Bailout and TBTF

In the bad state, the banker declares bankruptcy. The government has an incentive to recapitalize (bail out) the bank since it saves the bankruptcy cost.[17] However, there will be a cost to bail out banks. The government needs to tax depositors in order to finance the bailout funds. In reality, the taxation is distortionary. For simplicity, we assume that there is a fixed marginal cost in raising public funds, $\eta > 1$.[18] For the government, it is optimal to bail out the bank if the total cost of liquidation exceeds the total cost of raising the necessary public funds. Formally, the condition for a

[17] In reality, the justification can also come from the protection of small depositors as in Dewatripont and Tirole (1994).

[18] This is a simplified assumption as in Jeanne and Korinek (2020). Introducing more structure on the distortion cost of taxation will not change the insight of this paper.

bailout is:

$$\underbrace{\eta((1+r)D-\underline{\rho}I)}_{\text{cost of bail out}} \leq \underbrace{(1-\lambda)\underline{\rho}I}_{\text{cost of bankruptcy}},$$

where $D = I - E$ is the amount of risky debt and r is the interest rate on the debt.

The bailout is an important feature in financial crisis. Argubably, the government finds it difficult to commit no bailout ex ante. To capture this phenomenon, we impose the following assumption such that bailout occurs whenever the bad state is realized.

Assumption 2.

$$\frac{(1-\lambda)\underline{\rho}}{1-\underline{\rho}} > \eta.$$

Assumption 2 says that the cost of bailout is sufficiently small than the bankruptcy cost. Hence, the government cannot commit not to bail out in the event of bankruptcy.

Due to the existence of bailout, TBTF emerges. The maximization problem for the banker is given as

$$\begin{aligned} \Pi^{TBTF} &= \max_{E,I} e_0 - E + \alpha\Big[\rho(I)I - (I-E)\Big] \\ \text{s.t.} \quad & e_0 \geq E \\ & \rho(I)I \geq I - E \end{aligned}$$

The optimality conditions imply that $E = 0$ and the optimal level of investment satisfies

$$\Big[\rho\Big(I^{TBTF}\Big)I^{TBTF}\Big]' = 1$$

Note that bailout increases the marginal benefit of investment for the banker since she/he only cares about the good state. One can easily show that the size of investment I^{TBTF} exceeds the level of investment in the first-best level, I^* and Laissez Faire level, I^{LF}, which is the notion of TBTF. Furthermore, the banker finds it optimal not to use the internal equity since the banker uses risky debt to gamble the profit in the good state. In other words, the existence of bailout creates a moral hazard problem in our model.

Proposition 1. *Government Bailout and TBTF*

TBTF emerges due to the government's bailout. In equilibirum, the banker takes zero internal equity and borrows to understake an investment, whose size exceeds the first-best and Laissez Faire level.

Proof. See the discussion above. □

4 TBTF and Policy Responses

In this section, we evaluate a range of policy responses to address the issue of TBTF. In particular, we focus on capital requirement, size regulation and CoCos. To faciliate analysis, we define a second-best benchmark, where the government cannot commit no bailout but the cost of bailout is internalized by the banks.

4.1 Optimal Regulation for the Second-best

The second-best allocation is defined as the case where the bailout occurs and the cost of bailout is internalized by the agents. To this end, the maximization problem for the society is given as follows

$$\begin{aligned} U^{SB} &= \max_{I,E} e_0 + w - E + \alpha\Big[\rho(I)I - I + E\Big] - (1-\alpha)\eta\Big(I - E - \underline{\rho}I\Big) \\ \text{s.t.} &\quad E \leq e_0. \end{aligned}$$

where $(1-\alpha)\eta\Big(I - E - \underline{\rho}I\Big)$ captures the bailout cost in the bad state.

The optimality conditions require that

$$E = e_0$$

and

$$\alpha\Big[\rho\left(I^{SB}\right)I^{SB}\Big]' + (1-\alpha)\Big[1 - \eta(1-\underline{\rho})\Big] = 1$$

Proposition 2. *Implementation of the Second-best*
To implement the second-best allocation, we need a combination of capital requirement and size regulation. For size regulation, it can be implemented by either a price- or quantity- based regulation.

Proof. See Appendix A. □

Even if there is only one source of inefficiency in the model, i.e. the commitment issue of bailout, one needs two instruments to correct it. The reason is that the only inefficiency affects two decision margins of the banker, the issuance of internal equity and the risky debt. The government wants the size regulation to address the issue of TBTF so as to reduce the size of investment. Conditional on the optimal size of investment, the government also wants to reduce the cost of bailout, i.e. forcing the banker to put more internal equity in the project. As a result, a combination of capital requirement and size regulation is needed. In terms of the size regulation, there is an equivalent result between price- and quantity- based tools in the spirit of Weitzman (1974) since there is no asymmetric information between the government and the banker.

Corollary 1. *Capital requirement or size regulation alone is insufficient to implement the second-best allocation. The capital requirement forces the banker to put more stake into the project but fails to address the issue of TBTF. The size regulation addresses the issue of TBTF but does not minimize the cost of bailout.*

Proof. See Appendix B. □

Corollary 1 says that neither capital requirement nor size regulation can implement the second-best allocation. Consistent with the conventional wisdom, the size regulation is effective in controlling the size of banks. However, the banker still has an incentive to put zero capital into the project. The social cost of bailout is not minimized/internalized. Differently, capital requirement forces the banker to put more stake into the project and minimizes the cost of bailout in the bad state. However, it is ineffective to address the issue of TIBTF. The banker finds it optimal to increase the size of investment so as to gamble in the good state.

4.2 Optimal Regulation for the First-best

The main inefficiency in the second-best world is the issue of bankruptcy. In the first-best world, however, there is no need for bankruptcy. One way to avoid the costly bankruptcy is to increase the risk-sharing between the banker and depositors.

The issuance of CoCos is one option since the security converts from bond to equity in the bad state. One might expect that the banker might choose to issue CoCos so as to achieve the first-best level of investment. However, the following proposition shows that the capital requirement is also needed.

Proposition 3. *Implementation of the First-best*
To implement the first-best allocation, we need a combination of capital requirement and issuance of Contigent Convertible bonds.

Proof. See Appendix C. □

Proposition 3 says that the implementation of the first-best needs a combination of capital requirement and issuance of CoCo. The role of CoCo provides a better risk-sharing mechanism between the banker and depositors. However, CoCo alone cannot implement the first-best since the banker always finds it optimal to issue risky debt and thus incur the bailout. Imposing the capital requirement is one way to reduce the moral hazard problem caused by the bailout. In a way, the capital requirement complements the risk-sharing role of CoCos. The following corollary illustrates this point.

Corollary 2. *Contigent Convertible bonds alone cannot implement the first-best allocation. Instead, the banker finds it optimal to issue risky debt and trigger bailout.*

Proof. See Appendix D. □

4.3 Policy Implication

Our theoretical framework has several important policy implications. First, regulating TBTF is not simple. As illustrated in the model, the fundamental problem for TBTF lies in the government's inability to commit no bail-out during crises. Such an ex post intervention, although efficient during crises, creates an ex ante moral hazard problem. Importantly, no single policy instrument is able to completely eliminate its inefficiency. Second, capital requirement is needed for optimal regulation in either first- or second-best allocation. The nature of capital requirement is to let the banks put enough stake in the investment, which at the margin can

minimize the moral hazard problem. Size regulation can be effective in limiting the balance sheet but fails to minimize the bailout cost without the help of capital requirement. Similarly, banks have no incentive to issue CoCos without capital requirement. Third, the key policy message from the simple model is that one needs to combine multiple policy instruments to regulate the banking sector. But it is useful to delineate the relative scope of each policy before combining them together. Arguably, the optimal regulation analyzed in our framework is an ideal scenario that simplifies many real world complications. The ability for the regulators to implement those policy tools will significantly affect the efficiency of the economy. One needs to take that into account when regulating TBTF banks.

5 Conclusion

This paper analyzes the optimal regulation in a simple model with government bailout. TBTF emerges because the government cannot convincingly commit no bailout during crises. As a result, banks become excessively large in equilibrium. The TBTF allocation is inefficient because there is a deadweight loss from government bail-out during crises.

In this framework, our first result shows that no single policy instrument can resolve the inefficiency from TBTF. Capital requirement can force banks to put more stakes into their investment but fails to prevent banks to become large. Size regulation, however, cannot let banks internalize the social cost of bail-out. Moreover, banks never choose to issue CoCos without further regulation because issuing risky debt allows them to enjoy the TBTF subsidy. This analysis thus highlights the difficulty of regulating TBTF banks.

Our second result features the optimal policy regulation that combines multiple tools. To implement the first-best allocation that does not involve the costly bankruptcy, a combination of capital requirement and issuance of CoCos is needed. As the issuance of CoCos alone fails to incentivize banks, the existence of capital requirement, however, reduces the moral hazard problem incurred by the bailout and complements the risk-sharing role of CoCos.

Furthermore, capital requirement is also needed for the implementation of the

second-best allocation in which the government bail-out is unavoidable but its cost is internalized by the banks through the capital requirement regulation. Moreover, a size regulation is also needed to restrict the excessively large balance sheet.

Our paper has important policy implications. First, it provides a simple framework to evaluate different policy proposals. In particular, our model provides a welfare benchmark framework to design policy packages. Second, our paper points out two important market frictions that policymakers need to address — limited risk-sharing and moral hazard problem. To the extent that policy tools resolve such frictions, TBTF problem can be alleviated or completely removed.

There are several issues that we left for future research. First, our model is silent on the excessive risk-taking behavior by the banking sector. Introducing such a feature will enrich the analysis. It is interesting to explore such a case. Second, it is interesting to introduce TBTF in a full-fledged DSGE framework to understand its inefficiency quantitatively. In such an environment, understanding the tradeoff of regulatory policy instrument is worth investigating.

References

ACHARYA, V. V. (2009): "A Theory of Systemic Risk and Design of Prudential Bank Regulation," *Journal of Financial Stability*, 5, 224–255.

ACHARYA, V. V., T. F. COOLEY, M. RICHARDSON, AND I. WALTER (2010a): *Regulating Wall Street*, John Wiley & Sons, Inc.

ACHARYA, V. V., L. H. PEDERSEN, T. PHILIPPON, AND M. RICHARDSON (2010b): "Taxing Systemic Risk," in *Regulating Wall Street*, ed. by V. V. Acharya, T. F. Cooley, M. Richardson, and I. Walter, John Wiley & Sons, Inc., 121–142.

ACHARYA, V. V. AND T. YORULMAZER (2007): "Too Many to Fail—An Analysis of Time-inconsistency in Bank Closure Policies," *Journal of Financial Intermediation*, 16, 1–31.

ADMATI, A. AND M. HELLWIG (2014): *The Bankers' New Clothes: What's Wrong with Banking and What to Do about It-Updated Edition*, Princeton University Press.

AFONSO, G., J. A. C. SANTOS, AND J. TRAINA (2014): "Do "Too-Big-To-Fail" Banks Take on More Risk?" *Economic Policy Review*, 20, 41–58.

AIYAR, S., C. W. CALOMIRIS, AND T. WIELADEK (2015): "Bank Capital Regulation: Theory, Empirics, and Policy," *IMF Economic Review*, 63, 955–983.

ALTMAN, E. I. (1984): "A Further Empirical Investigation of the Bankruptcy Cost Question," *Journal of Finance*, 39, 1067–1089.

AVDJIEV, S., B. BOGDANOVA, P. BOLTON, W. JIANG, AND A. KARTASHEVA (2020): "CoCo Issuance and Bank Fragility," *Journal of Financial Economics*, 138, 593–613.

BAGEHOT, W. (1873): *Lombard Street: A Description of the Money Market*, London: HS King.

BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. R. YOUNG (2013): "Financial Crises and Macro-prudential Policies," *Journal of International Economics*, 89, 453–470.

BERNANKE, B. S. (1983): "Nonmonetary Effects of the Financial Crisis in Propagation of the Great Depression," *American Economic Review*, 73, 257–276.

——— (2013): "Semiannual Monetary Policy Report to the Congress," Board of Governors of the Federal Reserve System, testimony Before the Committee on Financial Services, US House of Representatives.

BREWER, E. AND J. JAGTIANI (2013): "How Much Did Banks Pay to Become Too-Big-To-Fail and to Become Systemically Important?" *Journal of Financial Services Research*, 43, 1–35.

BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

CALOMIRIS, C. W. AND R. J. HERRING (2013): "How to Design a Contingent Convertible Debt Requirement That Helps Solve Our Too-Big-To-Fail Problem," *Journal of Applied Corporate Finance*, 25, 39–62.

CETORELLI, N., J. MCANDREWS, AND J. TRAINA (2014): "Evolution in Bank Complexity," *Economic Policy Review*, 20, 85–106.

CHABOT, B. R. (2011): "The Cost of Banking Panics in an Age Before 'Too Big To Fail'," Working Paper 15, Federal Reserve Bank of Chicago.

CHEN, N., P. GLASSERMAN, B. NOURI, AND M. PELGER (2013): "CoCos, Bail-in, and Tail Risk," Working Paper 0004, Office of Financial Research.

CUBILLAS, E., A. I. FERNÁNDEZ, AND F. GONZÁLEZ (2017): "How Credible is a Too-Big-to-Fail Policy? International Evidence from Market Discipline," *Journal of Financial Intermediation*, 29, 46–67.

DÁVILA, E. AND A. WALTHER (2020): "Does Size Matter? Bailouts with Large and Small Banks," *Journal of Financial Economics*, 136, 1–22.

DEWATRIPONT, M. AND J. TIROLE (1994): *The Prudential Regulation of Banks*, Walras-Pareto Lectures, MIT Press.

DIAMOND, D. W. AND P. H. DYBVIG (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

ENNIS, H. M. AND H. S. MALEK (2005): "Bank Risk of Failure and the Too-Big-To-Fail Policy," *Federal Reserve Bank of Richmond Economic Quarterly*, 91, 21–44.

FARHI, E. AND J. TIROLE (2012): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," *American Economic Review*, 102, 60–93.

FENG, G. AND A. SERLETIS (2010): "Efficiency, Technical Change, and Returns to Scale in Large US Banks: Panel Data Evidence From an Output Distance Function Satisfying Theoretical Regularity," *Journal of Banking & Finance*, 34, 127–138.

FLANNERY, M. J. (2014): "Contingent Capital Instruments for Large Financial Institutions: A Review of the Literature," *Annual Review of Financial Economics*, 6, 225–240.

——— (2016): "Stabilizing Large Financial Institutions with Contingent Capital Certificates," *Quarterly Journal of Finance*, 6, 1650006.

FLEMING, M. J. AND A. SARKAR (2014): "The Failure Resolution of Lehman Brothers," *Economic Policy Review*, 20, 175–206.

FREIXAS, X. (1999): "Optimal Bail Out Policy, Conditionality and Creative Ambiguity," Discussion Papers DP327, Financial Markets Group.

FREIXAS, X. AND J.-C. ROCHET (1997): *Microeconomics of Banking*, vol. 2, MIT press Cambridge, MA.

GINÉ, X. AND M. KANZ (2018): "The Economic Effects of a Borrower Bailout: Evidence from an Emerging Market," *Review of Financial Studies*, 31, 1752–1783.

GOODHART, C. AND H. HUANG (1999): "A Model of the Lender of Last Resort," Working Paper 39, International Monetary Fund.

GROPP, R., C. GRÜNDL, AND A. GUETTLER (2014): "The Impact of Public Guarantees on Bank Risk-taking: Evidence From a Natural Experiment," *Review of Finance*, 18, 457–488.

HAUBRICH, J. G. (2020): "A Brief History of Bank Capital Requirements in the United States," *Economic Commentary*.

HETT, F. AND A. SCHMIDT (2017): "Bank Rescues and Bailout Expectations: The Erosion of Market Discipline during the Financial Crisis," *Journal of Financial Economics*, 126, 635–651.

HUGHES, J. P. AND L. J. MESTER (1998): "Bank Capitalization and Cost: Evidence of Scale Economies in Risk Management and Signaling," *Review of Economics and Statistics*, 80, 314–325.

IYER, R., T. LÆRKHOLM JENSEN, N. JOHANNESEN, AND A. SHERIDAN (2019): "The Distortive Effects of Too Big to Fail: Evidence from the Danish Market for Retail Deposits," *Review of Financial Studies*, 32, 4653–4695.

JACEWITZ, S. AND J. POGACH (2018): "Deposit Rate Advantages at the Largest Banks," *Journal of Financial Services Research*, 53, 1–35.

JEANNE, O. AND A. KORINEK (2018): "Managing Credit Booms and Busts: A Pigouvian Taxation Approach," *Journal of Monetary Economics*.

——— (2020): "Macroprudential Regulation versus Mopping up After the Crash," *Review of Economic Studies*, 87, 1470–1497.

JOHNSON, S. AND J. KWAK (2011): *13 Bankers: The Wall Street Takeover and the Next Financial Meltdown*, Vintage Books USA.

KEISTER, T. (2016): "Bailouts and Financial Fragility," *Review of Economic Studies*, 83, 704–736.

KOVNER, A., J. VICKERY, AND L. ZHOU (2014): "Do Big Banks Have Lower Operating Costs?" *Economic Policy Review*, 20, 1–27.

LAEVEN, M. L., L. RATNOVSKI, AND H. TONG (2014): "Bank Size and Systemic Risk," Staff Discussion Note 04, International Monetary Fund.

LIU, S., C. MA, AND H. SHEN (2021): "Sudden Stop with Local Currency Debt," *Working Paper*.

LORENC, A. G. AND J. Y. ZHANG (2020): "How Bank Size Relates to the Impact of Bank Stress on the Real Economy," *Journal of Corporate Finance*, 62, 101592.

MA, C. (2020a): "Financial Stability, Growth and Macroprudential Policy," *Journal of International Economics*, 122, 103259.

——— (2020b): "Self-regulation versus Government Regulation: An Externality View," *Journal of Regulatory Economics*, 58, 166–183.

MA, C. AND S.-J. WEI (2020): "International Equity and Debt Flows to Emerging Market Economies: Composition, Crises, and Controls," *NBER Working Paper No. 27129*.

MCANDREWS, J. J., D. P. MORGAN, J. A. C. SANTOS, AND T. YORULMAZER (2014): "What Makes Large Bank Failures so Messy and What Should be Done About it?" *Economic Policy Review*, 20, 229–244.

REBUCCI, A. AND C. MA (2020): "Capital Controls: A Survey of the New Literature," *Oxford Research Encyclopedia of Economics and Finance*.

SANTOS, J. A. C. (2014): "Evidence From the Bond Market on Banks' "Too-Big-To-Fail" Subsidy," *Economic Policy Review*, 20, 29–39.

SOMMER, J. H. (2014): "Why Bail-in? And How!" *Economic Policy Review*, 20, 207–228.

STERN, G. H. AND R. J. FELDMAN (2004): *Too Big To Fail: The Hazards of Bank Bailouts*, Brookings Institution Press.

STRAHAN, P. E. (2013): "Too Big To Fail: Causes, Consequences, and Policy Responses," *Annual Review of Financial Economics*, 5, 43–61.

TOBIAS, A. AND M. K. BRUNNERMEIER (2016): "CoVaR," *American Economic Review*, 106, 1705.

WEITZMAN, M. L. (1974): "Prices vs. Quantities," *Review of Economic Studies*, 41, 477–491.

WHEELOCK, D. C. AND P. W. WILSON (2012): "Do Large Banks Have Lower Costs? New Estimates of Returns to Scale for US Banks," *Journal of Money, Credit and Banking*, 44, 171–199.

WHITE, P. AND T. YORULMAZER (2014): "Bank Resolution Concepts, Tradeoffs, and Changes in Practices," *Economic Policy Review*, 20, 153–173.

A Proof of Proposition 2

Proof. Suppose that the government imposes a capital requirement $E \geq \bar{E}$ and a quantity-based size regulation $I \leq \bar{I}$. The maximization problem for the banker is given as follows.

$$\begin{aligned} \Pi(\bar{E},\bar{I}) &= \max_{E,I} e_0 - E + \alpha\Big(\rho(I)I - (I-E)\Big) \\ \text{s.t.} &\quad e_0 \geq E, E \geq \bar{E}, I \leq \bar{I}. \end{aligned}$$

The optimality conditions implies that

$$E = \bar{E},\ I = \min\left\{I^{TBTF}, \bar{I}\right\}.$$

By choosing $\bar{E} = e_0$ and $\bar{I} = I^{SB}$, the second-best allocation is implemented.

Similar procedure can be applied to a combination of capital requirement and price-based size regulation. The maximization problem for the banker is given as follows.

$$\begin{aligned} \Pi(\bar{E},\tau) &= \max_{E,I} e_0 - E - \tau I + T + \alpha\Big(\rho(I)I - (I-E)\Big) \\ \text{s.t.} &\quad e_0 \geq E, E \geq \bar{E}. \end{aligned}$$

where in equilibrium $T = \tau I$.

The optimality conditions implies that

$$E = \bar{E},\ \alpha\Big[\rho(I)I\Big]' = \tau + \alpha$$

By choosing $\bar{E} = e_0$ and $\tau = (1-\alpha)\left(1 - \underline{\rho}\right)\eta$, the second-best allocation is implemented.

□

B Proof of Corollary 1

Proof. Assume that the government only has the policy tool of capital requirement. The maximization problem is given by

$$\begin{aligned} \Pi(\bar{E}) &= \max_{E,I} e_0 - E + \alpha\Big(\rho(I)I - (I-E)\Big) \\ \text{s.t.} &\quad e_0 \geq E, E \geq \bar{E}. \end{aligned}$$

The optimality conditions for the banker are given by

$$E = \bar{E},\ \Big[\rho(I)I\Big]' = 1.$$

The Envelope theorem implies that $\Pi(\bar{E})' = -1+\alpha$. For the government, her maximization problem is to choose $\bar{E}$ to maximize the social welfare

$$\max_{\bar{E}} \Pi(\bar{E}) + w - (1-\alpha)\eta\Big(I(\bar{E}) - \bar{E} - \underline{\rho} I(\bar{E})\Big)$$

The optimality condition implies that

$$-1+\alpha-(1-\alpha)\eta(-1) > 0$$

And the government chooses $\bar{E} = e_0$.

Suppose the government only has the policy tool of size regulation. For the sake of argument, we only consider the quantity-based regulation. One can easily show that a price-based regulation can achieve the same allocation due to Weitzman (1974). The maximization problem is given by

$$\begin{aligned} \Pi(\bar{I}) &= \max_{E,I} e_0 - E + \alpha\Big(\rho(I)I - (I-E)\Big) \\ \text{s.t.} &\quad e_0 \geq E, I \leq \bar{I}. \end{aligned}$$

The optimality conditions implies that

$$E = 0,\ I = \min\Big\{I^{TBTF}, \bar{I}\Big\}.$$

The Envelope theorem implies that $\Pi(\bar{I})' = \alpha\Big(\rho(\bar{I})\bar{I}\Big)' - \alpha$. For the government, her maximization problem is to choose $\bar{I}$ to maximize the social welfare

$$\max_{\bar{I}} \Pi(\bar{I}) + w - (1-\alpha)\eta\Big(\bar{I} - \underline{\rho}\bar{I}\Big)$$

The optimality condition implies that

$$\alpha\Big(\rho(\bar{I})\bar{I}\Big)' + (1-\alpha)\Big(1 - \eta(1-\underline{\rho})\Big) = 1$$

And the government chooses $\bar{I} = I^{SB}$. Since $E = 0$, the government cannot implement the second-best. □

C Proof of Proposition 3

Proof. Suppose that the government has the policy of capital requirement and issuance of CoCos. In the bad state, CoCos become external equity, which saves the cost of bankruptcy. The maximization problem is given as

$$\begin{aligned}
\Pi(\bar{E}) \quad = \quad & \max_{E,B,I} e_0 - E + \alpha\Big(\rho(I)I - (I-E-B) - (1+r_B)B\Big) \\
\text{s.t.} \quad & e_0 \geq E, E + B \geq \bar{E}, \\
& \alpha(1+r_B)B + (1-\alpha)\Big[\underline{\rho}I - (I-E-B)\Big] = B, \\
& \underline{\rho}I - (I-E-B) \geq 0.
\end{aligned}$$

where the last inequality ensures that the banker can repay debt and no bankruptcy is declared.

Substituting the break-even condition into the problem changes the problem into the following one.

$$\begin{aligned}
\Pi(\bar{E}) \quad = \quad & \max_{E,B,I} e_0 + \alpha\rho(I)I + (1-\alpha)\underline{\rho}I - I \\
\text{s.t.} \quad & e_0 \geq E, E + B \geq \bar{E}, \\
& \underline{\rho}I - (I-E-B) \geq 0.
\end{aligned}$$

The optimality condition implies that

$$\alpha\Big[\rho(I^*)I^*\Big]' + (1-\alpha)\underline{\rho} = 1$$

Furthermore, $\{E,B\}$ are indetermined. One can choose $\bar{E} \in \Big[(1-\underline{\rho})I^*, I^*\Big]$ so as to satisfy the last inequality constraint. Importantly, the existence of $\bar{E}$ is to rule out the possibility that the banker finds it optimal to issue only risky debt and triggers TBTF. See the proof in Appendix D where there is no capital requirement $\bar{E}$.

□

D Proof of Corollary 2

Proof. Without regulatory constraint, the banker's maximization problem is as follows

$$\begin{aligned}
\Pi^{CoCo} \quad = \quad & \max_{E,B,I} e_0 - E + \alpha\Big(\rho(I)I - (I-E-B) - (1+r_B)B\Big) \\
\text{s.t.} \quad & e_0 \geq E, \\
& \alpha(1+r_B)B + (1-\alpha)\Big[\underline{\rho}I - (I-E-B)\Big] = B, \\
& \underline{\rho}I - (I-E-B) \geq 0.
\end{aligned}$$

Substituting the break-even condition into the problem changes the problem into the following one.

$$\begin{aligned}
\Pi^{CoCo} \quad = \quad & \max_{E,B,I} e_0 + \alpha\rho(I)I + (1-\alpha)\underline{\rho}I - I \\
\text{s.t.} \quad & e_0 \geq E, \\
& \underline{\rho}I - (I-E-B) \geq 0.
\end{aligned}$$

The optimality condition is given as follows

$$\alpha\Big[\rho(I^*)I^*\Big]' + (1-\alpha)\underline{\rho} = 1$$

The last constraint can be easily satisfied by setting $E+B = I^*$. Without loss of

generality, choose $E = 0, B = I^*$.

However, the banker finds it optimal to issue risky debt rather than CoCo since $\Pi^{CoCo} = e_0 + \alpha\rho(I^*)I^* + (1-\alpha)\underline{\rho}I^* - I^* < e_0 + \alpha\rho(I^*)I^* + (1-\alpha)I^* - I^* = e_0 + \alpha\Big(\rho(I^*)I^* - I^*\Big) < e_0 + \alpha\Big(\rho\Big(I^{TBTF}\Big)I^{TBTF} - I^{TBTF}\Big) = \Pi^{TBTF}$. □