Foreign Reserves and Capital Controls: A Financial Development Perspective*

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Abstract

We provide a financial development perspective on the joint use of foreign reserves and capital controls. Empirically, countries with intermediate levels of financial development have the highest reserve-to-GDP ratios, while the use of capital controls declines with financial development. To explain this pattern, we develop a small-open-economy model featuring liquidity shocks to foreign debt. Reserves and capital controls are jointly used to mitigate fire-sale externalities resulting from asset liquidation during liquidity crises. Our model captures the degree of financial development by the effective size of liquidity shocks, replicating the observed relationship between reserves, capital controls, and financial development.

Keywords: Capital Controls; Foreign Reserves; Sudden Stops; Liquidity Shocks; Financial Development.

JEL: F32, F41, F44.

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1 Introduction

Emerging markets and developing economies are subject to volatile capital flows driven by the global financial market and policies of major advanced economies (Rey 2015). For those economies, foreign reserves and capital controls are two important policy tools to smooth impacts from the global financial market (Basu et al. 2020). Accordingly, there is a growing literature on the optimal design of these policy tools (Bianchi and Lorenzoni 2022). However, there is wide cross-country variation in how actively each country uses these policy tools, and important questions remain unanswered: What is the key determinant of the optimal combination of capital controls and reserve policy? What explains the cross-country variation in these policy tools?

In this paper, we provide a financial development perspective on the joint use of foreign reserves and capital controls. Our analysis is motivated by Figure 1. It shows that the relationship between financial development and the reserve-to-GDP ratio is non-monotonic across countries. Countries with an intermediate level of financial development tend to have a higher reserve-to-GDP ratio than countries with either high or low financial development. In contrast, the use of capital controls declines monotonically as financial development increases.

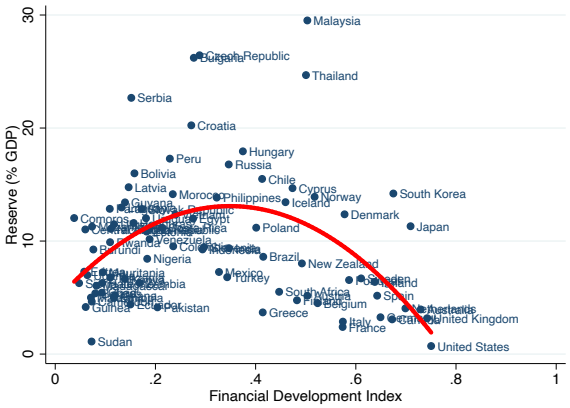
We develop a dynamic small-open-economy model to understand these empirical facts. Our contributions are twofold. On the empirical side, we contribute to the reserve-demand literature (e.g., Obstfeld et al. 2010) by documenting a non-monotonic relationship between reserves and financial development. This contrasts with the monotonically declining relationship between capital controls and financial development. On the theoretical side, we provide a new model to explain these empirical patterns jointly. In particular, our model features a fire-sale externality and an occasionally binding liquidity constraint that requires early repayment of foreign debt before new borrowing. Financial development determines the fraction of debt that must be repaid early during a liquidity shock, effectively setting the size of liquidity shocks. Our analysis shows that this simple model can provide a unified explanation for the observed cross-country patterns.

In our model, households produce and consume tradable goods, borrow from abroad, hold liquid foreign assets (private foreign reserves), and invest in productive assets. Productive assets accumulate over time, and the model features endogenous growth, following the literature that connects business cycles and growth (Comin and Gertler 2006, Queralto 2020, Benigno, Fornaro, and Wolf 2022). Endogenous growth allows us to capture both the persistent impact of crises (Cerra and Saxena 2008, Reinhart and Rogoff 2009) and cur-

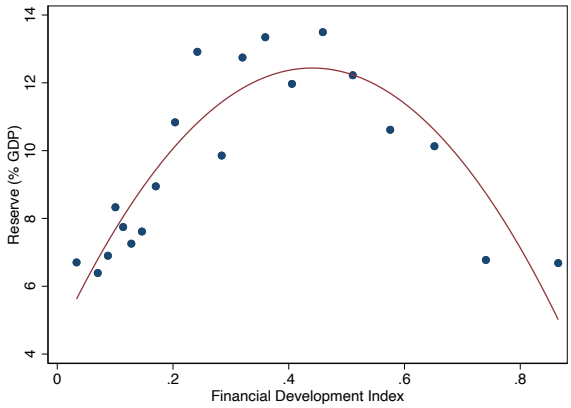
Figure 1 Reserves, Capital Control Index, and Financial Development Index

A: Foreign Reserves and Financial Development Index

A1: Cross-sectional

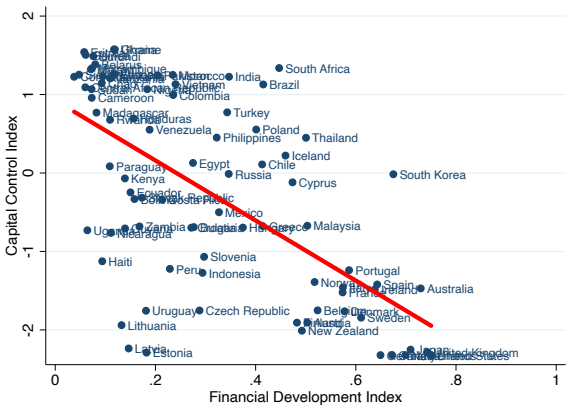


A2: Panel (Bin Scatter)

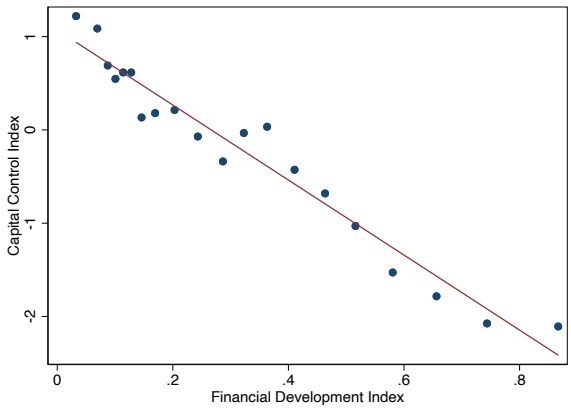


B: Capital Control Index and Financial Development Index

B1: Cross-sectional



B2: Panel (Bin Scatter)



Note. Panel A (B) presents the relationship between reserves (capital control index, i.e. the negative of Chinn-Ito Index in Chinn and Ito 2006) and the financial development index for a panel of 87 economies in 1980-2019. The Financial Development Index is taken from the IMF and is constructed to summarize the development of financial institutions and financial markets in terms of depth, access, and efficiency (Svirydzenka 2016). A1 and B1 plot the cross-sectional relationship where each data point is the time-average for a country. A2 and B2 are bin scatter plots for the panel dataset.

rent account reversals driven by permanent income losses during crises, without relying on collateral borrowing constraints (Aguiar and Gopinath 2007, Guntin et al. 2023).

The key element of the model is a liquidity shock. At the beginning of each period, before new borrowing and production, households may be required to repay a fraction of their foreign debt with an exogenous probability. When this occurs, households can liquidate productive assets to meet the required repayment, but at a low fire-sale price (Aguiar and

Gopinath 2005). To avoid costly liquidation, households accumulate foreign reserves ex ante as a liquidity buffer. Our model thus highlights the liquidity role of foreign reserves (Arce, Bengui, and Bianchi 2025, Céspedes and Chang 2020, Jeanne and Sandri 2023).

The magnitude of asset liquidation during a liquidity shock depends on the liquidity shortage, defined as required debt repayment minus reserve holdings. Households can reduce costly liquidation by decreasing debt and increasing reserves ex ante, but the latter is more efficient. A one-unit increase in reserves directly reduces the next period's liquidity shortage by the same amount, while a one-unit decrease in debt only reduces it by the fraction of debt subject to liquidity shocks, which may be less than one. Thus, reserves have a *liquidity advantage* over debt in reducing liquidity risk. However, reserve holdings have an opportunity cost as their interest rate is lower than that of debt (Jeanne and Rancière 2011). Households balance this liquidity advantage against the opportunity cost when deciding on debt and reserve levels.

Policy intervention is needed due to a fire-sale externality arising from asset liquidation. Foreign buyers' demand for liquidated assets is downward-sloping, and the price declines as the volume of liquidated assets increases. Domestic households take this price as given, leading to a fire-sale externality.[1] Specifically, reducing debt and increasing reserves ex ante can decrease required liquidation and raise the liquidation price, which allows a further reduction in liquidation. However, households do not internalize this effect, resulting in excessive borrowing and insufficient reserve holdings ex ante. The optimal policy consists of a capital-control debt tax and public reserve accumulation.[2]

The degree of financial development is represented by the size of liquidity shocks, that is, the fraction of foreign debt subject to early repayment during liquidity shocks. Although countries may face similar external disturbances, the *effective* size of liquidity shocks varies with the level of financial development. This interpretation is based on two rationales. First, well-developed financial markets provide access to alternative financing and hedging instruments, thus reducing the pressure for immediate external debt repayment. This notion of financial development aligns with theoretical works such as Chang and Velasco (2001), Mendoza et al. (2009), Maggiori (2017), as well as empirical results in Section 2. Second, empirical evidence suggests that bank-loan maturities, including those of foreign banks, are significantly longer in countries with stronger creditor protection and better contract enforce-

[1] Because assets are sold to foreign buyers, this price externality also has a terms-of-trade interpretation. We use the term "fire-sale externality" to emphasize that asset sales are forced by liquidity shortages.

[2] In Appendix D.4, a subsidy on private reserve holdings can implement the same constrained efficient allocation as public reserve accumulation when combined with a capital control tax on foreign debt.

ability (Qian and Strahan 2007, Bae and Goyal 2009). For these two reasons, we associate higher financial development with smaller effective liquidity shocks and vice versa.[3]

Our analysis shows how the size of liquidity shocks—a proxy for financial development—shapes optimal reserve holdings and capital controls. When the size of liquidity shocks is small (high financial development), there is little need to hold large reserves. Conversely, when the size of liquidity shocks is large (low financial development), the liquidity advantage of reserves diminishes because reducing debt also effectively alleviates the liquidity shortage. The prospect of larger shocks induces households to borrow less, which further reduces the need for reserves. As a result, optimal reserve holdings are highest when the size of liquidity shocks is intermediate.[4] In contrast, capital control is always required as long as there is a risk of liquidity shortages. In our quantitative model, the optimal capital control tax rate increases monotonically with the size of liquidity shocks, reflecting a larger wedge arising from the fire-sale externality.

We calibrate the decentralized economy without policy to Chile, an emerging economy with an intermediate level of financial development that has experienced sudden stops. The parameter governing the size of liquidity shocks is disciplined by the observed output deviation during crises. Neither reserves nor capital controls are calibration targets. Holding these parameters fixed, we then compare the decentralized economy with the planner's allocation.

The optimal policy combines public reserve accumulation with a tax on foreign borrowing to reduce the severity of fire sales. Under this implementation, private reserve holdings are zero, so the planner's reserve holdings correspond to official reserves. The optimal reserve-to-GDP ratio is about 12%, reasonably close to Chile's 15% despite not being targeted in the calibration. Together, reserve accumulation and capital controls lower the share of assets liquidated during crises from 4.5% in the decentralized economy to only 1% under the planner's allocation. The joint policy thus mitigates persistent output losses.

The model also reproduces the joint empirical pattern linking reserves and capital controls to financial development. To examine this relationship, we solve the model across different values of the parameter governing the size of liquidity shocks and compute optimal reserves and debt tax rates in stochastic simulations. Optimal reserves are highest when the size of liquidity shocks is intermediate and lower when shocks are either small or large. The reserve-to-GDP ratio peaks at 12.2%, close to the peak of approximately 13.0% in the

[3]Appendices H and I provide a micro-foundation for this interpretation and an extended model with endogenous debt maturity.

[4]This feature holds not only for the planner's allocation but also for the decentralized economy.

empirical fit in Figure 1. Meanwhile, the optimal debt tax rises monotonically with the size of liquidity shocks. Interpreting smaller shocks as greater financial development, the model thus generates both the reserve hump and declining capital controls observed in the data. The comparison for capital controls is qualitative, since the empirical index records *de jure* restrictions on capital flows, whereas the model specifies a tax rate on foreign borrowing. We also examine the robustness of this joint policy pattern and compare our model with existing models. These exercises highlight the roles of the liquidity constraint requiring early debt repayment and the associated fire-sale externality in explaining the observed relationships between financial development and the two policies.

The welfare gain from the joint policy is approximately 0.5% of permanent consumption at the benchmark calibration and increases with the size of liquidity shocks in our quantitative exercise. Our welfare analysis shows that the magnitude of these gains depends on both the persistent productivity losses that policy can prevent and the sensitivity of fire-sale prices to asset liquidation. Endogenous growth allows policy to affect welfare through persistent changes in productivity, but does not by itself imply large welfare gains.

Literature Review This paper contributes to a broad literature on capital controls and foreign reserve policy for a precautionary motive. Bianchi and Lorenzoni (2022) survey the recent literature. One strand of literature focuses on capital controls to correct excessive foreign borrowing. A common assumption is that a pecuniary externality arising from collateral borrowing constraints induces over-borrowing by private agents and calls for a tax on private debt. Papers in this literature include Bianchi (2011), Benigno et al. (2013, 2016), Bianchi and Mendoza (2018), Jeanne and Korinek (2020), and Ma (2020), among others. Unlike these papers, we model sudden stops as liquidity shocks to foreign debt.[5] Current account reversals occur due to a permanent income loss rather than a collateral borrowing constraint, consistent with Aguiar and Gopinath (2007) and Guntin et al. (2023).[6]

Another strand of literature focuses on foreign reserves. Papers in this literature introduce different assumptions to motivate reserve accumulation, such as shocks to borrowing limits (Jeanne and Rancière 2011, Céspedes and Chang 2020, Matsumoto 2022), liquidity shocks (Hur and Kondo 2016), capital flow shocks (Cavallino 2019), sovereign default and en-

[5]Our rationale for capital controls is consistent with Ben Zeev (2017), who documents a meaningful shock-absorbing role for capital controls in emerging market economies facing global credit supply shocks.

[6]We build on Aguiar and Gopinath's (2007) emphasis on persistent income changes by making these changes endogenous to liquidity-driven asset liquidation, with the associated fire-sale externality providing a rationale for studying the joint use of reserves and capital controls.

dogenous borrowing costs (Hernández 2018, Bianchi, Hatchondo, and Martinez 2018, Bianchi and Sosa-Padilla 2020), self-fulfilling currency crises (Bocola and Lorenzoni 2020), growth externalities (Benigno et al. 2022), and collateral constraints on foreign borrowing (Shousha 2017). Jeanne and Sandri (2023) develop a model with liquidity shocks and pecuniary externalities to show that foreign reserve policy is necessary for countries with intermediate levels of financial development, similar to our result. In contrast to these papers that study capital controls and reserve policies separately, we study the optimal combination of foreign reserve policy and capital controls in a unified framework.

Several recent papers study the relationship between capital controls and reserve policies. Arce, Bengui, and Bianchi (2025) show that public reserve accumulation can be used as a macroprudential policy tool similar to capital controls against sudden stops. Davis et al. (2021a,b) and Fanelli and Straub (2021) assume financial frictions on private foreign debt similar to Gabaix and Maggiori (2015) and show that foreign reserve policy can be used as a substitute for capital controls to manage private capital flows. In these papers, reserve policy is at best a perfect substitute for capital controls, and could be a less efficient policy tool if reserve policy is associated with a higher cost.

Our model differs from these papers by rationalizing the *joint* use of capital controls and reserve policy. Aizenman (2011) also studies reserves and a tax on external borrowing in a two-period model with liquidity risk and fire-sale externalities. Our infinite-horizon framework examines how this policy mix varies with financial development and whether it can explain the observed cross-country relationships. The contribution lies in explaining these relationships, beyond providing a rationale for the two instruments. Our fire-sale mechanism is also related to Lorenzoni (2008) and Dávila and Korinek (2018), who study pecuniary externalities as a rationale for policy intervention.

Lutz and Zessner-Spitzenberg (2023) develop a closely related model that rationalizes the joint use of the two policies by adding a cash-in-advance constraint for trade financing to a collateral-constraint model in the tradition of Bianchi (2011). Our framework differs in two respects: liquidity needs arise from early repayment of outstanding debt rather than expenditure on intermediate inputs, and policy corrects a fire-sale externality from asset liquidation rather than an externality arising from collateral borrowing constraints. These differences allow our model to explain the joint policy pattern: the liquidity constraint generates the reserve hump, while capital controls address the fire-sale externality and decline

with financial development in our quantitative model.[7] The link to development also appears in Ottonello et al. (2024), whose model makes reserve accumulation most desirable as an industrial policy at intermediate levels of economic development.

Our paper is also related to the literature on gross capital flows. Broner et al. (2013a) document a positive correlation between gross inflows and outflows, which Davis and van Wincoop (2018) attribute to financial globalization. Caballero and Simsek (2020) explain this correlation using liquidity shocks similar to ours and show that capital retrenchment can stabilize the economy when foreign capital flows out. Our model differs from these papers by emphasizing the interplay between private capital inflows and public reserve outflows, consistent with the empirical pattern in emerging economies as shown by Avdjiev et al. (2022), and models by Arce et al. (2025) and Jeanne and Sandri (2023).

On the empirical side, we contribute to the reserve-demand literature by documenting the hump-shaped relationship between reserve holdings and financial development across countries. Earlier studies generally use linear specifications to distinguish precautionary, mercantilist, and domestic-liquidity motives. For example, Aizenman and Lee (2007) evaluate precautionary and mercantilist explanations for reserve accumulation. Obstfeld, Shambaugh, and Taylor (2010) emphasize domestic financial liabilities, measured by M2 relative to GDP, together with financial openness and the exchange-rate regime. Dominguez (2010) also considers M2 relative to GDP, imports, capital-account openness, crises, and narrower measures of financial development. Chinn and Ito (2006) examine how financial openness and institutions relate to financial development and provide the capital-account-openness measure used in our analysis. These studies establish important determinants of reserves and financial development, but do not directly test the hump-shaped relationship.

The rest of the paper is organized as follows. Section 2 presents empirical facts about financial development, reserves, and capital controls. Section 3 outlines the model. Section 4 calibrates the model and conducts quantitative analyses. Section 5 demonstrates how financial development impacts the optimal policy. Section 6 concludes.

[7] Appendix K compares our policy patterns with those in Bianchi (2011), Arce, Bengui, and Bianchi (2025), and Lutz and Zessner-Spitzenberg (2023). In these quantitative exercises, varying collateral pledgeability or trade-financing needs does not reproduce the reserve hump together with declining capital controls.

2 Motivating Empirical Facts

This section presents motivating empirical facts on financial development, foreign reserves, capital controls, and external liabilities. We document three patterns:

Fact 1. *The relationship between financial development and the reserve-to-GDP ratio is non-monotonic. Countries with an intermediate level of financial development tend to have higher reserve-to-GDP ratios than other countries.*

Fact 2. *Financial development is negatively correlated with the capital controls index. Countries with higher financial development impose fewer restrictions on capital flows.*

Fact 3. *Financial development is positively correlated with total external liabilities and external debt liabilities relative to GDP. Countries with higher financial development tend to have higher ratios of both measures to GDP.*

To document these facts, we combine data from several sources for 87 economies from 1980 to 2019: the financial development index constructed by the IMF, the External Wealth of Nations data set from Lane and Milesi-Ferretti (2007), and the capital controls index constructed by Chinn and Ito (2006). We also include country-level controls from the World Bank's World Development Indicators database and the Worldwide Governance Indicators, together with the exchange rate regime classification constructed by Shambaugh (2004). Appendix A provides the country list and variable definitions.

Our primary focus is the measurement of financial development. Ideally, it should capture the capacity of a country's financial system to help domestic agents manage external shocks. The IMF's financial development measure is constructed to capture how developed financial institutions and financial markets are in terms of depth (size and liquidity), access (the ability of individuals and companies to access financial services), and efficiency (the ability of institutions to provide financial services at low cost and with sustainable revenue, together with the level of activity in capital markets). Unlike traditional proxies for financial depth, such as the private credit-to-GDP ratio or stock market capitalization-to-GDP ratio, the IMF's financial development index takes into account the complex multidimensional nature

Table 1 Reserves, Capital Controls, and External Liabilities: Relationships with Financial Development, 1980–2019

	Reserve/GDP		Capital Control Index		External Liabilities/GDP		External Debt Liabilities/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.23***	0.91***	-0.57***	-0.24**	0.36***	0.50**	0.27***	0.63**
	(0.08)	(0.26)	(0.06)	(0.12)	(0.10)	(0.23)	(0.10)	(0.25)
Financial Development^2	-0.32***	-0.23***						
	(0.06)	(0.08)						
GDP per capita (log)		-0.16		-0.28		-0.87**		-0.81**
		(0.34)		(0.26)		(0.34)		(0.31)
Trade (% GDP)		0.18		-0.24**		0.24***		0.08
		(0.12)		(0.09)		(0.08)		(0.11)
Institutional Quality		0.20		-0.55**		-0.39*		-0.26
		(0.27)		(0.21)		(0.20)		(0.25)
Peg		-0.06		0.00		-0.02		0.02
		(0.14)		(0.08)		(0.07)		(0.09)
CA (% GDP)		0.07		-0.02		-0.03		0.01
		(0.04)		(0.02)		(0.05)		(0.05)
Constant	0.30***	0.29*	0.04	-0.04	-0.04	0.16	-0.06	-0.01
	(0.09)	(0.14)	(0.06)	(0.07)	(0.06)	(0.11)	(0.06)	(0.10)
Time FE	Y	Y	Y	Y	Y	Y	Y	Y
Country FE	N	Y	N	Y	N	Y	N	Y
Observations	3212	1853	3141	1852	3137	1819	3138	1813
Adjusted R^2	0.239	0.681	0.386	0.869	0.176	0.747	0.086	0.697

Note. The dependent variable is the reserve-to-GDP ratio in columns (1) and (2), the capital controls index (the negative value of the Chinn-Ito Index constructed by Chinn and Ito 2006) in columns (3) and (4), total external liabilities/GDP in columns (5) and (6), and external debt liabilities/GDP in columns (7) and (8). The independent variables include a financial development index constructed by the IMF, GDP per capita (log), trade-to-GDP ratio, the institutional quality measure, the exchange rate regime constructed by Shambaugh (2004), and the current account-to-GDP ratio. We standardize all variables (except for the exchange rate regime). Standard errors are clustered by country and year and reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Variable definitions are provided in Appendix A.

of financial development (Svirydzenka 2016).[8,9]

Columns (1) and (2) in Table 1 present the relationship between foreign reserves and

[8]We conduct robustness checks using two traditional measures of financial development—the domestic credit-to-GDP ratio and stock market capitalization-to-GDP ratio—and by examining sub-indices of the Financial Development Index. The results are broadly consistent with the baseline, though some coefficients lose statistical significance, possibly reflecting endogeneity, measurement issues, or limited sample size. See Online Appendix Tables A3 and A4 for details.

[9]Financial development is related to economic development, but the two are not interchangeable. Appendix N.1 examines this distinction and shows that financial development provides information beyond measures of economic development in explaining the empirical relationships.

financial development.[10] Consistent with Figure 1, the relationship is non-monotonic: countries with an intermediate level of financial development tend to have higher reserve-to-GDP ratios than other countries. Column (1) controls for year fixed effects that absorb common global shocks. Column (2) adds country fixed effects to absorb time-invariant country characteristics, together with controls for GDP per capita (a proxy for economic development), trade (a proxy for economic openness), institutional quality measures, exchange rate regimes, and the current account-to-GDP ratio. As is conventional, we cluster the standard errors at the country and year level. The non-monotonic relationship remains statistically significant and economically meaningful with these controls. For ease of interpretation, we standardize all variables in the regression except for the exchange rate regime dummy variable, and construct the squared financial development term from the standardized index. Using the estimates in column (2), an increase in financial development from its sample mean to one standard deviation above the mean is associated with a 0.68-standard-deviation increase in the reserve-to-GDP ratio. This combines a positive linear contribution of 0.91 standard deviations and a negative quadratic contribution of 0.23 standard deviations. Transformed back into the original units of the IMF Financial Development Index, column (2) implies a turning point of 0.776.[11]

Columns (3) and (4) present the relationship between the capital controls index and financial development. We use the negative of the Chinn-Ito index (Chinn and Ito 2006) because it provides a measure of *de jure* restrictions on capital flows dating back to 1980. Consistent with Figure 1, countries with higher financial development tend to impose fewer restrictions on capital flows.[12] The negative relationship is statistically significant and economically meaningful. In column (3), a one-standard-deviation increase in financial development is associated with a 0.57-standard-deviation decrease in the capital controls index.

[10] Online Appendix Section B, Table B3, and Figure B1 examine the role of observations from the United States and euro-area economies, as well as country-years with active Federal Reserve arrangements. The estimated magnitude and precision of the conditional panel relationship are sensitive to euro-area observations. However, these observations do not fully account for the pattern, as the non-monotonic relationship persists in the cross section.

[11] The precise location of the peak depends on whether the regression uses within- or between-country variation. The turning point of 0.776 implied by column (2) of Table 1 lies within the observed support of the index, with 175 country-year observations from 15 economies at or above it. Appendix Table B2 reports two complementary specifications: one with time fixed effects but no country fixed effects and one using country-average data, the latter being more directly comparable to the unconditional relationship in Figure 1. The corresponding turning points are 0.472 and 0.367, respectively.

[12] We also examine whether this relationship is non-monotonic by including the squared financial development index in the regression. Its coefficient is not statistically significant, consistent with a linear specification. Online Appendix Table A2 reports the results.

The estimated relationship is smaller in magnitude and less precisely estimated after including country fixed effects and country-level controls in column (4), but remains statistically significant. This is consistent with capital controls being slow-moving and correlated with country characteristics (Acosta-Henao et al. 2020, Ma and Wei 2020).

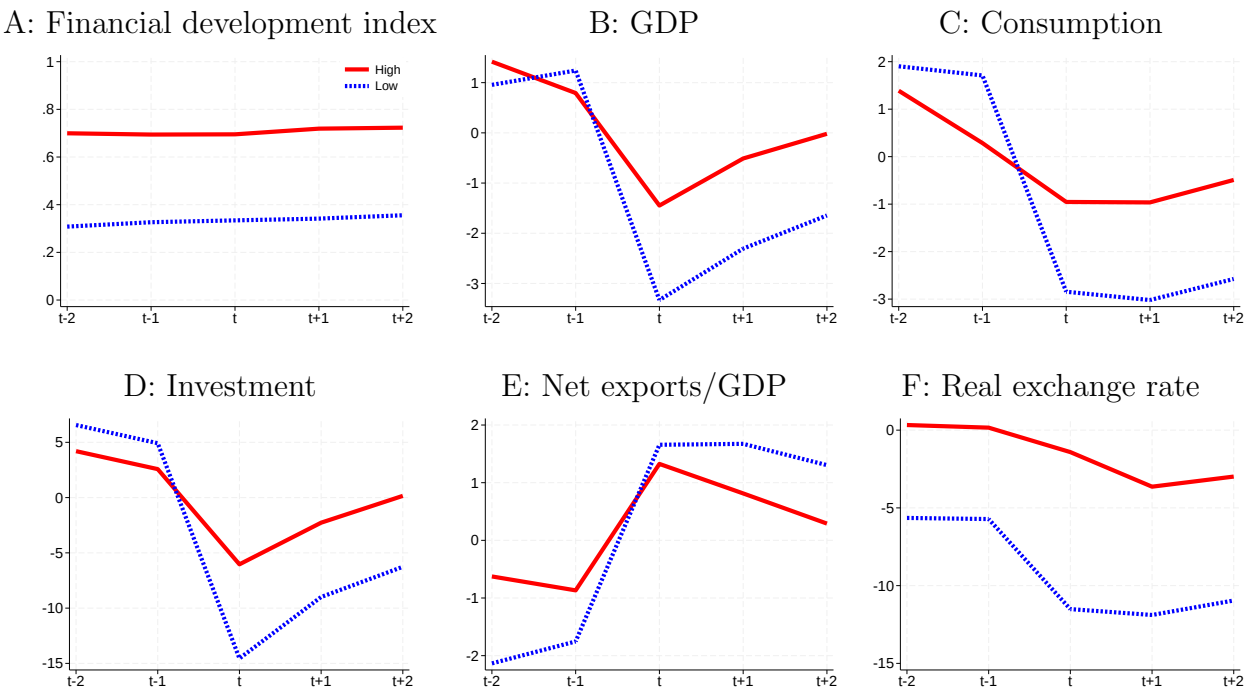
Columns (5)–(8) present the relationships between financial development and total external liabilities and external debt liabilities. Countries with higher financial development tend to have higher ratios of both measures to GDP. These positive relationships hold with year fixed effects in columns (5) and (7), and with the addition of country fixed effects and country-level controls in columns (6) and (8). In column (5), a one-standard-deviation increase in financial development is associated with a 0.36-standard-deviation increase in total external liabilities relative to GDP.[13]

These relationships motivate our interpretation of financial development as shaping the *effective* size of liquidity shocks. We next examine whether crisis dynamics are consistent with this interpretation. Advanced financial systems can ease liquidity shortages by providing financing to meet early repayment obligations and enabling agents to hedge liquidity risk. This interpretation is consistent with studies linking financial development to additional borrowing during crises (Chang and Velasco 2001) and longer debt maturities (Qian and Strahan 2007, Bae and Goyal 2009).

Our model formalizes this mechanism by allowing financial development to determine the fraction of debt requiring early repayment during a liquidity shock. Countries facing similar external disturbances may therefore experience different crisis severity, depending on their financial systems' capacity to absorb liquidity pressures. Figure 2 provides evidence consistent with this interpretation using sudden-stop episodes identified by Korinek and Mendoza (2014). We classify episodes by financial development two years before the crisis and find that economies with lower financial development experience larger declines in GDP, consumption, and investment, as well as greater real depreciation. The financial development index changes little over the five-year event window, consistent with its interpretation as a persistent characteristic. These comparisons do not establish causality or rule out endogeneity. Online Appendix Figure O1 reports the same exercise using group medians and shows the same qualitative crisis-time pattern.

[13] Appendix Figure O2 presents the debt counterpart of Figure 1, using a cross-country scatter of country averages and a binscatter of country-year observations.

Figure 2 Financial Development and the Severity of Sudden Stop Dynamics



Note. Sudden-stop episodes and macroeconomic data are from Korinek and Mendoza (2014). The 56 episodes are split into 28 low- and 28 high-financial-development episodes using the financial development index two years before the crisis ($t-2$). Lines report group means at each event time. GDP, consumption, and investment are real per-capita series expressed as 100 times their log deviations from HP trends. Net exports/GDP is expressed as a percentage-point deviation from its HP trend. The real exchange rate is expressed as a percentage deviation from its country sample mean; a decline indicates depreciation. The financial development index is shown in levels.

3 Model

We develop a model to explain the salient empirical patterns documented in the previous section. Our small-open-economy model features liquidity shocks and endogenous growth. Financial development is represented by the size of liquidity shocks, that is, the fraction of foreign debt that must be repaid immediately when a shock occurs.

3.1 Setup

Our small open economy is inhabited by a unit measure of identical infinitely lived households. They produce and consume tradable goods, borrow from abroad, hold liquidity in

safe foreign assets (reserves), and invest in productive assets. Their utility is given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

where β is the discount factor and the utility function u is strictly increasing and strictly concave. The budget constraint is

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^{\ell} \tag{2}$$

where c_t denotes consumption of tradable goods, b_t is foreign bond holdings, s_t is reserve holdings, and z_t is investment in productive assets. R_t is the gross interest rate on foreign bonds, and R^s is the fixed gross interest rate on reserves. Households are not allowed to borrow using reserves, implying that s_t cannot be negative. In the quantitative analysis below, we set parameter values such that b_t is always negative. We therefore refer to b_t as foreign debt, or simply debt, henceforth. a_t is productive asset holdings and also output because we assume a linear production function $y_t = a_t L$ with fixed labor supply $L = 1$. $q_t a_t^{\ell}$ is the amount of resources obtained by liquidating a part of the asset holdings when a liquidity shock occurs. Further details are provided below.

Productive assets a_t in the model are broad assets that include capital and technology used for the production of tradable goods. Their stock can also be interpreted as the productivity level of the economy. The amount of productive assets a_t grows endogenously as households invest z_t units of tradable goods. The law of motion for a_t is given as follows:

$$a_t = a_{t-1} + \eta (z_{t-1})^{\gamma} \left(a_{t-1} + \kappa a_{t-1}^* \right)^{1-\gamma} - a_t^{\ell} \tag{3}$$

a_{t-1}^* is the level of foreign productive assets, which is assumed to grow at a fixed rate $1 + \bar{g}$. The term in the second set of parentheses implies that both domestic and foreign assets, a_{t-1} and a_{t-1}^*, promote the accumulation of productive assets. κ captures the degree of technological spillover from foreign technology embedded in foreign assets, as in Gavazzoni and Santacreu (2020) and Gornemann, Guerrón-Quintana, and Saffie (2020). The parameters $\gamma \in (0, 1)$ and $\eta > 0$ govern the accumulation process. With spillovers from exogenous a_t^*, domestic productive assets a_t fluctuate endogenously around the path of a_t^* without persistently diverging from it. This implies that the long-run average growth rate of a_t is

exogenously given by $1+\bar{g}$.[14] We assume that households internalize that current assets a_t facilitate future growth through (3), ensuring that there is no externality associated with growth. a_t^ℓ denotes the amount of liquidated assets. As explained below, households may need to liquidate a part of their assets to repay foreign debt when a liquidity shock occurs.

There are two stochastic shocks. One is a shock to the interest rate R_t given by

$$R_t \equiv R(b_t/a_t, \varepsilon_t^R) = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\frac{b_t}{a_t} - \bar{b}\right) - 1\right] \tag{4}$$

where R^b is the baseline interest rate, ε_t^R is a stochastic shock, and the second term is a debt-elastic component with $\psi^b > 0$ and $\bar{b} \in \mathcal{R}$ as in Schmitt-Grohé and Uribe (2003). We introduce the debt-elastic component to keep debt relative to output stationary as we vary financial development.[15] We assume that households internalize the effect of b_t and a_t on the interest rate R_t, so that there is no externality through the interest rate.

The other shock is a liquidity shock, the key component of our model. At the beginning of each period, before households obtain new borrowing b_t and output a_t, households may be required by foreign lenders to repay a fraction $\theta_t \in [0,1]$ of the existing debt b_{t-1}, where θ_t is a stochastic variable. Thus, θ_t determines the rollover risk associated with external borrowing. Any stochastic process for $\theta_t \in [0,1]$ is consistent with our model, but for simplicity and clarity of the mechanism, we assume that θ_t is a binary stochastic variable that takes the value $\theta \in [0,1]$ with an exogenous probability and zero otherwise. In this case, the parameter θ determines the size of liquidity shocks.

When the liquidity shock occurs, households can use reserves s_{t-1} from the last period to make early repayments of $-\theta b_{t-1}$. If the reserves s_{t-1} are not enough, households must liquidate a part of their productive assets to finance the liquidity shortage defined by $-\theta b_{t-1} - s_{t-1} > 0$.[16] Because households are homogeneous and equally short of liquidity, foreign agents are the only potential buyers of assets. Their demand determines the liquidation price, which will be explained below. Let a_t^ℓ denote the amount of liquidated assets and q_t denote its price.

[14] If there is no spillover, a_t could deviate substantially from a_t^*. Large deviations of a_t from a_t^* lead to an extremely high or low liquidation price q_t, because q_t is affected by a_t^* as shown below in (15).

[15] The debt-elastic component in (4) serves a technical role by keeping the debt-to-GDP ratio stationary as we vary the financial development parameter. Appendix M examines the robustness of our results to this specification and shows that the main quantitative findings are preserved.

[16] We assume that households cannot default on their foreign debt.

The proceeds from asset liquidation, $q_t a_t^\ell$, must be sufficient to cover the liquidity shortage.

$$q_t a_t^\ell \geq -\theta b_{t-1} - s_{t-1}. \tag{5}$$

We call this inequality a liquidity constraint. The inequality sign indicates that households can potentially liquidate more assets than necessary for the early repayment, including in normal times. We also assume that households cannot buy productive assets from foreigners.

$$q_t a_t^\ell \geq 0. \tag{6}$$

Financial Development We interpret θ, the size of liquidity shocks, as reflecting the degree of financial development, where low θ corresponds to high financial development. Although countries may face similar disturbances in global financial markets, a well-developed domestic financial system provides alternative financing and risk management tools, thereby reducing the *effective* size of liquidity shocks. This is what θ in our model is meant to capture. More specifically, θ in our model is closely linked to two aspects of financial development studied in the literature. The first is the degree of commitment and enforceability of financial contracts. Chang and Velasco (2001) develop a model of bank runs à la Diamond and Dybvig (1983) for emerging economies and show that limited commitment to foreign debt repayment may induce foreign lenders to refuse additional loans for rollover, thereby triggering a liquidity crisis. Our model is in line with this mechanism if we interpret high θ as a low degree of commitment to foreign debt repayment. A low degree of commitment implies a limited amount of additional foreign loans for rollover, requiring a large fraction of foreign debt to be repaid using domestic resources. Similarly, Mendoza et al. (2009) and Maggiori (2017) model financial development as the degree of financial contract enforceability. In Mendoza et al. (2009), a high degree of enforceability allows for better insurance against idiosyncratic shocks. Our model is also consistent with this interpretation. Appendix Section H provides a micro-foundation for this interpretation of θ by assuming that financial development lowers the cost of risk hedging.

Another strand of literature focuses on the relationship between financial development and the maturity of bank loans. Qian and Strahan (2007) and Bae and Goyal (2009) show that average bank-loan maturities are longer in countries with stronger creditor protection. These studies include foreign banks in their samples, and Qian and Strahan (2007) show that foreign banks' lending decisions are even more sensitive to the legal and institutional envi-

ronment than those of domestic banks. Broner et al. (2013b) show that emerging economies rely more on short-term borrowing than developed countries because long-term borrowing is more costly in emerging economies. These papers support our interpretation of θ as a measure of financial development because θ is a fraction of debt subject to rollover risk and can be interpreted as a share of short-term debt. In Appendix Section I, we extend our baseline model by endogenizing θ as a fraction of short-term debt and show that the main results of the paper are robust.[17]

Optimization problem Households choose $\{c_t, b_t, s_t, z_t, a_t, a_t^\ell\}$ to maximize their expected utility (1) subject to the budget constraint (2), the law of motion for productive assets (3), the debt-elastic interest rate (4), the liquidity constraint (5), and the non-negativity constraints on liquidation (6) and reserves, taking the liquidation price q_t as given. The recursive optimization problem is

$$V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a_{t-1}^*) = \max_{c_t, b_t, s_t, z_t, a_t^\ell, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a_t^*)$$

subject to

$$c_t + \frac{b_t}{R(b_t/a_t, \varepsilon_t^R)} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^\ell$$
$$a_t = a_{t-1} + \eta z_{t-1}^\gamma \left(a_{t-1} + \kappa a_{t-1}^*\right)^{1-\gamma} - a_t^\ell$$
$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1}$$
$$q_t a_t^\ell \geq 0$$
$$\frac{s_t}{R^s} \geq 0$$

where $R(b_t/a_t, \varepsilon_t^R)$ is the borrowing-rate function defined in (4). Θ_t represents a set of stochastic shocks $\Theta_t = \left\{\theta_t, \varepsilon_t^R\right\}$.

[17] The interpretation differs slightly across the two models. While in the benchmark model the capital control tax rate increases with θ as shown in Section 5, in the model with endogenous maturity the coverage of capital controls, which is the share of short-term debt, increases with θ.

The first-order conditions in Appendix D.1 imply:

$$u'(c_t) = \beta \mathbb{E}_t \left[\xi_{t+1} \eta \gamma \left(\frac{z_t}{a_t + \kappa a_t^*} \right)^{\gamma - 1} \right] \tag{7}$$

$$\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t) \tag{8}$$

$$\xi_t = u'(c_t) \left[1 + \left(\frac{b_t/a_t}{R_t} \right)^2 \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b} \right) \right] + \beta \mathbb{E}_t \left[\xi_{t+1} \left\{ 1 + \eta(1-\gamma) \left(\frac{z_t}{a_t + \kappa a_t^*} \right)^{\gamma} \right\} \right] \tag{9}$$

$$u'(c_t) = \beta \underbrace{\frac{R_t}{1 + \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b} \right) \frac{b_t/a_t}{R_t}}}_{\equiv \tilde{R}_t} \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \theta_{t+1} \right] \tag{10}$$

$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right] + \nu_t \tag{11}$$

(7) is the Euler equation for investment z_t. ξ_t is the multiplier on the law of motion for productive assets, and captures the shadow value of one unit of productive assets. (8) is the first-order condition for liquidation a_t^ℓ. ψ_t is the multiplier on the liquidity constraint, and φ_t is the multiplier on the non-negativity constraint. (9) is the first-order condition regarding asset a_t. The shadow value of one unit of productive assets, ξ_t, reflects the marginal utility of additional consumption and the contribution to next-period asset accumulation. (10) and (11) are the Euler equations regarding debt and reserves, respectively. ν_t is the multiplier on the non-negativity constraint on reserve holdings.

The first-order condition regarding liquidation (8) needs some explanation. ψ_t is the multiplier for the liquidity constraint (5), and φ_t is the multiplier for the non-negativity constraint on liquidation (6). The complementary slackness conditions are given by

$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1},\ \psi_t \geq 0,\ \psi_t (q_t a_t^\ell + \theta_t b_{t-1} + s_{t-1}) = 0 \tag{12}$$

$$q_t a_t^\ell \geq 0,\ \varphi_t \geq 0,\ \varphi_t (q_t a_t^\ell) = 0 \tag{13}$$

Since both multipliers are non-negative, (8) implies $\xi_t / u'(c_t) \geq q_t$: the private value of an asset is no lower than its liquidation price. The latter is determined by foreign buyers' demand, as explained below. Households' liquidation decisions depend on the comparison between these two values. We explain each case separately:

1. If the private asset value exceeds the liquidation price, $\xi_t / u'(c_t) > q_t$, at least one of ψ_t or φ_t is strictly positive. There are three sub-cases.

(a) Required repayment exceeds reserves, $-\theta_t b_{t-1} > s_{t-1}$. Then liquidation a_t^ℓ must be positive to fill the liquidity shortage, implying $\varphi_t = 0$. Then (8) gives $\psi_t > 0$, indicating that the liquidity constraint (5) is binding, $q_t a_t^\ell = -\theta b_{t-1} - s_{t-1}$. Households liquidate just enough assets to cover the liquidity shortage.

(b) Reserves exceed required repayment, $-\theta_t b_{t-1} < s_{t-1}$. This case includes normal times with $\theta_t = 0$ and $s_{t-1} > 0$. The liquidity constraint (5) is slack, implying $\psi_t = 0$ and $\varphi_t > 0$, and no liquidation $a_t^\ell = 0$.

(c) Reserves exactly cover required repayment, $-\theta_t b_{t-1} = s_{t-1}$. At this boundary, $a_t^\ell = 0$ and both constraints bind. The optimal liquidation condition (8) determines only the sum of the multipliers, and both can be positive. We retain this case in the theory, although it does not arise in our quantitative analysis.

2. If the private asset value is equal to the liquidation price, $\xi_t/u'(c_t) = q_t$, then both multipliers are zero. The optimality conditions permit asset sales beyond those needed to meet the liquidity shortage, including in normal times with $\theta_t = 0$.

In equilibrium, asset liquidation a_t^ℓ and its price q_t must be consistent with foreign demand. As we explain below, foreign agents face a fixed cost of entering the asset market and are unwilling to buy assets in normal times under our calibration. Moreover, in the quantitative analysis, when a liquidity shock occurs, the asset liquidation price q_t is substantially lower than the private asset value $\xi_t/u'(c_t)$. This corresponds to case 1(a): households liquidate just enough assets to cover the liquidity shortage. In this case, the multiplier ψ_t is interpreted as the private value of one unit of liquidity. $\psi_t = \xi_t/q_t - u'(c_t)$ indicates that one additional unit of liquidity allows households to reduce asset liquidation by $1/q_t$ units, whose value is ξ_t/q_t. At the same time, a reduction in asset liquidation reduces the proceeds available in the budget, which is captured by the last term $-u'(c_t)$.

In the Euler equation regarding debt (10), the interest rate is adjusted to internalize the effect of debt on the interest rate through (4). We denote this adjusted interest rate by $\tilde{R}_t$ henceforth. This Euler equation can be understood as follows. By giving up one unit of consumption at period t, households can reduce debt by $\tilde{R}_t$ units and increase resources at $t+1$ by the same amount. The first term on the right-hand side captures the expected marginal utility value of these resources. In addition, when $\theta_{t+1} = \theta$ and there is a liquidity shortage $-\theta b_t - s_t > 0$ at $t+1$, a reduction in debt by $\tilde{R}_t$ units will reduce the liquidity shortage by $\theta \tilde{R}_t$ units. The last term in (10) captures this benefit, evaluated at the private value of liquidity ψ_{t+1}.

The Euler equation regarding reserves (11) can be understood in the same way. By giving up one unit of consumption at period t, households can increase reserves and resources at $t+1$ by R^s units, whose value is captured by the first term on the right-hand side. R^s units of reserves at $t+1$ will also reduce the liquidity shortage by R^s units if there is a liquidity shortage, captured by the second term. Note that ψ_{t+1} is not multiplied by θ_{t+1}, in contrast to the Euler equation regarding debt (10), because R^s units of reserves will reduce the liquidity shortage one-for-one by R^s units. This difference between (10) and (11) plays a critical role in the model, which is explained in the next subsection. ν_t is a Lagrange multiplier for the non-negativity constraint on reserves and can be positive only when $s_t = 0$.

Foreign agents Because domestic households are homogeneous, foreign agents are the only potential buyers of liquidated assets. We assume that foreign agents are less efficient than domestic agents in producing goods from domestic assets. In particular, they combine liquidated assets a_t^ℓ with their own assets a_t^* to produce tradable goods. Taking the asset liquidation price q_t as given, their profit maximization problem is given as follows.

$$\pi_t^* = \max\left\{0,\ \max_{a_t^\ell}\left\{(a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - q_t a_t^\ell - F a_t^*\right\}\right\} \tag{14}$$

where $0 < \zeta < 1$ is a production-function parameter and $F a_t^*$ is the fixed cost of entering the market for liquidated assets. Foreign agents enter the market only if their maximized profit is large enough to cover the entry cost. Conditional on entry, the first-order condition gives a demand equation for liquidated assets:

$$q_t = (1-\zeta)\left(\frac{a_t^*}{a_t^\ell}\right)^\zeta \tag{15}$$

The demand for liquidated assets is downward-sloping: the price declines as more assets are liquidated. Importantly, atomistic households take q_t as given and do not internalize the effect of their liquidation on the price through (15). This creates a terms-of-trade externality: households could collectively raise the price by reducing the supply of assets. However, as mentioned earlier, in the quantitative analysis, households liquidate their assets only when a liquidity shock occurs and the liquidity constraint binds. These are forced sales to cover the liquidity shortage, so households cannot reduce liquidation below the amount required for repayment. This amount is determined by debt and reserves chosen in the previous period, because the liquidity shortage is given by $-\theta_t b_{t-1} - s_{t-1}$. This implies that the ex ante

decisions on debt and reserves are distorted by this pecuniary externality. We call this a fire-sale externality because forced asset sales lower the price and require further liquidation.

The demand equation in (15) also implies that q_t becomes very high when a_t^ℓ is small. This gives households an incentive to sell a small amount of assets even if there is no liquidity shortage. To prevent such asset sales in normal times, we set the entry cost parameter F such that foreign buyers are willing to buy liquidated assets only when a liquidity shock occurs and a_t^ℓ is large enough to cover the fixed cost.[18]

Decentralized equilibrium The decentralized equilibrium of this economy consists of decision functions $\{c_t,\ b_t,\ s_t,\ z_t,\ a_t,\ a_t^\ell\}$, the price function $\{q_t\}$, and the multipliers $\{\psi_t,\ \varphi_t,\ \xi_t,\ \nu_t\}$ as functions of endogenous and exogenous state variables $\{b_{t-1},\ s_{t-1},\ z_{t-1},\ a_{t-1};\ \Theta_t, a_{t-1}^*\}$, that satisfy

1. Equations (2), (3), (7), (8), (9), (10), (11), (12), (13),
2. Reserve non-negativity: $s_t/R^s \geq 0$ with $\nu_t(s_t/R^s) = 0$ and $\nu_t \geq 0$,
3. Foreign buyers' optimality condition and asset-market clearing:
   (a) If $a_t^\ell > 0$, q_t is given by (15), and $(a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - q_t a_t^\ell \geq F a_t^*$.
   (b) If $a_t^\ell = 0$, q_t is given by the domestic value $\xi_t/u'(c_t)$.

given the stochastic process for Θ_t and the exogenous path $a_t^* = (1+\bar{g})a_{t-1}^*$.

3.2 Reserve Holdings

We now examine how households choose reserve holdings. Combining the two Euler equations (10) and (11), we obtain the key equation of the model:

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \theta_{t+1}\tilde{R}_t)\psi_{t+1}] + \nu_t \tag{16}$$

We set parameter values such that $\tilde{R}_t > R^s$ always holds, as is typically the case in emerging economies.[19] The left-hand side captures the opportunity cost of holding re-

[18] At the same time, F should not be too high, as high F would make π_t^* negative even when asset liquidation is triggered by liquidity shortages. We confirmed that $F = 0.0062$ satisfies these conditions for any value of θ in our quantitative model.

[19] If $\tilde{R}_t = R^s$, then households choose debt and reserves such that there is no liquidity shortage in any state at $t+1$, implying $\psi_{t+1} = 0$. Then all the terms in (16) become zero. In this case, the model is reduced to a standard small open-economy model in which only the net foreign asset position $b_t + s_t$ matters.

serves. Households can use one unit of tradable goods either to buy R^s units of reserves or reduce $\tilde{R}_t$ units of debt. If households choose the former, they receive R^s units at $t+1$ but lose the opportunity to reduce the interest payment on debt $\tilde{R}_t$. This gap is the opportunity cost of holding reserves.

The first term on the right-hand side captures a relative advantage of holding reserves over reducing debt in liquidity management. As explained above, by holding R^s units of reserves, households can reduce a liquidity risk $-\theta b_t - s_t$ (a potential liquidity shortage at $t+1$) by R^s units, and its expected value is given by $\beta \mathbb{E}_t[R^s \psi_{t+1}]$. In contrast, by reducing $\tilde{R}_t$ units of debt, households can reduce a liquidity risk by $\theta \tilde{R}_t$ units, and its expected value is $\beta \mathbb{E}_t[\theta_{t+1} \tilde{R}_t \psi_{t+1}]$. If θ satisfies $R^s > \theta \tilde{R}_t$, then increasing reserves will reduce the liquidity risk more effectively than reducing debt. This gap is the relative advantage of holding reserves over reducing debt in liquidity management, and we call it the *liquidity advantage* of reserves. Recall that ψ_{t+1} is the private value of liquidity, which is positive only when there is a liquidity shortage and the liquidity constraint binds. It follows that the liquidity advantage has a positive value only when there is a positive probability that a liquidity shock triggers a liquidity shortage $-\theta b_t - s_t > 0$ at $t+1$. The key equation (16) says that households choose debt and reserves to equalize the opportunity cost and the liquidity advantage of reserves. When the cost exceeds the benefit even at zero reserves, households choose $s_t = 0$, and the Lagrange multiplier for the non-negativity constraint on reserves $\nu_t > 0$ fills the gap between the cost and benefit.

The value of θ is a key determinant of the liquidity advantage of reserves and thus the level of reserve holdings. To see this, we derive the following three propositions.

Proposition 1. *If $\theta = 0$, households do not hold reserves, $s_t = 0$ $\forall t$.*

Proof. If $\theta = 0$, there is no liquidity shortage, and $\psi_{t+1} = 0$ for every period. The first term on the right-hand side of (16) is zero, which implies $\nu_t > 0$ and thus $s_t = 0$. □

The intuition is straightforward: if there is no liquidity risk at all, there is no reason to hold reserves because reserve holdings carry an opportunity cost.

Proposition 2. *If $\theta \geq R^s / \tilde{R}_t$, households do not hold reserves, $s_t = 0$ $\forall t$.*

Proof. If this condition holds, the first term on the right-hand side of (16) is non-positive. Because the left-hand side is positive, $\nu_t > 0$ and thus $s_t = 0$. □

Intuitively, if θ is close to one, reserves lose their liquidity advantage over debt because reducing debt is as effective as increasing reserves in reducing liquidity risk. Households simply reduce debt to manage liquidity risks as reserves come with an opportunity cost.

Proposition 3. *Households never hold enough reserves to eliminate a liquidity risk. In other words, whenever $-\theta b_t > 0$, households choose b_t and s_t such that $-\theta b_t - s_t > 0$ holds.*

Proof. Suppose to the contrary that households hold enough reserves to eliminate a liquidity risk, $-\theta b_t \leq s_t$. Then there is no liquidity shortage and $\psi_{t+1} = 0$ at $t+1$. In addition, $\nu_t = 0$ because $s_t \geq -\theta b_t > 0$. This implies that the right-hand side of (16) is zero. But the left-hand side is positive, and this cannot be an equilibrium. □

Intuitively, once reserves are sufficient to eliminate liquidity risk, an additional unit provides no further liquidity benefit but still carries an opportunity cost. Households therefore accept some liquidity risk to balance the marginal cost and benefit of reserve holdings.

An important implication of these propositions is that households hold positive reserves only when θ lies between 0 and $R^s/\tilde{R}_t$. More generally, the amount of reserves is small when θ is close to zero or close to $R^s/\tilde{R}_t$. On the one hand, if θ is close to zero, the size of the early repayment $-\theta b_t$ is small. Proposition 3 implies that households never hold enough reserves to fully cover the early repayment, always satisfying $-\theta b_t > s_t$. As a result, reserve holdings become small. On the other hand, if θ is close to $R^s/\tilde{R}_t$, the liquidity advantage of reserves is small. In this case, households reduce debt rather than increase reserves to manage liquidity risk. In addition, high θ implies that liquidity risk is highly sensitive to debt, discouraging households from borrowing much ex ante. Both forces therefore reduce reserve holdings.

In contrast, when θ takes an intermediate value, the size of an early repayment to be covered by reserves is relatively large, and the liquidity advantage of reserves is also high. In this case, reserve holdings can become large. This mechanism leads to a non-monotonic relationship between reserve holdings and the value of θ. This feature holds not only in the decentralized economy but also in the planner's allocation studied in the next subsection. Since lower θ corresponds to greater financial development, this non-monotonic relationship is consistent with the empirical finding in Section 2. We show this relationship quantitatively in Section 5.

3.3 Social Planner's Allocation

We next examine the social planner's allocation. Unlike decentralized households, the planner internalizes the decline in the liquidation price q_t as the amount of liquidated assets a_t^ℓ

increases. The planner's recursive optimization problem is

$$V^{SP}(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) = \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V^{SP}(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t)$$

subject to

$$\begin{gathered} c_t + \frac{b_t}{R(b_t/a_t, \varepsilon^R_t)} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q(a^\ell_t) a^\ell_t \\ a_t = a_{t-1} + \eta z^\gamma_{t-1} \left(a_{t-1} + \kappa a^*_{t-1}\right)^{1-\gamma} - a^\ell_t \\ q(a^\ell_t) a^\ell_t \geq -\theta_t b_{t-1} - s_{t-1} \\ q(a^\ell_t) a^\ell_t \geq 0 \\ \frac{s_t}{R^s} \geq 0 \end{gathered}$$

where $q(a^\ell_t)$ is given by the foreign demand equation (15). The first-order conditions are given in Appendix D.2. The key first-order condition is the one regarding liquidation a^ℓ_t, corresponding to (8) in the decentralized equilibrium:

$$\psi^{SP}_t + \varphi^{SP}_t = \frac{\xi_t}{q_t - \zeta q_t} - u'(c_t) \tag{17}$$

The difference from (8) is $-\zeta q_t$ in the denominator, obtained by $(\partial q_t / \partial a^\ell_t) a^\ell_t = -\zeta q_t < 0$. This implies that the social value of liquidity ψ^{SP}_t is higher than the private value ψ_t when the liquidity constraint binds, holding everything else constant. Intuitively, an additional unit of liquidity will reduce liquidation and increase the liquidation price q_t, thereby reducing liquidation even further. The planner internalizes this effect and assigns a higher value to liquidity compared to decentralized households.

Although the externality appears in the first-order condition for liquidation, it does not change the amount liquidated conditional on the same inherited debt and reserves when the liquidity constraint binds. In that case, debt b_{t-1} and reserves s_{t-1} chosen in the previous period determine the liquidity shortage and hence the required liquidation through (5). The planner values liquidity more highly but cannot liquidate less than the amount needed for repayment. The planner reduces liquidation through ex ante debt and reserve choices.

The externality instead distorts households' ex ante debt and reserve choices, before the liquidity constraint binds. Recall that the private value of liquidity ψ_{t+1} appears in the right-hand sides of the Euler equations (10) and (11), and affects how much households borrow

and hold reserves. The corresponding Euler equations for the planner are given by

$$u'(c_t) = \beta \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi^{SP}_{t+1} \theta_{t+1} \right] \tag{18}$$

$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi^{SP}_{t+1} \right] + \nu_t \tag{19}$$

where ψ^{SP}_{t+1} is given by (17). The gap between ψ^{SP}_{t+1} and ψ_{t+1} implies that households underestimate the value of liquidity when the liquidity constraint binds in the next period. As a result, they have an incentive to borrow too much and hold too few reserves relative to the planner's allocation. Combining (18) and (19) gives the counterpart of (16) with the social value of liquidity replacing the private value. The reserve-holding propositions in Section 3.2 therefore also apply to the planner's allocation.

Policy implementation The fire-sale externality distorts households' ex ante choices of both debt and reserves. The planner's constrained efficient allocation can be implemented through a tax on foreign borrowing and official reserve accumulation. The following proposition states the joint implementation result.

Proposition 4 (Policy implementation)**.** *The planner's constrained efficient allocation can be implemented by official reserve holdings equal to the planner's reserve choice and a tax on foreign borrowing given by* (21)*, with lump-sum taxes and transfers balancing the government budget. Under this implementation, private reserve holdings are zero.*

Appendix D.3 provides the formal argument, following Lutz and Zessner-Spitzenberg (2023). We explain the role of each instrument below, beginning with official reserves.

Official reserves The government finances official reserve accumulation through lump-sum taxes on households and makes these reserves available through lump-sum transfers when a liquidity shock creates a liquidity shortage. Public and private reserves are perfect substitutes for households because both provide resources for early debt repayment.

This substitutability initially leads households to reduce private reserves one-for-one as official reserves increase, following a Ricardian equivalence logic. However, private reserves cannot fall below zero. Given the implementing debt tax, households still undervalue liquidity relative to the planner. When the planner's reserve choice is positive, households would therefore choose a lower amount of total reserves. By accumulating the socially optimal amount, the government implements the planner's reserve choice, with private reserve hold-

ings equal to zero. Under this implementation, the planner's reserves are entirely official.[20]

Capital control tax A tax on foreign borrowing corrects the borrowing incentive created by the gap between the private and social values of liquidity. With a debt tax τ_t^b, the household's debt Euler equation becomes

$$u'(c_t) = \beta(1+\tau_t^b)\tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1}\right] \tag{20}$$

Matching this condition to the planner's Euler equation (18) gives:

$$1+\tau_t^b = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP}\theta_{t+1}\right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1}\right]} \tag{21}$$

The tax is positive whenever future liquidity shortages generate a fire-sale externality:

Proposition 5. *If $\theta = 0$, the optimal debt tax rate is zero, $\tau_t^b = 0\ \forall t$. If $\theta > 0$, the optimal debt tax rate is strictly positive, $\tau_t^b > 0$, whenever there is a positive probability that the liquidity constraint binds and asset liquidation occurs in the next period.*

Proof. If $\theta = 0$, the right-hand side of (21) is equal to one, proving $\tau_t^b = 0$. If $\theta > 0$, the binding liquidity constraint and positive asset liquidation imply $\psi_t^{SP} > \psi_t > 0$ and $\varphi_t = \varphi_t^{SP} = 0$ by (8) and (17). A positive probability of these events at $t+1$ implies $\psi_{t+1}^{SP} > \psi_{t+1} > 0$ at some state in the next period. This makes the right-hand side of (21) strictly greater than one, proving $\tau_t^b > 0$. □

Implications for financial development The two instruments respond differently to the size of liquidity shocks. The reserve-holding mechanism in Section 3.2 also applies to official reserves under the joint policy. When θ is small, required early repayment is small and there is little need for reserves. When θ approaches $R^s/\tilde{R}_t$, reserves lose their liquidity advantage over debt reduction, and reserve holdings fall. Larger shocks also discourage borrowing ex ante, further reducing the need for reserves. At intermediate values of θ, both the potential liquidity shortage and the liquidity advantage of reserves can be substantial. This mechanism generates the non-monotonic reserve profile examined quantitatively in Section 5.

[20] Appendix D.4 shows that a subsidy on private reserve holdings, combined with the optimal debt tax, can implement the same constrained efficient allocation.

Capital controls can remain positive even when optimal reserves are zero. Proposition 5 implies that the planner taxes foreign borrowing even at $\theta = 1$, provided that the liquidity constraint binds and asset liquidation occurs with positive probability in the next period. Section 5 shows that the optimal debt tax increases monotonically with θ in our quantitative model. Appendix G establishes this result analytically in a simplified two-period model. Holding the private and social values of liquidity and marginal utility fixed, a higher θ increases the tax in (21) by increasing the exposure of debt to the liquidity externality.[21]

Since lower θ corresponds to greater financial development, the joint policy generates a hump-shaped reserve profile and a declining capital control tax as financial development increases. These patterns provide the model counterparts to the empirical relationships in Section 2, which we compare quantitatively in Section 5.

4 Quantitative Analyses

This section calibrates the model and compares the decentralized economy with the planner's allocation. We solve the model numerically using a global method to accommodate the occasionally binding liquidity constraint and the nonnegativity constraint on reserves. Details of the numerical solution are presented in Appendix J.

4.1 Calibration

We calibrate the decentralized economy rather than the planner's allocation for two reasons. First, the size of liquidity shocks, θ, is an important determinant of the reserve-to-GDP ratio in the model. Because reserve holdings are non-monotonic in θ, the reserve-to-GDP ratio does not uniquely identify this parameter. Second, calibrating the decentralized economy is common in the literature and facilitates comparison with other papers (Lutz and Zessner-Spitzenberg 2023). We calibrate the decentralized economy without targeting the reserve-to-GDP ratio and compare the model-implied reserve-to-GDP ratio with the data.

One period in the model corresponds to one year. We assume log utility $u(c_t) = \ln(c_t)$ for households. Standard parameters are set to conventional values from the literature, while the remaining parameters are set to match data moments from Chile between 1980 and 2019. We choose Chile because it is an emerging economy that has experienced sudden stops and

[21] The two-period model also features an indirect channel: a higher θ increases liquidation and raises ψ_{t+1}^{SP} more than ψ_{t+1}. This channel need not operate in the same way in the full model, where debt and reserves respond endogenously to θ. In the two-period exercise, the initial state is held fixed across values of θ.

because its reserve-to-GDP ratio and capital control index lie close to the fitted lines in Panels A1 and B1 of Figure 1.

Table 2 summarizes the externally determined parameters. The baseline gross interest rate on foreign debt is $R^b = 1.04$, a standard value in the literature. We assume no interest on reserves and set $R^s = 1$. We set the reference debt-to-GDP ratio $\bar{b}$ to the calibration target of 0.50, so that the debt-elastic component of the borrowing rate is zero at this reference ratio. The discount factor β is calibrated jointly with the remaining parameters to match mean debt-to-GDP. Investment curvature is $\gamma = 0.8$, following Comin and Gertler (2006). The exogenous growth rate of foreign productive assets is $\bar{g} = 2.95\%$, Chile's average growth rate over 1980–2019. The productivity spillover coefficient is $\kappa = 0.1$, the estimate in Gornemann et al. (2020). The foreign production parameter ζ determines the elasticity of the fire-sale price to the amount of liquidated assets. Aguiar and Gopinath (2005) estimate the elasticity of the fire-sale price to firm liquidity to be 0.64 during the Asian currency crisis. Given the price equation (15) and the liquidity shortage $q_t a_t^\ell = -\theta_t b_{t-1} - s_{t-1}$, the absolute elasticity of q_t with respect to the liquidity shortage is $\zeta/(1-\zeta)$. Matching the empirical elasticity of 0.64 gives $\zeta = 0.389$.[22]

Table 2 Externally Determined Parameters

Parameter		Value	Source
R^b	Gross interest rate on debt	1.04	Standard
R^s	Gross interest rate on reserves	1	Standard
$\bar{b}$	Reference debt-to-GDP ratio	0.50	Calibration target
γ	Investment curvature	0.8	Comin and Gertler (2006)
$\bar{g}$	Average growth rate	0.0295	Chile, 1980–2019
κ	Productivity spillover	0.1	Gornemann et al. (2020)
ζ	Foreign production curvature	0.389	Aguiar and Gopinath (2005)
ε^R	Interest rate shock	0.0196	Mendoza (2010)

The model has two stochastic shocks: an interest rate shock ε_t^R and a liquidity shock θ_t. For the interest rate shock, we follow Mendoza (2010) and assume that ε_t^R takes either of two values $\{\varepsilon^R, -\varepsilon^R\}$ with $\varepsilon^R = 0.0196$. The liquidity shock takes the value θ or 0. We assume that when a liquidity shock occurs, the interest rate shock takes the high value ε^R. The joint shock therefore has three possible realizations, $(\varepsilon_t^R, \theta_t) = \{(\varepsilon^R, 0), (-\varepsilon^R, 0), (\varepsilon^R, \theta)\}$.

[22] The price elasticities with respect to liquidity holdings and liquidity shortages need not be equal. We therefore examine sensitivity to ζ in Appendix L.3.

These realizations follow a three-state Markov process. As described below, we calibrate the liquidity shock probability to $\pi = 0.0584$. From a state without a liquidity shock, the interest rate shock remains unchanged with probability $0.6(1-\pi)$, and switches with probability $0.4(1-\pi)$. The implied autocorrelation, 0.57, is consistent with Mendoza (2010).

Given the externally determined parameters, we jointly calibrate the remaining five parameters to match the moments in Table 3. The discount factor β is set by targeting the mean debt-to-GDP ratio in Chile. The sensitivity of the borrowing rate to debt, ψ^b, targets the standard deviation of the current account-to-GDP ratio in Chile. The investment efficiency parameter η is set to match the mean investment-to-GDP ratio in Chile. Given the limited number of sudden stops in Chile, the liquidity shock probability targets the 5.5% frequency used by Eichengreen et al. (2006) and Bianchi (2011).

Finally, we discipline the size of liquidity shocks, θ, using the mean deviation of output from its Hodrick–Prescott (HP)-filtered trend at sudden stop episodes. In our sample constructed from Korinek and Mendoza (2014), this deviation is -2.4%. The calibrated value $\theta = 0.45$ matches this target. Holding the other parameters at their benchmark values, the output deviation in the decentralized economy becomes more negative as θ rises, as shown in Appendix Figure O3. This monotonic relationship helps discipline θ using crisis severity.

Table 3 Calibrated Parameters

	Parameter	Value	Target moment	Data	Model
β	Discount factor	0.9195	Mean debt/GDP	0.50	0.49
ψ^b	Debt sensitivity of spread	0.05	S.D. of CA/GDP	0.040	0.047
η	Investment efficiency	0.098	Mean investment/GDP	0.23	0.23
π	Liquidity shock probability	0.0584	Sudden-stop frequency	5.5%	5.4%
θ	Size of liquidity shock	0.45	Crisis output deviation	-2.4%	-2.4%

Note. The five parameters are calibrated jointly in the decentralized economy. Debt, investment, and current-account ratios are expressed as fractions of GDP; CA denotes the current account. The output target is the mean percentage deviation from the HP-filtered trend at the sudden-stop date.

4.2 Business Cycle Moments and Crisis Dynamics

Table 4 reports unconditional moments for the decentralized economy (DE) and the social planner's allocation (SP). The planner holds substantially more reserves, 12% of GDP compared with 3% in DE, while average debt falls from 49% to 44% of GDP. The planner's reserve ratio is close to Chile's 15%, although reserves are not a calibration target. This

untargeted comparison provides a useful reference for the magnitude of reserve holdings. The mean capital control tax is 3.1%, and the crisis probability falls from 5.4% in DE to 1.7% under SP. Thus, the policy package changes the composition of the external balance sheet and reduces crisis frequency.[23]

The modest difference in average debt does not imply that capital controls are unimportant. Reserve accumulation without a debt tax encourages households to borrow more, partly offsetting its stabilization effect. Capital controls limit this borrowing response, allowing official reserves to reduce exposure to liquidity shocks. Conversely, a debt tax alone induces households to reduce private reserves, partly offsetting the reduction in borrowing. Appendix F illustrates these adjustments by applying each instrument separately. The two instruments jointly correct excessive borrowing and insufficient reserve holdings.

Table 4 Stochastic Simulation Moments

	Decentralized economy		Social planner	
	Mean	S.D.	Mean	S.D.
Consumption	0.78	0.02	0.79	0.02
Investment	0.23	0.08	0.21	0.09
Debt (b_t/a_t)	−0.49	0.16	−0.44	0.20
Reserves	0.03	1.19	0.12	0.41
Current account	−0.01	0.05	−0.01	0.04
Capital control tax on debt		—		3.1%
Crisis probability		5.4%		1.7%

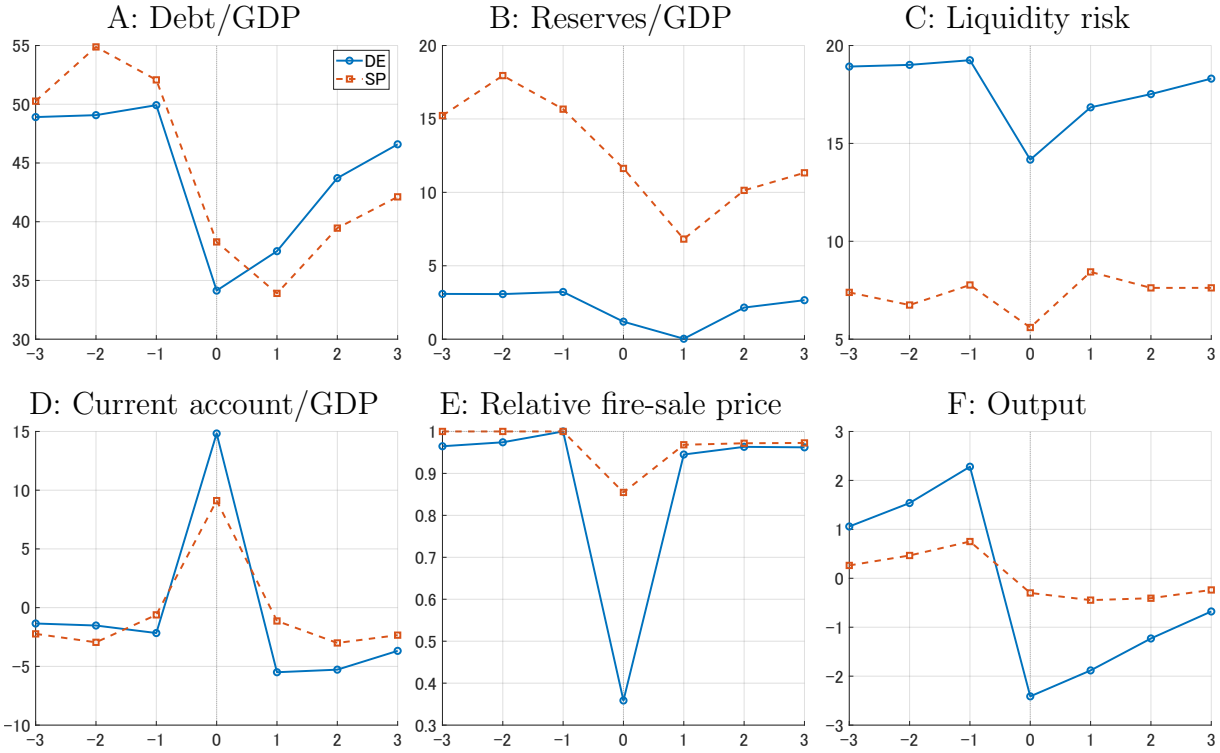
Note. Each simulation has 100,000 periods; the first 10,000 are discarded. Consumption, investment, debt, reserves, and the current account are expressed as ratios to GDP; negative bond holdings indicate debt. For consumption, investment, debt, and reserves, standard deviations are divided by the absolute mean of each variable. The current-account standard deviation is in GDP units. A crisis occurs in either allocation when current account/GDP exceeds its long-run mean by more than twice the DE standard deviation.

Figure 3 shows how the joint policy mitigates crises. The panels average over crises identified separately under each allocation, whereas Table 4 includes both crisis and normal periods. The planner can therefore hold more debt before a typical crisis despite having lower unconditional average debt. Its reserve buffer is also larger, and the difference in reserves more than offsets the difference in debt. The potential pre-crisis liquidity shortage, $-\theta b_t - s_t$,

[23] Appendix E reports the decision rules for debt and reserves. Appendix C shows that debt and reserve flows are positively correlated and both increase when interest rates are low, consistent with the patterns observed in emerging economies.

is about 7% of GDP under SP, versus nearly 20% in DE. This smaller shortage limits asset liquidation and the decline in fire-sale prices. At the crisis date, the relative fire-sale price falls to about 0.85 under SP and 0.36 in DE. The current-account-to-GDP ratio reaches about 9% and 15%, respectively, and output contracts less under SP.

Figure 3 Crisis Dynamics under DE and SP



Note. Panels show average dynamics around crises identified separately in simulations of the decentralized economy (DE, blue solid lines) and the planner's allocation (SP, red dashed lines). The horizontal axis gives years relative to the crisis date, $t = 0$. Panels A–D are expressed as percentages of GDP. Panel C reports the potential liquidity shortage, $-\theta b_t - s_t$, if a liquidity shock occurs in the next period. Panel E reports the fire-sale price relative to the simulation mean of the domestic value of one unit of the asset, $\xi_t/u'(c_t)$, with normal times normalized to one. Panel F reports the average percentage deviation of output from its HP-filtered trend. Table 4 defines the crisis criterion.

Endogenous growth connects asset liquidation to these macroeconomic responses. Liquidation reduces productive capacity and permanent income, inducing households to cut consumption and contributing to the current account reversal. The loss of productive assets also leaves output below its pre-crisis trend. Together, reserve accumulation and capital controls lower the share of assets liquidated during crises from 4.5% in DE to only 1% under SP. By limiting liquidation, the joint policy preserves productive capacity and mitigates

persistent output losses.[24] These benchmark dynamics explain how the policy package operates. Section 5 then compares crisis dynamics across financial-development levels with their empirical counterparts in Figure 2 and studies the joint policy pattern.

5 Financial Development and Optimal Policy

The central question is whether our model can reproduce the joint empirical pattern in Section 2: reserves are hump-shaped in financial development, while capital controls decline. We vary the effective size of liquidity shocks, θ, and first assess whether it captures the observed differences in crisis severity. We then compare the policy predictions with the data, explain the reserve hump, and examine robustness and welfare.

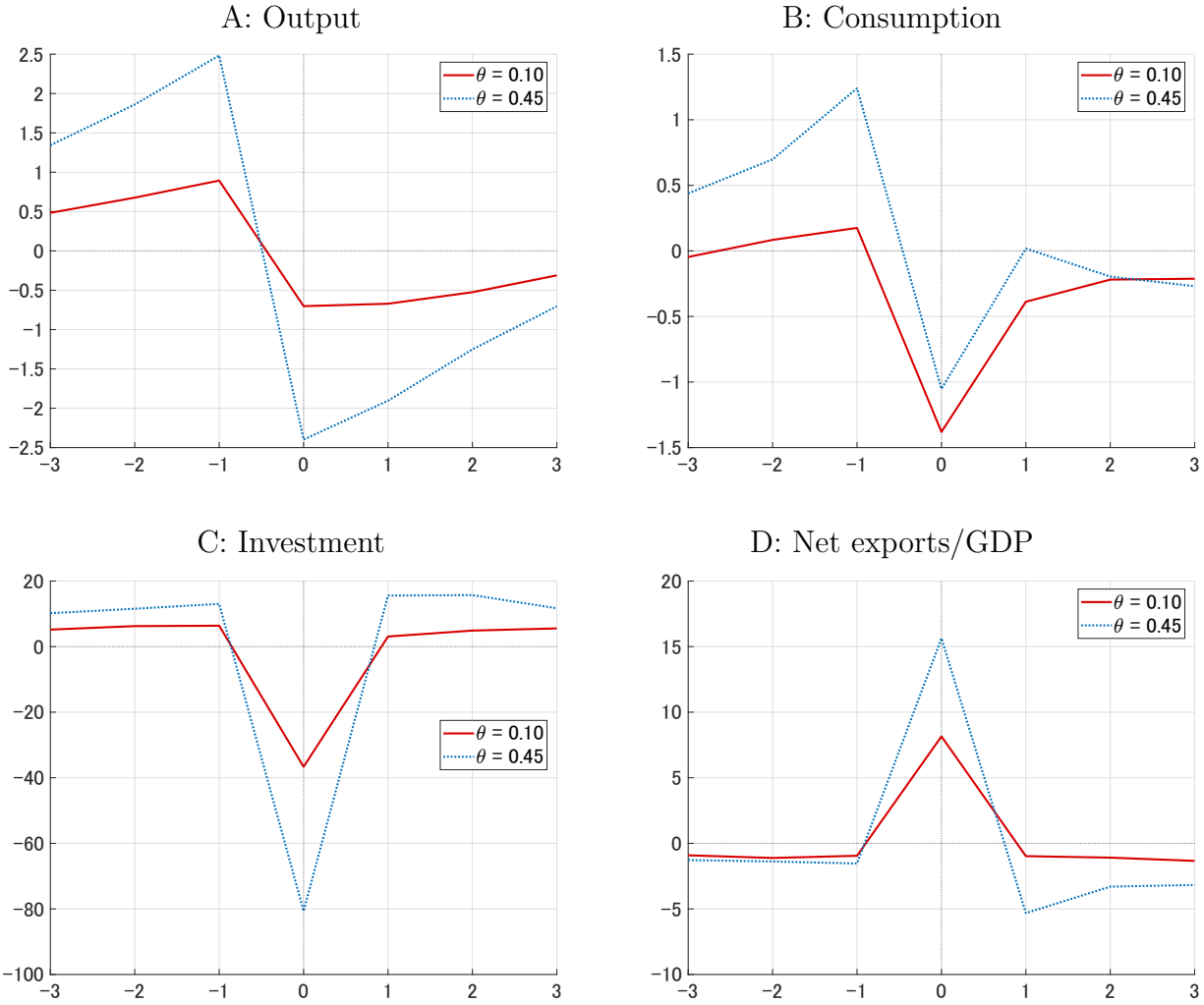
Liquidity-shock severity. As discussed in Section 3, greater financial development reduces early repayment needs through better financing opportunities and risk management, corresponding to a lower θ. We calibrate this parameter to crisis severity rather than reserves (Section 4.1). Holding other parameters fixed, a higher θ produces a larger output shortfall relative to trend at the sudden-stop date in the decentralized economy (Appendix Figure O3). This relationship supports using θ to capture crisis severity.

Figure 4 compares crisis dynamics at $\theta = 0.10$ and 0.45, corresponding to high and low financial development, with the empirical patterns in Figure 2. Lower financial development generates a larger boom-bust cycle and a deeper output contraction. Consumption and investment fall more from their pre-crisis levels, while net exports/GDP rises more. These responses match the qualitative differences across financial-development groups. The single-good model does not include a real exchange rate.

The comparison also reveals some quantitative differences: investment falls more sharply and rebounds more quickly than in the data. In our model, however, investment captures a broad range of growth-enhancing expenditures, following Ma (2020), rather than physical investment alone. We therefore focus on the qualitative decline in investment during crises rather than its precise magnitude. Given its simplicity, we view the model as capturing the broad features of crises, particularly how they differ with financial development.

[24] This permanent-income mechanism is consistent with Aguiar and Gopinath (2007). Guntin et al. (2023) show that even top-income households with liquid assets reduce consumption substantially during sudden stops. Persistent output losses are documented by Cerra and Saxena (2008) and Reinhart and Rogoff (2009). Related endogenous-growth models, including Benigno et al. (2022), also generate persistent output effects and potentially sizable welfare gains. Our emphasis is on how financial development shapes the joint use of reserves and capital controls.

Figure 4 Crisis Dynamics across θ

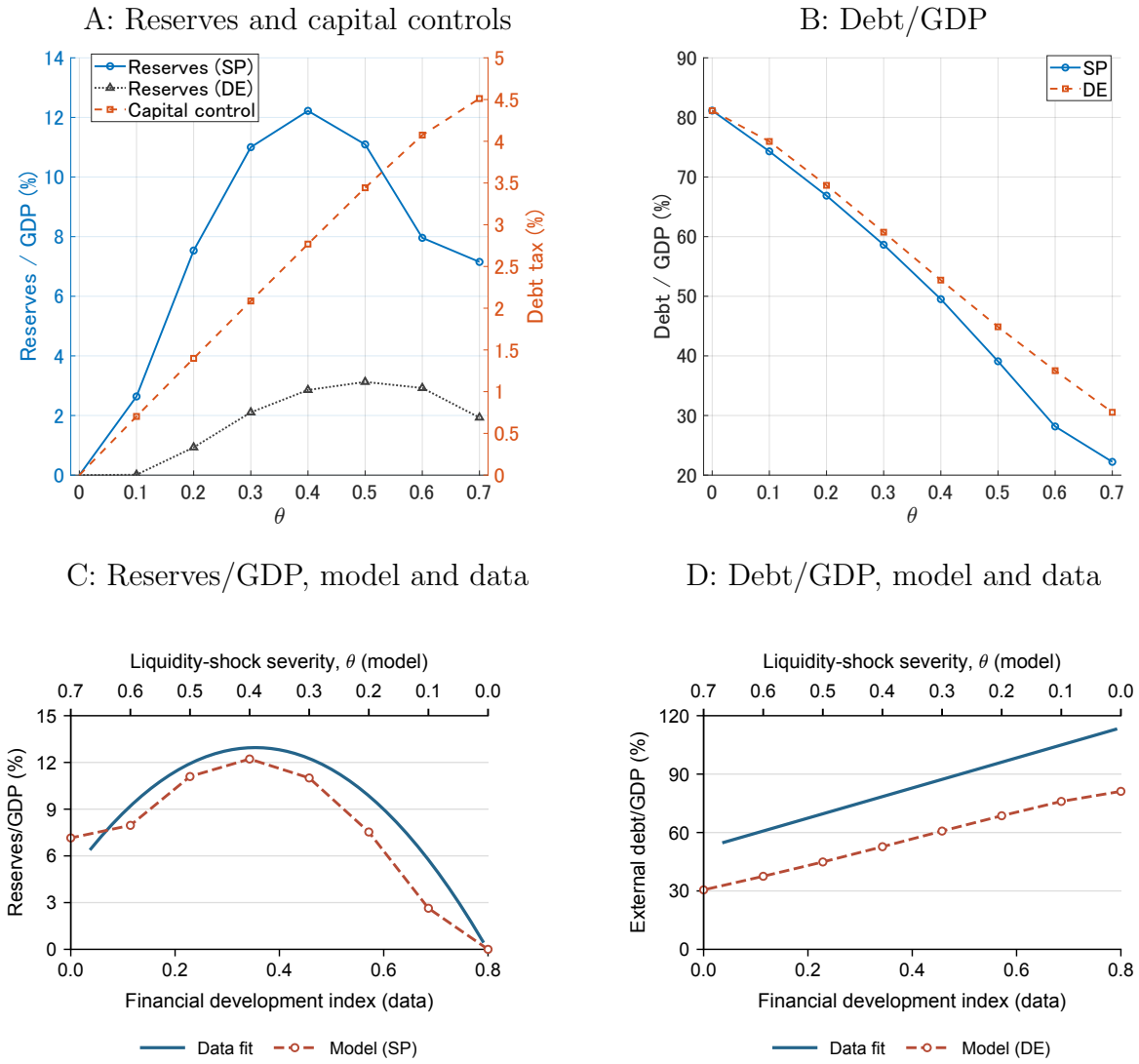


Note. The panels show crisis dynamics under the planner's allocation for $\theta = 0.10$ (high financial development) and 0.45 (low financial development). The horizontal axes show periods relative to the sudden stop at 0. Output and consumption are log deviations from HP-filtered trends, in percent. Investment is measured as $z_t - q_t a_t^{\ell}$, net of asset liquidation, and shown as a percentage deviation from its HP-filtered trend. Net exports/GDP is expressed as a percentage-point deviation from its HP-filtered trend.

5.1 Reserve Holdings and Capital Controls

The model generates the joint policy pattern without targeting reserves or capital controls. Figure 5 presents the model predictions alongside their empirical counterparts. Panels A–B report means from 100,000-period simulations of the decentralized equilibrium (DE) and the social planner's allocation (SP) over $\theta \in [0, 0.7]$. Reserves are non-monotonic in both allocations, while the capital control tax rises and debt falls with θ. Since lower θ represents

Figure 5 Financial Development and Policy: Model and Data



Note. Ratios and tax rates are in percent. Panels A–B report simulated means across θ in the decentralized equilibrium (DE) and social planner's allocation (SP); the capital control tax is for SP. Lower θ represents greater financial development. Panels C–D compare SP reserves and DE debt with fitted curves for the 85 economies in Figure 1 with financial development data, using available-year country averages over 1980–2019. Reserves exclude gold; external debt covers debt liabilities to nonresidents. The reserve fit is quadratic and the debt fit linear, over the observed index range. Financial development increases to the right in Panels C–D: the data index rises on the bottom axes and θ falls on the top axes. The scales are aligned for comparison without estimating a mapping between them.

greater financial development, the model reproduces the reserve hump, declining capital controls, and rising borrowing as financial development improves.

Panels C–D assess the quantitative magnitudes. Panel C compares official reserves in

the data with the planner's reserves, which correspond to official holdings under our policy implementation. The peaks are similar: 13.0% of GDP in the empirical fit and 12.2% in the model, attained at $\theta = 0.40$. Panel D compares external debt with household borrowing in DE. Model debt/GDP rises from 30.5% to 81.1% as θ falls from 0.7 to 0, compared with an empirical fitted range of 54.9% to 113.0%. Thus, the model captures the increase in borrowing but generates lower debt levels.

Together, these comparisons show that the model captures the qualitative policy pattern and the magnitude of the reserve hump without targeting them. Although debt is a calibration target at the Chilean benchmark, the cross-country profiles are untargeted. The comparison varies θ while holding the other parameters fixed; the horizontal axes are aligned without estimating a mapping to the financial development index. For capital controls, we compare the direction of the relationship because the empirical restrictions index and the model's borrowing tax are different measures.

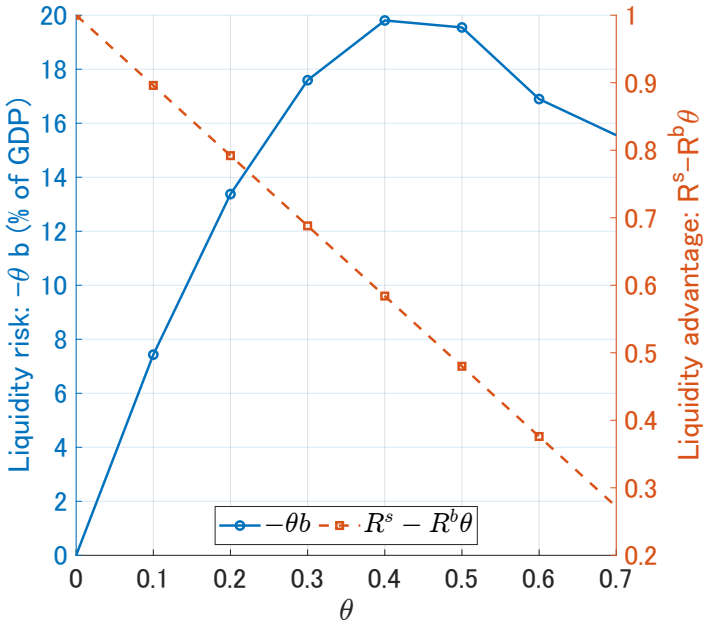
To understand the reserve hump, consider the interaction between repayment needs and the liquidity advantage of reserves, as discussed in Section 3.2.[25] Figure 6 shows the early-repayment requirement before reserves, $-\theta b$, on the left axis and the liquidity advantage, $R^s - R^b\theta$, evaluated at the baseline borrowing rate, on the right. At low θ, reserves have a large liquidity advantage, but small repayment needs limit reserve demand.

As θ rises, repayment needs increase and peak at $\theta = 0.4$. The liquidity advantage remains positive at this intermediate value, encouraging reserve accumulation. At higher θ, the liquidity advantage of reserves declines, while households reduce debt in response to greater liquidity risk. Both forces lower reserve demand. Capital controls rise with θ because greater repayment needs increase exposure to the fire-sale externality. This explains why the two policies respond differently to financial development.

Robustness and extensions. We next ask whether other dimensions of financial development can account for the joint policy pattern and whether the pattern survives changes in the calibration. We vary the liquidity-shock probability, the reference debt ratio, and the sensitivity of borrowing rates to debt, which capture alternative aspects of liquidity risk and financing conditions. We also vary the growth calibration. In these exercises, changing the other parameters does not reproduce the joint pattern generated by varying θ. This supports

[25] Appendix K compares this pattern with quantitative exercises in Bianchi (2011), Arce, Bengui, and Bianchi (2025), and Lutz and Zessner-Spitzenberg (2023). Varying collateral pledgeability or trade-financing needs in those exercises does not reproduce the reserve hump together with declining capital controls.

Figure 6 Determinants of Reserve Holdings



Note. The horizontal axis is the early-repayment fraction θ. The left axis reports the early-repayment requirement $-\theta b$ as a percentage of GDP, where b is mean debt/GDP under the model's sign convention. The right axis reports the liquidity advantage $R^s - R^b\theta$, evaluated at the baseline borrowing rate.

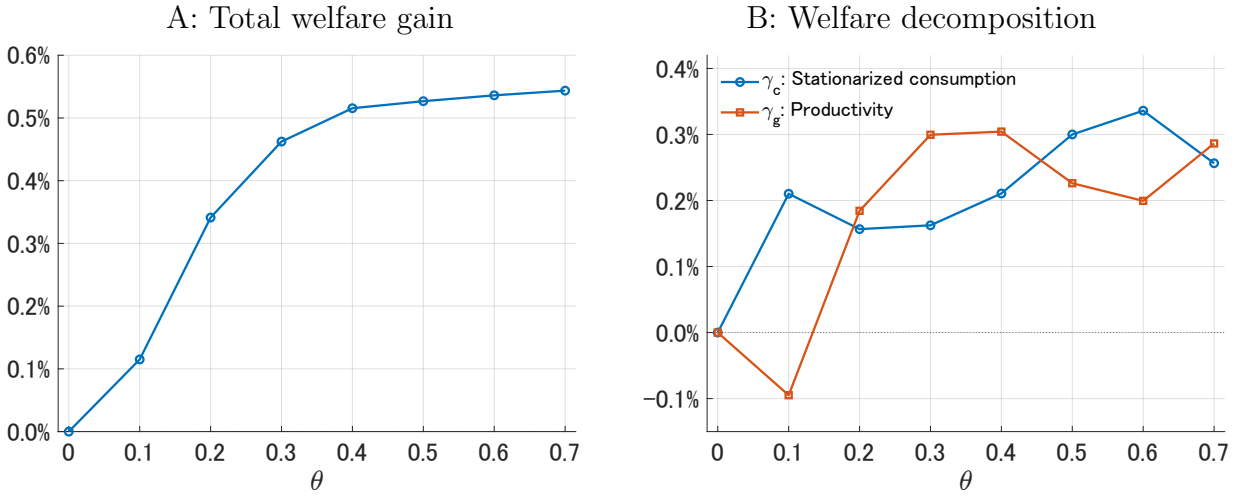
the role of crisis severity in linking financial development to policy choices. Appendices O and M report the results.

We also examine whether the cross-country pattern arises along an economy's development path. Unlike the baseline comparisons, which hold financial development fixed within each economy, this extension allows θ to decline as domestic productive assets catch up with the foreign frontier (Appendix N.2). This introduces a financial-development externality alongside the fire-sale externality. Along the planner's transition, reserves/GDP rises and then falls, while the capital control tax declines after the initial adjustment. The extension thus connects the cross-country policy pattern to the evolution of policy within an economy.

5.2 Welfare Analysis

We next examine how financial development affects the welfare gains from policy. We measure the gain from SP as the permanent percentage increase in DE consumption that makes

Figure 7 Welfare Gains and Decomposition



Note. The horizontal axis is the early-repayment fraction θ. Panel A reports the consumption-equivalent welfare gain from SP relative to DE, measured as a permanent percentage increase in consumption. Panel B reports the stationary-consumption component γ_c and productivity component γ_g.

households as well off as under SP. For each θ, we average state-specific gains using the long-run frequencies from a 100,000-period DE simulation. Following Ma (2020), we decompose the gain into stationary-consumption and productivity components.[26]

Figure 7 reports total gains and their decomposition. Gains rise with θ, implying larger gains at lower levels of financial development. At the benchmark $\theta = 0.45$, they are slightly above 0.5%, with roughly equal contributions from the two components. The benchmark gain exceeds those in Bianchi (2011) and Ma (2020). To understand these gains, we examine two features of our model. First, the distortion arises from a fire-sale externality rather than a collateral externality. We therefore examine the parameters that govern its strength. Second, our liquidity constraint links asset liquidation to endogenous growth, creating scope for policy to prevent losses of productive capacity.

Two parameters govern exposure to the fire-sale externality and its strength. A higher θ raises repayment needs for a given debt position, increasing exposure to liquidation. The parameter ζ determines how sharply the fire-sale price falls as assets are sold, governing the price effect that households do not internalize. Welfare gains rise with ζ under endogenous growth, exogenous growth, and no growth (Appendix L.3). Thus, the strength of the underlying fire-sale externality matters independently of the growth specification.

[26] Appendix L defines the components and reports the detailed comparisons.

The second channel operates through productive capacity: liquidation reduces both current assets and future output. Reserves protect these assets during crises but use resources that could finance investment in normal times. At $\theta = 0.45$, the benefit of avoiding liquidation dominates, producing a positive productivity component. This differs from the negative component in the baseline of Ma (2020), where crises affect growth through investment decisions. At $\theta = 0.1$, the scope to prevent liquidation losses is smaller and our productivity component also becomes negative (Appendix L.1). The larger benchmark gain therefore reflects both the fire-sale externality and the productivity losses that policy can prevent; endogenous growth alone does not ensure a large gain.

6 Conclusion

In this paper, we provide a financial development perspective to explain the joint use of foreign reserves and capital controls. We first document a non-monotonic relationship between financial development and the reserve-to-GDP ratio, with countries at intermediate levels of financial development holding the most reserves relative to GDP. In contrast, the relationship between financial development and capital controls is monotonic, and countries with high financial development use capital controls less actively. We then build a small-open-economy model with liquidity shocks and endogenous growth. A fire-sale externality arising from asset liquidation causes households to over-borrow and under-accumulate reserves, justifying the joint use of public reserve accumulation and capital controls. Our model reproduces the reserve hump and the qualitative decline in capital controls across countries.

Our paper has significant policy implications. It highlights how structural factors such as financial development shape policy design. Our welfare analysis shows that the scope for policy to prevent persistent productivity losses depends on liquidity-shock severity. Endogenous growth contributes to the benchmark gain, while the sensitivity of the fire-sale price also matters for welfare under exogenous growth and no growth.

We leave several open questions for future research. Extending the analysis to a two-good economy could inform exchange rate policy. Policy interventions addressing aggregate-demand or additional terms-of-trade externalities offer another direction.

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Internet Appendix

'Foreign Reserves and Capital Controls: A Financial Development Perspective'

(Intended for online publication only)

by C. Ma, and H. Matsumoto

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A Data Appendix

A.1 Country List

The sample used in Figure 1 and Table 1 follows Bianchi and Lorenzoni (2022). We exclude countries with extreme reserve-to-GDP ratios and external liability-to-GDP ratios, such as Lebanon, Liberia, Luxembourg, Malta, Saudi Arabia, Singapore, and Switzerland. Our final sample thus includes 87 economies from 1980 to 2019, including 30 advanced economies: Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Latvia, Lithuania, Netherlands, New Zealand, Norway, Portugal, San Marino, Slovak Republic, Slovenia, South Korea, Spain, Sweden, United Kingdom, United States; 31 emerging economies: Belarus, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Croatia, Ecuador, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Paraguay, Peru, Philippines, Poland, Romania, Russia, Serbia, South Africa, Thailand, Turkey, Ukraine, Uruguay, Venezuela, Vietnam; and 26 low-income economies: Bolivia, Burundi, Cameroon, Central African Republic, Chad, Comoros, Eritrea, Ethiopia, Ghana, Guinea, Guyana, Haiti, Honduras, Kenya, Madagascar, Malawi, Mauritania, Mozambique, Nicaragua, Rwanda, Sierra Leone, Somalia, Sudan, Tanzania, Uganda, Zambia.

The sample used in Table C1 consists of 47 countries: Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Côte d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Georgia, Ghana, Hungary, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russia, Senegal, South Africa, South Korea, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam.

A.2 Variable Construction

CA (% GDP) Current account balance as a share of GDP, from the World Development Indicators.

Capital controls index The negative of the Chinn–Ito index from Chinn and Ito (2006).

EMBI spread (in log) The log of one plus the EMBI spread from the World Bank's Global Economic Monitor Database.

Euro-area indicators The post-adoption euro indicator equals one beginning in the year in which an economy adopts the euro.

External debt (% GDP) Total external debt liabilities divided by GDP, based on Lane and Milesi-Ferretti (2007).

External liability (% GDP) Total external liabilities divided by GDP, based on Lane and Milesi-Ferretti (2007).

Federal Reserve line access An indicator equal to one if any Federal Reserve reciprocal-currency or dollar-liquidity arrangement is active during part of a country-year. The measure covers the historical reciprocal-currency arrangements through 1998, the Canadian and Mexican arrangements under the North American Framework Agreement, the temporary arrangements after September 11, 2001, the dollar-liquidity facilities established from 2007, and the standing facilities established in 2013. Source: Federal Reserve and FOMC records and Bahaj et al. (2024).

Financial development index The IMF index measures the development of financial institutions and markets in each country.

GDP per capita The log of gross domestic product divided by midyear population from the World Development Indicators.

Institutional quality The average of the six Worldwide Governance Indicators from the World Bank Institute, including control of corruption, government effectiveness, political stability and absence of violence/terrorism, rule of law, regulatory quality, and voice and accountability.

Reserves (% GDP) Foreign reserves (excluding gold) divided by GDP, based on Lane and Milesi-Ferretti (2007).

Reserve flows (% GDP) Changes in reserve stocks excluding gold (% GDP) from Alfaro et al. (2014).

Trade (% GDP) Exports and imports of goods and services as a share of GDP from the World Development Indicators.

Peg An indicator for whether a country has a fixed exchange rate from Shambaugh (2004).

Private flows (% GDP) Total private capital inflows (% GDP) from Alfaro et al. (2014).

A.3 Summary Statistics

Table A1 Summary Statistics

Panel A: Variables used in Figure 1, Table 1, and Table B3					
	Obs	Mean	S.D.	Min	Max
Reserves (% GDP)	3279	9.58	8.58	0	68.35
Capital control index	3188	-0.23	1.59	-2.32	1.92
External liability (% GDP)	3204	1.11	1.05	0.16	10.23
External debt (% GDP)	3205	0.74	0.66	0.11	5.6
Financial development index	3400	0.31	0.24	0	0.97
GDP per capita (in log)	3219	8.11	1.71	4.55	11.54
Trade/GDP	3126	0.68	0.39	0.06	3.21
Institutional quality	2088	0.11	1	-2.41	1.95
Peg	3118	0.34	0.47	0	1
Current account/GDP	3222	-0.03	0.07	-0.84	0.31
Post-adoption euro indicator	3480	0.08	0.27	0.00	1.00
Active Fed-line indicator	3480	0.16	0.36	0.00	1.00
Panel B: Variables used in Table C1					
	Obs	Mean	S.D.	Min	Max
Private flow (% GDP)	1275	6.24	12.91	-161.68	126.09
Reserve flow (% GDP)	1274	1.51	3.70	-17.11	26.27
EMBI (in log)	844	1.51	0.64	0.00	4.60
GDP per capita (in log)	1499	8.03	0.99	4.55	10.42
Trade/GDP	1471	0.74	0.36	0.00	2.20
Institutional quality	1128	-0.20	0.58	-1.90	1.22
Peg	1432	0.39	0.49	0.00	1.00
Current account (% GDP)	1414	-1.69	6.30	-26.21	38.79

Table A2 Capital Controls and Financial Development: Testing the Nonlinear Relationship

	Capital Control Index	
	(1)	(2)
Financial Development	-0.61***	-0.28*
	(0.08)	(0.16)
Financial Development^2	0.05	0.02
	(0.05)	(0.06)
GDP per capita (log)		-0.26
		(0.26)
Trade (% GDP)		-0.24**
		(0.09)
Institutional Quality		-0.55**
		(0.21)
Peg		0.01
		(0.08)
CA (% GDP)		-0.02
		(0.02)
Constant	-0.01	-0.06
	(0.09)	(0.08)
Time FE	Y	Y
Country FE	N	Y
Observations	3141	1852
R^2	0.396	0.877

Note. The dependent variable is the capital control index (the negative of the Chinn–Ito Index constructed by Chinn and Ito 2006). The independent variables include the IMF Financial Development Index, its squared term, log GDP per capita, the trade-to-GDP ratio, institutional quality, the exchange rate regime constructed by Shambaugh (2004), and the current account-to-GDP ratio. All variables are standardized except for the exchange rate regime. Standard errors are clustered by country and year and reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. All variable constructions are described in Appendix A.

Table A3 Reserves and Financial Development: Robustness

Financial Development:	Domestic Credit		Stock Market Capitalization		Financial Institutions		Financial Markets		Financial Institutions Depth	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Financial Development	0.10	0.23	-0.06	0.56***	0.25***	0.79***	0.16*	0.27	0.23**	0.54**
	(0.14)	(0.16)	(0.19)	(0.14)	(0.08)	(0.17)	(0.09)	(0.32)	(0.10)	(0.23)
Financial Development^2	-0.10*	-0.13**	0.03	-0.07**	-0.35***	-0.21**	-0.23***	-0.08	-0.25***	-0.12*
	(0.05)	(0.05)	(0.04)	(0.03)	(0.07)	(0.09)	(0.05)	(0.09)	(0.06)	(0.06)
Time FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country FE	N	Y	N	Y	N	Y	N	Y	N	Y
Observations	2347	1542	1401	980	3212	1853	3212	1853	3212	1853
R^2	0.161	0.784	0.105	0.808	0.239	0.704	0.207	0.689	0.209	0.693

Financial Development:	Financial Institutions Access		Financial Institutions Efficiency		Financial Markets Depth		Financial Markets Access		Financial Markets Efficiency	
	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Financial Development	0.25***	0.73***	0.08	0.16*	0.19	0.27	0.11	0.31	0.09	0.13
	(0.07)	(0.21)	(0.08)	(0.09)	(0.12)	(0.18)	(0.09)	(0.20)	(0.10)	(0.22)
Financial Development^2	-0.29***	-0.23*	0.02	0.04	-0.19***	-0.08	-0.12**	-0.07	-0.14*	-0.06
	(0.08)	(0.13)	(0.05)	(0.06)	(0.04)	(0.05)	(0.05)	(0.05)	(0.08)	(0.08)
Time FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country FE	N	Y	N	Y	N	Y	N	Y	N	Y
Observations	3212	1853	3212	1853	3212	1853	3212	1853	3212	1853
R^2	0.206	0.704	0.155	0.691	0.201	0.690	0.170	0.690	0.166	0.688

Note. The dependent variable is the reserve-to-GDP ratio. Columns (1)–(20) use alternative measures of financial development: domestic credit to the private sector (% GDP) from WDI in columns (1)–(2); stock market capitalization (% GDP) in columns (3)–(4); the IMF Financial Institutions Index in columns (5)–(6); the IMF Financial Markets Index in columns (7)–(8); Financial Institutions sub-indices (depth, access, efficiency) in columns (9)–(14); and Financial Markets sub-indices (depth, access, efficiency) in columns (15)–(20). See Svirydzenka (2016) for details. Controls include log GDP per capita, trade-to-GDP, institutional quality, the exchange rate regime from Shambaugh (2004), and the current account-to-GDP ratio (omitted for brevity). All variables are standardized except the exchange rate regime. Standard errors are clustered by country and year and reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Variable definitions are in Appendix A.

Table A4 Capital Controls and Financial Development: Robustness

Financial Development:	Domestic Credit		Stock Market Capitalization		Financial Institutions		Financial Markets		Financial Institutions Depth	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Financial Development	-0.49***	-0.10	-0.08	-0.03	-0.61***	-0.22*	-0.44***	-0.09	-0.55***	-0.18*
	(0.08)	(0.07)	(0.16)	(0.05)	(0.05)	(0.12)	(0.07)	(0.09)	(0.05)	(0.10)
Time FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country FE	N	Y	N	Y	N	Y	N	Y	N	Y
Observations	2313	1541	1383	980	3141	1852	3141	1852	3141	1852
R^2	0.334	0.896	0.023	0.881	0.443	0.877	0.283	0.876	0.405	0.876

Financial Development:	Financial Institutions Access		Financial Institutions Efficiency		Financial Markets Depth		Financial Markets Access		Financial Markets Efficiency	
	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Financial Development	-0.55***	-0.16	-0.33***	-0.00	-0.43***	-0.12	-0.39***	-0.14*	-0.34***	0.02
	(0.06)	(0.10)	(0.07)	(0.05)	(0.06)	(0.09)	(0.06)	(0.07)	(0.08)	(0.06)
Time FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country FE	N	Y	N	Y	N	Y	N	Y	N	Y
Observations	3141	1852	3141	1852	3141	1852	3141	1852	3141	1852
R^2	0.387	0.877	0.172	0.875	0.266	0.876	0.244	0.877	0.217	0.875

Note. The dependent variable is the capital control index (the negative of the Chinn–Ito Index from Chinn and Ito 2006). Columns (1)–(20) use alternative measures of financial development: domestic credit to the private sector (% GDP) from WDI in columns (1)–(2); stock market capitalization (% GDP) in columns (3)–(4); the IMF Financial Institutions Index in columns (5)–(6); the IMF Financial Markets Index in columns (7)–(8); Financial Institutions sub-indices (depth, access, efficiency) in columns (9)–(14); and Financial Markets sub-indices (depth, access, efficiency) in columns (15)–(20). See Svirydzenka (2016) for details. Controls include log GDP per capita, trade-to-GDP, institutional quality, the exchange rate regime from Shambaugh (2004), and the current account-to-GDP ratio (omitted for brevity). All variables are standardized except the exchange rate regime. Standard errors are clustered by country and year and reported in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Variable definitions are in Appendix A.

B Foreign Reserves and Financial Development: Non-Monotonic Relationship

Table 1 documents a non-monotonic relationship between foreign reserves and financial development. This section examines the relationship in three ways. First, we compare our specification with the empirical reserve-demand literature. Second, we report the turning point in the original units of the IMF index and examine its sensitivity to alternative specifications. Third, we study economies for which reserve demand may be affected by reserve-currency status, euro-area membership, or access to Federal Reserve swap lines.

B.1 Comparison with the Literature

The empirical literature emphasizes several determinants of reserve demand. Aizenman and Lee (2007) study precautionary and mercantilist motives using measures of economic size, trade, exchange-rate volatility, real exchange-rate misalignment, financial openness, and crises. Obstfeld et al. (2010) emphasize domestic financial liabilities, measured by M2 relative to GDP, together with financial openness and the exchange-rate regime. Dominguez (2010) also considers M2 relative to GDP, imports, capital-account openness, crises, and measures of financial development. These studies generally estimate linear reserve-demand relationships and do not test the quadratic relationship.

We begin with column (2) of Table 1 and add the literature-based controls sequentially. Columns (1)–(4) of Table B1 show that population, exchange-rate variables, and financial openness leave the inverted-U pattern close to the baseline estimate.

Column (5) adds M2 relative to GDP. M2 and the IMF index capture different financial channels. M2 measures liquid domestic liabilities that may generate a demand for foreign currency during a crisis. The IMF index instead summarizes the depth, access, and efficiency of both financial institutions and financial markets. Including both variables therefore allows us to distinguish the domestic-liquidity channel from the broader level of financial development. The coverage of M2, however, is limited. To expand coverage, we use the WDI broad-money-to-GDP ratio whenever it is available. For countries with no WDI observations, we construct the same ratio using broad money and nominal GDP, reported in matching local-currency units, from the JST Macrohistory Database (Jordà et al., 2017). In column (5), the quadratic coefficient is -0.190 and remains significant at the 5% level. Because M2 coverage changes the sample, we also estimate the same specification without M2 in an unreported regression. Its quadratic coefficient is -0.191 and remains statistically significant. Thus, conditioning on M2 has little effect on the estimated curvature.

Table B1 Foreign Reserves and Financial Development: Additional Controls

	Baseline	Population	Exchange Rate	Financial Openness	M2/GDP
	(1)	(2)	(3)	(4)	(5)
Financial Development	0.910***	0.972***	0.920***	0.970***	0.713***
	(0.256)	(0.293)	(0.256)	(0.264)	(0.235)
Financial Development^2	−0.233***	−0.231**	−0.236***	−0.238***	−0.190**
	(0.082)	(0.083)	(0.082)	(0.084)	(0.084)
Time FE	Y	Y	Y	Y	Y
Country FE	Y	Y	Y	Y	Y
Observations	1,853	1,853	1,853	1,852	1,610
Adjusted R^2	0.681	0.682	0.681	0.685	0.764

Note. The dependent variable is foreign reserves relative to GDP. The baseline controls are log GDP per capita, the trade-to-GDP ratio, institutional quality, the peg indicator, and the current account-to-GDP ratio. Financial Development is the standardized IMF Financial Development Index, and its squared term is constructed after standardization. In column (5), M2/GDP is logged and standardized. Every specification includes country and time fixed effects. Standard errors are clustered by country and year and reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

B.2 Turning Points

Because our baseline regression in Table 1 uses the standardized financial-development index, we report the turning point implied by the regression estimates in the original units of the IMF Financial Development Index. Let $FD^z_{it} = (FD_{it} - \overline{FD})/s_{FD}$ denote the standardized index used in Table 1. For a regression containing FD^z_{it} and $(FD^z_{it})^2$, the turning point in standardized units is

$$FD^{z,*} = -\frac{\widehat{\beta}_1}{2\widehat{\beta}_2}.$$

We transform this value back into the original IMF index using $FD^* = \overline{FD} + s_{FD} FD^{z,*}$. Standardizing the index changes the reported regression coefficients but not the fitted curve or the turning point expressed in the original units.

Table B2 compares three specifications. In the baseline panel regression with country and time fixed effects, the estimated turning point is 0.776. The point estimate lies within the observed support of the index, and there are 175 country-year observations from 15 economies at or above it. When we retain time fixed effects but remove country fixed effects using the same 1,853 observations, the turning point falls to 0.472. The country-average cross-sectional regression implies a turning point of 0.367. The precise location of the peak is therefore sensitive to whether the regression uses within-country or between-country variation. We view these turning-point estimates as complementary descriptions of the same non-monotonic pattern.

Table B2 Turning Points in the IMF Financial Development Index

	Baseline FE (1)	Time FE Only (2)	Cross-Sectional (3)
Financial Development	0.910***	0.451**	34.435**
	(0.256)	(0.204)	(14.818)
Financial Development^2	−0.233***	−0.329***	−46.927***
	(0.082)	(0.073)	(15.673)
Turning point (original IMF index)	0.776	0.472	0.367
Time FE	Y	Y	N/A
Country FE	Y	N	N/A
Observations	1,853	1,853	82
Adjusted R^2	0.681	0.239	0.312

Note. Columns (1) and (2) use standardized reserves-to-GDP ratios and a standardized financial development index, with the squared term constructed from the standardized index. They use the same panel observations and baseline controls; column (2) removes only country fixed effects. Column (3) uses country-average reserves-to-GDP ratios in percentage points and the country-average financial development index in its original units. All turning points are reported on the original zero-to-one IMF index scale. The cross-sectional controls are the log of average GDP per capita and country averages of trade relative to GDP, institutional quality, the peg indicator, and the current-account-to-GDP ratio. Robust standard errors are reported for the cross-sectional regression. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

B.3 Excluding the United States, Euro Area, and Swap-Line Economies

The upper end of the financial-development distribution contains economies with unusual access to international liquidity. The United States issues the dominant reserve currency, euro-area countries share a common monetary authority after entry, and Federal Reserve swap lines provide foreign-currency liquidity during periods of stress. These institutional arrangements may reduce national demand for foreign reserves.

We address these concerns using both sample exclusions and indicator controls. The exclusion specifications remove the relevant countries or country-years. The indicator specifications retain the full sample and allow post-adoption euro-area membership or an active Federal Reserve arrangement to shift the level of reserve demand.

Panel A shows that excluding the United States leaves the estimates essentially unchanged. Excluding complete euro-country histories reduces the quadratic coefficient from −0.233 to −0.114, while excluding active Fed-line years reduces it to −0.146; both estimates are statistically insignificant. The two treatments overlap substantially: 211 of the 316 active Fed-line observations come from countries that eventually adopt the euro, and 193 of the 268 post-adoption euro observations have active Fed-line access, either through earlier arrangements or through the ECB.

Panel B helps explain why the quadratic term becomes statistically insignificant when

Table B3 Foreign Reserves and Financial Development: Euro-Area and Federal Reserve Swap-Line Treatments

Panel A: Sample exclusions

	Baseline (1)	Exclude US (2)	Exclude Euro Area (3)	Exclude Fed-Line Years (4)	Combined (5)
Financial Development	0.910***	0.906***	0.641***	0.646***	0.631***
	(0.256)	(0.257)	(0.202)	(0.196)	(0.205)
Financial Development^2	−0.233***	−0.232**	−0.114	−0.146	−0.066
	(0.082)	(0.084)	(0.082)	(0.087)	(0.105)
Time FE	Y	Y	Y	Y	Y
Country FE	Y	Y	Y	Y	Y
Baseline controls	Y	Y	Y	Y	Y
Observations	1,853	1,830	1,462	1,537	1,334
Adjusted R^2	0.681	0.675	0.758	0.751	0.742

Panel B: Full-sample indicator controls

	Baseline (1)	Euro Indicator (2)	Fed-Line Indicator (3)	Both Indicators (4)
Financial Development	0.910***	0.740***	0.764***	0.690***
	(0.256)	(0.176)	(0.221)	(0.175)
Financial Development^2	−0.233***	−0.096	−0.207**	−0.098
	(0.082)	(0.076)	(0.079)	(0.074)
Post-adoption euro indicator	—	−1.889***	—	−1.682***
		(0.263)		(0.229)
Active Fed-line indicator	—	—	−0.849***	−0.401***
			(0.209)	(0.107)
Time FE	Y	Y	Y	Y
Country FE	Y	Y	Y	Y
Baseline controls	Y	Y	Y	Y
Observations	1,853	1,853	1,853	1,853
Adjusted R^2	0.681	0.769	0.715	0.775

Note. The dependent variable is foreign reserves relative to GDP. All columns otherwise follow column (2) of Table 1. "Exclude Euro Area" excludes the complete histories of eventual euro adopters. "Exclude Fed-Line Years" covers country-years with any active Federal Reserve arrangement, including legacy reciprocal-currency and NAFA arrangements. The combined specification in Panel A excludes the United States, the complete histories of euro adopters, and active Fed-line years for the remaining countries. In Panel B, the indicators equal one only while the corresponding status or arrangement is active. Standard errors are clustered by country and year and reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

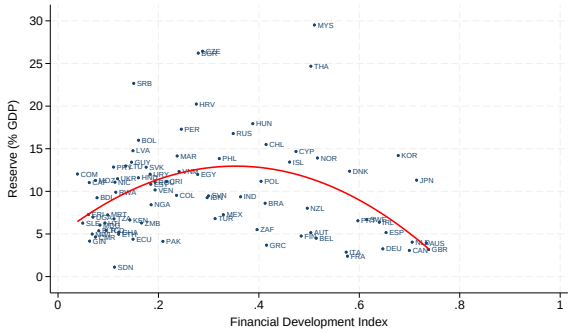
euro-area countries or active Fed-line observations are excluded, while the estimate remains negative. Both indicators are negative and precisely estimated, consistent with lower national reserve holdings when alternative liquidity arrangements are available. The euro in-

dicator accounts for most of the attenuation: it reduces the quadratic coefficient to −0.096, whereas the Fed-line indicator alone leaves a coefficient of −0.207, significant at the 5% level; adding both produces a coefficient of −0.098. This pattern is consistent with high-financial-development euro-area observations holding relatively low national reserves after monetary union. Thus, the panel estimates are sensitive to euro-area observations.

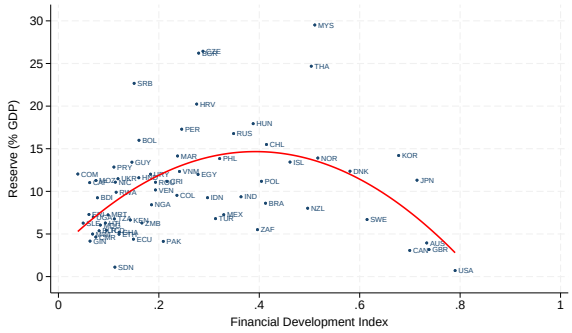
However, the non-monotonic relationship is not entirely driven by either euro membership or Fed-line access. Figure B1 reproduces the cross-sectional relationship under the corresponding sample restrictions. The four panels respectively exclude the United States, the complete histories of euro adopters, active Fed-line country-years, and all three groups using the combined restriction. The quadratic coefficient is negative with a p-value below 0.001 in every case. Excluding euro-country histories therefore does not explain the cross-sectional hump: the quadratic coefficient becomes more negative, from −65.38 in Figure 1 to −74.97. The combined restriction also preserves the hump, with a quadratic coefficient of −58.56.

Figure B1 Foreign Reserves and Financial Development under Alternative Sample Restrictions

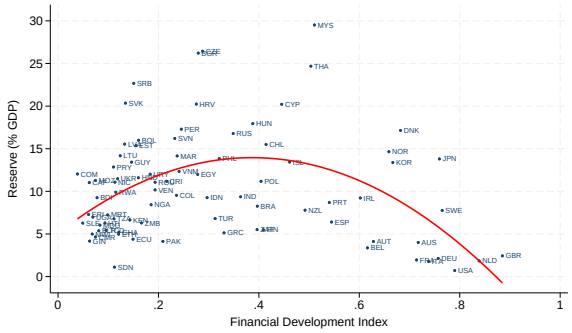
Panel A: Exclude US



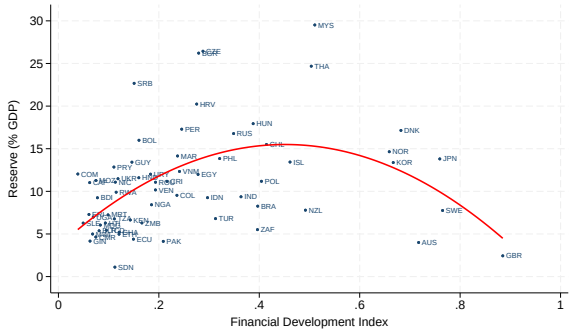
Panel B: Exclude Euro-Area Countries



Panel C: Exclude Active Fed-Line Observations



Panel D: Exclude All



Note. Each point is a country average computed after applying the stated restriction; the lines are quadratic fits across country averages. The quadratic coefficients are −65.62 in Panel A, −74.97 in Panel B, −59.23 in Panel C, and −58.56 in Panel D; the corresponding heteroskedasticity-robust standard errors are 14.97, 14.72, 12.19, and 13.30. The p-value is below 0.001 in every panel. Panel D excludes the United States and the complete histories of euro adopters, then removes active Fed-line observations for the remaining countries.

C Procyclical Gross Flows with Interest Rates

Table C1 Capital Flows, Reserve Flows, and Borrowing Costs

	Private Flows (% GDP)		Private Flows (% GDP)		Reserve Flows (% GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Reserve Flows (% GDP)	0.25***	0.28***				
	(0.07)	(0.06)				
EMBI Spread (in log)			-0.32***	-0.12*	-0.22***	-0.26***
			(0.08)	(0.07)	(0.06)	(0.08)
GDP per capita		0.29**		0.39		-0.28
		(0.14)		(0.27)		(0.17)
Trade (% GDP)		0.02		0.04		0.14
		(0.07)		(0.10)		(0.17)
Institutional Quality		0.13		0.35		0.14
		(0.12)		(0.22)		(0.18)
Peg		-0.12		-0.10		-0.05
		(0.09)		(0.11)		(0.19)
CA (% GDP)		-0.37***		-0.28***		0.34***
		(0.06)		(0.08)		(0.09)
Constant	-0.00***	0.04	0.05***	-0.04	0.02***	0.10
	(0.00)	(0.04)	(0.00)	(0.09)	(0.00)	(0.09)
Year FE	N	Y	N	Y	N	Y
Country FE	Y	Y	Y	Y	Y	Y
Observations	1269	871	663	627	664	628
Adjusted R^2	0.101	0.407	0.159	0.354	0.077	0.234

Note. The dependent variable is private flows (% GDP) in columns (1)-(4) and reserve flows (% GDP) in columns (5) and (6), respectively. The independent variables include reserve flows (% GDP), EMBI spread (in log), GDP per capita (log), trade-to-GDP ratio, the institutional quality measure, the exchange rate regime constructed by Shambaugh (2004), and the current account-to-GDP ratio. We standardize all variables (except for the exchange rate regime variable) for ease of comparison. Standard errors are clustered by country and reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. All variable constructions are reported in Appendix A.

C.1 Empirical Study

In this section, we document the business cycle features of private capital flows and reserve flows. We use data from 47 emerging economies from 1987 to 2019. Data on private and reserve flows are taken from the updated dataset in Alfaro et al. (2014). The

Emerging Markets Bond Index (EMBI) spreads measure external borrowing costs and are taken from the World Bank's Global Economic Monitor. Other independent variables are the same as those in Table 1.

Two patterns emerge in private flows, reserve flows, and borrowing costs. First, private capital flows are positively correlated with reserve accumulation, consistent with Jeanne and Sandri (2023). Columns (1) and (2) in Table C1 show that the relationship is robust to country-fixed effects, year-fixed effects, and important country-level characteristics. Economically, a one-standard-deviation increase in reserve flows is associated with a 0.25-standard-deviation increase in private flows as in column (1). Moreover, the point estimate barely changes with additional controls in column (2).

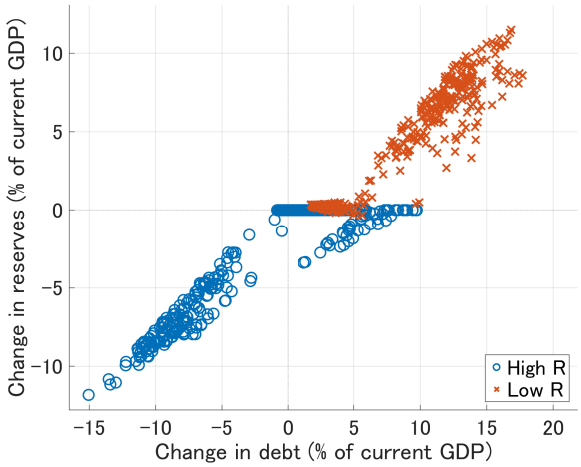
Second, both private capital and reserve flows are negatively correlated with the borrowing cost measure, as shown in columns (3)-(6) in Table C1. The EMBI spreads capture the *cost* difference for a country between external borrowing and holding reserves. When the spread is high, both the capital flows and the reserve flows decline. Again, the relationship holds even after controlling for other important country-level characteristics. Economically, a one-standard-deviation increase in EMBI spread is associated with a 0.12-standard-deviation decline in private flows as in column (4), and a 0.26-standard-deviation decline in reserve flows in column (6). This suggests that the interest rate shock is important in driving both private and reserve flows.

C.2 Model Analysis

Our benchmark model can replicate the observed correlation across debt, reserves, and borrowing costs. Figure C1 plots changes in debt and reserves as a ratio to GDP, $(-b_t + b_{t-1})/a_t$ and $(s_t - s_{t-1})/a_t$, at each period in a stochastic simulation of the decentralized economy. Dots in the top-right (bottom-left) quadrant indicate periods when both debt and reserves increase (decrease). There are two observations, both of which are consistent with procyclical gross capital flows documented in Section C.1 and also in Broner et al. (2013a) and Avdjiev et al. (2022). First, changes in debt and reserves are positively correlated, indicated by the fact that most dots are located in the top-right and bottom-left quadrants. Second, both debt and reserves increase when the interest rate is low, indicated by solid diamonds, and both decrease when the interest rate is high, indicated by circles and asterisks.

The reason for the negative correlation between the interest rate and changes in debt is straightforward, as households borrow more when the interest rate is low. There are two reasons why reserves increase when the interest rate is low. First, as households borrow more, the liquidity risk $-\theta b_t - s_t$ increases, raising the value of liquidity and inducing households to hold more reserves. Second, a low interest rate on debt reduces the opportunity cost of holding reserves and increases the liquidity advantage of reserves, further encouraging reserve accumulation. The same logic applies in the opposite direction when the interest rate is high. The dynamics under SP show the same patterns.

Figure C1 Changes in debt and reserves in model simulation



Note. This figure plots changes in debt and reserves at each period in a stochastic simulation of the decentralized economy. The horizontal axis shows changes in debt $(-b_t + b_{t-1})/a_t$, and the vertical axis shows changes in reserves $(s_t - s_{t-1})/a_t$.

D Model Appendix

D.1 Decentralized Equilibrium

In the derivations below, R_t denotes $R(b_t/a_t, \varepsilon_t^R)$ defined in (4); households internalize its dependence on b_t and a_t. The Lagrangian for the household's maximization problem is set up as follows:

$$\begin{aligned}\mathcal{L}_t = & \, u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a_t^*) \\ & - \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a_t^\ell \right] \\ & - \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^\gamma (a_{t-1} + \kappa a_{t-1}^*)^{1-\gamma} + a_t^\ell \right] \\ & + \psi_t \left[q_t a_t^\ell + \theta_t b_{t-1} + s_{t-1} \right] \\ & + \varphi_t q_t a_t^\ell \\ & + \nu_t \frac{s_t}{R^s}\end{aligned}$$

The first-order conditions are as follows:

$$c_t : \lambda_t = u'(c_t) \tag{A1}$$

$$b_t : \lambda_t = \beta \frac{R_t}{1 + \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right) \frac{b_t/a_t}{R_t}} \mathbb{E}_t V_b(t+1) \tag{A2}$$

$$s_t : \lambda_t - \nu_t = \beta R^s \mathbb{E}_t V_s(t+1) \tag{A3}$$

$$z_t : \lambda_t = \beta \mathbb{E}_t V_z(t+1) \tag{A4}$$

$$a_t^\ell : \psi_t q_t + \varphi_t q_t = \xi_t - q_t \lambda_t \tag{A5}$$

$$a_t : \xi_t = \lambda_t \left[1 + \left(\frac{b_t/a_t}{R_t} \right)^2 \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b} \right) \right] + \beta \mathbb{E}_t V_a(t+1) \tag{A6}$$

$$\psi_t \left[q_t a_t^\ell + \theta_t b_{t-1} + s_{t-1} \right] = 0, \; \psi_t \geq 0 \tag{A7}$$

$$\varphi_t q_t a_t^\ell = 0, \; \varphi_t \geq 0 \tag{A8}$$

$$\nu_t \frac{s_t}{R^s} = 0, \; \nu_t \geq 0 \tag{A9}$$

The envelope conditions are given as follows:

$$V_b(t) = \lambda_t + \psi_t \theta_t \tag{A10}$$

$$V_s(t) = \lambda_t + \psi_t \tag{A11}$$

$$V_z(t) = \xi_t \eta \gamma \left(\frac{z_{t-1}}{a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma - 1} \tag{A12}$$

$$V_a(t) = \xi_t \left[1 + \eta(1-\gamma) \left(\frac{z_{t-1}}{a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma} \right] \tag{A13}$$

Evaluating the envelope conditions one period ahead and substituting them into the first-order conditions, we obtain the equilibrium conditions in the text, (7), (8), (9), (10), and (11). Foreign productive assets a^*_t follow the exogenous law of motion $a^*_t = (1+\bar{g})a^*_{t-1}$.

The decentralized equilibrium is defined by allocations $\{c_t,\ b_t,\ s_t,\ z_t,\ a^\ell_t,\ a_t,\ a^*_t\}_{t=0}^{\infty}$, the Lagrange multipliers $\{\psi_t,\ \varphi_t,\ \xi_t,\ \nu_t\}_{t=0}^{\infty}$, and the liquidation price $\{q_t\}_{t=0}^{\infty}$ that satisfy (2), (3), (15), (7), (8), (9), (10), (11), (A7), (A8), (A9), and the law of motion for a^*_t, given the initial conditions and sequences of exogenous shocks $\{\varepsilon^R_t, \theta_t\}_{t=0}^{\infty}$.

D.2 Social Planner's Problem

The only difference from the decentralized equilibrium is that the planner internalizes that the liquidation price q_t is a function of liquidation a^ℓ_t as in (15). Accordingly, the setup of the planner's problem is identical to the maximization problem in the decentralized economy, except that q_t is a function of a^ℓ_t. Formally, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L}^{SP}_t = {} & u(c_t) + \beta \mathbb{E}_t V^{SP}(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\
& - \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t(a^\ell_t; a^*_{t-1}) a^\ell_t \right] \\
& - \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^\gamma (a_{t-1} + \kappa a^*_{t-1})^{1-\gamma} + a^\ell_t \right] \\
& + \psi^{SP}_t \left[q_t(a^\ell_t; a^*_{t-1}) a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\
& + \varphi^{SP}_t q_t(a^\ell_t; a^*_{t-1}) a^\ell_t \\
& + \nu_t \frac{s_t}{R^s}
\end{aligned}$$

The first-order condition for liquidation a^ℓ_t is:

$$\psi^{SP}_t + \varphi^{SP}_t = \frac{\xi_t}{q_t + (\partial q_t / \partial a^\ell_t) a^\ell_t} - u'(c_t) \tag{A14}$$

$$= \frac{\xi_t}{(1-\zeta) q_t} - u'(c_t) \tag{A15}$$

This is (17) in the main text. As explained in the main text, the social value of liquidity ψ_t^{SP} is higher than the private value of liquidity ψ_t because the planner internalizes the effect of liquidation a_t^ℓ on the liquidation price q_t, which is captured by $\partial q_t / \partial a_t^\ell < 0$.

D.3 Public Reserve Holdings

In Section 3.3, we show that the planner's allocation can be decentralized by a tax on debt and official reserve holdings. In this subsection, we show how these policy tools achieve the planner's constrained efficient allocation. A similar argument appears in Lutz and Zessner-Spitzenberg (2023).

Consider a Ramsey planner's problem whose policy instruments are a tax on debt and direct reserve holdings. In particular, this planner can collect a tax from households in a lump sum, buy foreign reserves at the interest rate R^s, and rebate the proceeds to households in a lump sum. The planner can also provide reserves to households when a liquidity shock hits the economy. Let $\hat{s}_t$ denote the planner's reserve holdings. Given this setup, the households' budget constraint can be written as follows:

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + \frac{\hat{s}_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + \hat{s}_{t-1} + q_t a_t^\ell + T_t \tag{A16}$$

where T_t is a lump-sum transfer by the planner to finance the tax on debt. Liquidation is subject to the liquidity constraint:

$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1} - \hat{s}_{t-1} \tag{A17}$$

Private and public reserves are subject to nonnegativity constraints:

$$s_t \geq 0 \tag{A18}$$

$$\hat{s}_t \geq 0 \tag{A19}$$

Given these policies, the first-order conditions in the decentralized equilibrium remain the same as those in the main text (except the Euler equation regarding debt due to the tax), because public reserve holdings do not distort the households' decisions. The Ramsey planner chooses all the endogenous variables including public reserve holdings $\hat{s}_t$ to maximize the households' expected utility, given the households' first-order conditions as the implementability constraints, and internalizing the effect of liquidation a_t^ℓ on the liquidation price q_t through (15).

In the budget constraint (A16) and the liquidation constraint (A17), private and public reserve holdings appear only in the form of $s_t + \hat{s}_t$. This means that s_t and $\hat{s}_t$ are perfect substitutes for households, and they only care about the sum of private and public reserves in their decisions. Let s_t^0 denote reserve holdings chosen by households when $\hat{s}_t = 0 \ \forall t$, i.e. no public reserves. As the planner increases public reserves $\hat{s}_t$ from zero, households reduce private reserves to achieve $s_t + \hat{s}_t = s_t^0$, because s_t^0 is the individually optimal amount of total

reserves. But this is possible only if $s_t \geq 0$ and equivalently $\hat{s}_t \leq s_t^0$. Once public reserves $\hat{s}_t$ exceed the individually optimal reserves s_t^0, households choose $s_t = 0$, and the planner can choose the total amount of reserves, all of which are public.

Now, suppose the planner introduces the optimal tax on debt τ_t^b given by (21). Given this policy, private reserves chosen by households are smaller than the planner's optimal amount of reserves because of the gap between ψ_{t+1} and ψ_{t+1}^{SP} discussed in Section 3.3. This means that the planner can choose the optimal amount of reserves, and private reserves are zero. These policies achieve the socially optimal allocation discussed in Section 3.3.

D.4 Subsidy on Private Reserve Holdings

Alternatively, the same optimal allocation can be achieved by a debt tax and a subsidy on private reserve holdings. Although official reserve holdings are empirically more relevant, studying the optimal reserve subsidy is useful because it indicates the size of a wedge that the externality creates in the reserve Euler equation. In this policy scheme, any fiscal surplus or deficit resulting from the policies is balanced through lump-sum transfers to or from households. Let τ_t^s denote a subsidy on reserve holdings. With the reserve subsidy, the reserve Euler equation for decentralized households becomes

$$u'(c_t) = \beta(1+\tau_t^s)R^s\mathbb{E}_t\left[u'(c_{t+1}) + \psi_{t+1}\right] + \nu_t \tag{A20}$$

with the optimal subsidy rate given by

$$1 + \tau_t^s = \frac{\mathbb{E}_t\left[u'(c_{t+1}) + \psi_{t+1}^{SP}\right]}{\mathbb{E}_t\left[u'(c_{t+1}) + \psi_{t+1}\right]} \tag{A21}$$

Following the same proof in Proposition 5, we can prove that $\tau_t^s > 0$ whenever $\theta > 0$ and there is a positive probability that asset liquidation occurs in the next period. However, $\tau_t^s > 0$ does not necessarily imply that the planner holds a positive amount of reserves. Proposition 2 holds for the planner's allocation, and optimal reserve holdings are zero when $\theta \geq R^s/\tilde{R}_t$. This can be understood from the key equation (16) with ψ_{t+1} replaced with ψ_{t+1}^{SP}. Because ψ_{t+1}^{SP} affects both Euler equations for debt and reserves, higher ψ_{t+1}^{SP} does not change the sign of the liquidity advantage of reserves over debt, and the sign is still determined by $R^s - \theta\tilde{R}_t$. When this is zero or negative, the planner holds no reserves even if $\tau_t^s > 0$.

There is an interesting observation on the reserve subsidy. We show that the reserve subsidy rate needs to be substantially higher than the debt tax rate. Combining the two Euler equations with a tax and a subsidy (20) and (A20), we obtain the following equation, which is a variant of the key equation (16):

$$\beta((1+\tau_t^b)\tilde{R}_t - (1+\tau_t^s)R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[((1+\tau_t^s)R^s - \theta_{t+1}(1+\tau_t^b)\tilde{R}_t)\psi_{t+1}] + \nu_t \tag{A22}$$

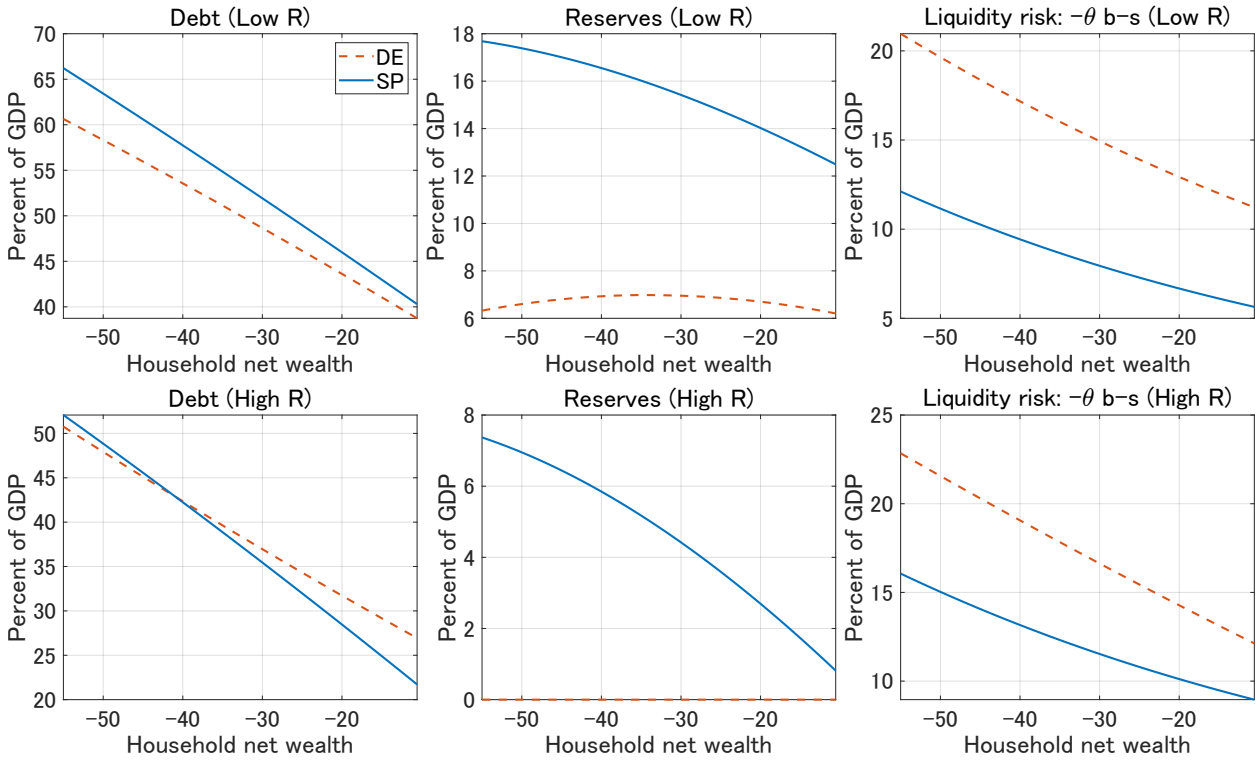
As we discussed in Section 3.2, the left-hand side is the opportunity cost of holding reserves,

and the right-hand side is the liquidity advantage of reserve holdings. When $\tau_t^s > \tau_t^b$, the opportunity cost becomes lower and the liquidity advantage becomes larger. Both effects induce households to hold a larger amount of reserves and take a safer position. In contrast, setting a debt tax rate higher than a reserve subsidy rate would discourage private reserve holdings by increasing the opportunity cost and reducing the liquidity advantage.

E Decision Rules

Figure E1 plots the decision rules for debt b_t, reserves s_t, and associated liquidity risk $-\theta b_t - s_t$ in the decentralized economy (DE) and the social planner's allocation (SP) in the baseline economy with $\theta = 0.45$. The horizontal axis shows households' net wealth, which is one of the state variables in the model. The top three panels are the decision rules when the borrowing interest rate R_t is low, and the bottom three correspond to those when the interest rate is high. The left two panels show that DE borrows more than SP in most states. Although this ordering reverses when the interest rate is high and net wealth is substantially negative, this does not mean that capital controls are not important, as decentralized households would borrow substantially more if the planner accumulates reserves but does not impose capital controls. The case of one policy instrument is examined in Appendix Section F. The middle two panels show that the planner accumulates substantially more reserves than DE. In particular, the bottom panel shows that decentralized households would hold zero reserves when the interest rate is high. A high interest rate implies a high opportunity cost of holding reserves, as indicated by the key equation (16). The right two panels present the size of liquidity risk, measured by $-\theta b_t - s_t$, as a result of debt and reserve choices. The liquidity risk is always lower under SP than under DE due to larger reserve holdings. As discussed in the main text, the planner internalizes the fire-sale externality, thereby choosing lower liquidity risk by holding more reserves than decentralized households.

Figure E1 Decision Rules



Note. These panels plot the decision rules for debt, reserves, and associated liquidity risk $-\theta b - s$ in the decentralized equilibrium and the social planner's solution. The top three panels are when the interest rate is low and the bottom three correspond to when it is high.

F Case of One Policy Instrument

We show in the main text that the planner needs to use two policy instruments to achieve the socially optimal allocation, one to correct over-borrowing and the other to correct insufficient reserve holdings. In this section, we study the case where only one policy instrument is available. We consider three policy instruments separately: a tax on foreign debt, a subsidy on private reserve holdings, and official reserve accumulation. In all cases, we set $\theta = 0.45$ and compare simulation moments and welfare impacts with those under the decentralized equilibrium and the planner's allocation in the main text.

The first policy is a tax on foreign debt. In this case, the planner cannot directly affect households' decisions on reserve holdings. The optimal tax design thus needs to internalize its impact on households' decisions on reserves. This introduces time inconsistency in the policy design because expectations of future policies affect households' decisions today. We could consider a time-consistent policy in a Markov perfect equilibrium as in Bianchi and Mendoza (2018), but instead, we introduce the same tax policy (21) as in the main text. While this is not the optimal tax policy in the case of one policy instrument, this exercise helps to underscore the importance of using two policy instruments jointly by simply dropping one of them. Table F1 presents the simulation means of debt, reserves, liquidity risk $-\theta b - s$, and the tax rate under this policy regime along with those under the decentralized equilibrium and the planner's allocation in the main text. When the debt tax is introduced, debt is substantially smaller than debt in DE and SP, and reserves become zero. As we discussed in Section D.4, a debt tax without a reserve policy increases the opportunity cost of holding reserves and reduces the liquidity advantage of reserves, both of which discourage private reserve holdings. Households partially offset the stabilization effect of a debt tax by reducing reserves, but reserve holdings cannot be negative. Thus, the resulting liquidity risk is lower than in DE but higher than in SP. The welfare impact of this policy is a 0.2% gain in permanent consumption.

The second policy is a reserve subsidy. We again consider the same subsidy policy (A21) as in Appendix D.4. Table F1 shows that this policy induces much larger debt and reserves. A reserve subsidy without a debt tax lowers the opportunity cost of reserves and increases the liquidity advantage of reserves, thereby inducing large reserve holdings. At the same time, high reserves lower liquidity risk and the private value of liquidity ψ_{t+1}, inducing large foreign debt. Although this policy induces households to take a safer position with large reserves and low liquidity risk, its welfare impact is only a 0.1% gain in permanent consumption. There are two main reasons for this result. First, large debt increases interest payments to foreign lenders because the interest rate is elastic to the amount of debt, and this is a large loss in domestic resources. Second, 25% of GDP is invested in low-yielding reserves ($R^s = 1$ in our model), which is an inefficient investment and causes a large opportunity cost of reserves. These costs push down welfare gains resulting from this policy.

The third policy is official reserve accumulation. As in the main text, we assume that the planner accumulates reserves through a lump-sum tax on households and makes them available through a lump-sum transfer when there is a liquidity shortage. We assume the

Table F1 The Case of One Policy Instrument

	DE	SP	Debt tax	Reserve subsidy	Official reserves
Debt	−0.49	−0.44	−0.29	−0.68	−0.57
Reserve	0.03	0.12	0	0.25	0.14
Liquidity risk	0.19	0.08	0.13	0.06	0.12
Mean τ_t^b	...	3.1%	3.7%	...	...
Mean τ_t^s	...	6.7%	...	6.7%	...
Welfare gain	...	0.5%	0.2%	0.1%	0.3%

Note. Debt and reserves are measured in terms of the ratios to GDP. The numbers in this table are simulation means under each policy regime. Welfare gain is a permanent consumption gain/loss compared to the expected utility under the decentralized equilibrium.

same decision rules on reserves as the planner in the main text. Namely, given the state of the economy, the planner here chooses the same amount of reserves as the planner in the main text does. The only difference from the planner in the main text is that the planner here does not impose a tax on debt. Table F1 shows that households under this policy borrow more than in DE and SP, and the planner holds reserves corresponding to 14% of GDP. The resulting liquidity risk is 0.12, which is lower than in DE but higher than in SP. Again, households partially offset the planner's reserve policy by taking larger foreign debt. The welfare impact of this policy is a 0.3% gain in permanent consumption. As with the reserve subsidy, large debt increases interest payments, while large reserves imply a large opportunity cost. But these costs are lower than those under reserve subsidy, resulting in relatively large welfare gains.

G Two-Period Model

In this section, we introduce a two-period model and show analytical results to facilitate understanding of the full model in the main text. In particular, we prove that the optimal tax rate on debt τ_b is increasing in θ in this two-period model, and explain the intuition.

G.1 Model Setup

We consider a two-period model with $t = 0$ and 1. At $t = 0$, households choose consumption c_0, foreign bonds b_0 (negative b_0 is borrowing), and reserves s_0. Households own a_0 units of productive assets, with a_0 exogenously given and used for production at $t = 1$. There is no production or endowment at $t = 0$, and households need to borrow to consume and buy reserves, implying $b_0 < 0$.

At $t = 1$, households produce goods using productive assets. However, at the beginning of period 1 before production, a liquidity shock may hit the economy with probability π. A liquidity shock forces households to repay a θ fraction of the period-0 debt b_0. Households make this repayment using reserves s_0 and liquidating some of their assets if reserves are not enough for the repayment. Liquidation of assets is costly because liquidated assets can be sold only at a fire-sale price. We assume that the liquidation price q_1 decreases in the amount of liquidated assets a_1^ℓ, satisfying $q_1'(a_1^\ell) < 0$ for any a_1^ℓ. We also assume that liquidation proceeds $q_1(a_1^\ell)a_1^\ell$ are increasing in a_1^ℓ. This assumption implies that the elasticity of q_1 is not too high, so that households can make the repayment by liquidation when reserves are not enough. These two assumptions are satisfied by the functional form assumed in the main text. Individual households take q_1 as given when they make decisions.

Later at $t = 1$, households produce goods and consume. Production is linear in the amount of assets and given by $y = A(a_0 - a_1^\ell)$ with productivity $A > 0$.

The period-by-period budget constraints are given as follows:

$$t = 0 : c_0 + \frac{b_0}{R^b} + \frac{s_0}{R^s} = 0 \tag{A23}$$

$$t = 1 : c_1 = A(a_0 - a_1^\ell) + b_0 + s_0 + q_1 a_1^\ell \tag{A24}$$

We assume $R^b > R^s$. Liquidation a_1^ℓ needs to satisfy:

$$q_1 a_1^\ell \geq -\theta_1 b_0 - s_0 \tag{A25}$$

$$q_1 a_1^\ell \geq 0 \tag{A26}$$

where θ_1 is stochastic and takes θ with probability π and 0 with probability $1 - \pi$.

We assume the households' utility function as follows:

$$U(c_0, c_1^N, c_1^L) = \log(c_0) + \beta \left[(1 - \pi)c_1^N + \pi c_1^L\right]$$

β is a discount factor, c_1^N is consumption when there is no liquidity shock at $t = 1$, and c_1^L is

consumption when a liquidity shock hits at $t = 1$. We assume linear utility in period 1 for analytical tractability.

G.2 Period 1 with No Liquidity Shock ($\theta_1 = 0$)

We solve the decentralized equilibrium of the model backward from period 1. The state variables at period 1 are debt b_0, reserves s_0, and a realization of a liquidity shock θ_1. We first consider the case of no liquidity shock, $\theta_1 = 0$. The maximization problem is given by

$$V^N(b_0, s_0) = \max_{c_1^N, a_1^\ell} c_1^N$$

subject to the constraints. The Lagrangian is given by

$$\begin{aligned}\mathcal{L}^N = & c_1^N \\ & - \lambda_1^N \left[c_1^N - A(a_0 - a_1^\ell) - b_0 - s_0 - q_1 a_1^\ell\right] \\ & + \psi_1^N \left[q_1 a_1^\ell + s_0\right] \\ & + \varphi_1^N \left[q_1 a_1^\ell\right]\end{aligned}$$

Combining the first-order conditions regarding c_1^N and a_1^ℓ leads to the following equation:

$$\psi_1^N + \varphi_1^N = \frac{A}{q_1} - 1$$

Note that the liquidity constraint (A25) never binds because $\theta_1 = 0$ and $s_0 \geq 0$, implying $\psi_1^N = 0$. We assume $A > q_1(a_1^\ell)$ for any a_1^ℓ, implying $\varphi_1^N > 0$ and $a_1^\ell = 0$. This implies that households do not liquidate unless it is necessary.

Envelope conditions are given by

$$V_b^N(b_0, s_0) = 1 \tag{A27}$$
$$V_s^N(b_0, s_0) = 1 \tag{A28}$$

where the subscripts b and s indicate the variable with which a partial derivative is taken. The second condition utilizes $\psi_1^N = 0$.

G.3 Period 1 with Liquidity Shock ($\theta_1 = \theta$)

Next, we consider the case of a liquidity shock hitting the economy, $\theta_1 = \theta$. The value function is given by

$$V^L(b_0, s_0) = \max_{c_1^L, a_1^\ell} c_1^L$$

subject to the constraints. The Lagrangian is

$$\begin{aligned}\mathcal{L}^L = &\max_{c_1^L, a_1^\ell} c_1^L \\ &- \lambda_1^L \left[c_1^L - A(a_0 - a_1^\ell) - b_0 - s_0 - q_1 a_1^\ell\right] \\ &+ \psi_1^L \left[q_1 a_1^\ell + \theta b_0 + s_0\right] \\ &+ \varphi_1^L \left[q_1 a_1^\ell\right]\end{aligned}$$

Again, combining the first-order conditions leads to the same equation:

$$\psi_1^L + \varphi_1^L = \frac{A}{q_1} - 1 \tag{A29}$$

If $-\theta b_0 > s_0$ and reserves are not enough to cover the early repayment (which is always the case as shown below), there is positive liquidation $a_1^\ell > 0$ and $\varphi_1^L = 0$. Assuming $A > q_1$, ψ_1^L is given by (A29), and households liquidate just enough assets to cover the liquidity shortage, implying $q_1 a_1^\ell = -\theta b_0 - s_0$.

If $-\theta b_0 \leq s_0$ and reserves are enough to cover the early repayment (which never happens in the equilibrium as shown below), then $a_1^\ell = 0$ and $\psi_1^L = 0$.

Envelope conditions are given as follows:

$$V_b^L(b_0, s_0) = 1 + \psi_1^L \theta \tag{A30}$$

$$V_s^L(b_0, s_0) = 1 + \psi_1^L \tag{A31}$$

G.4 Period 0

We go back to period $t = 0$. The value function is given by

$$V_0 = \max_{c_0, b_0, s_0} \log(c_0) + \beta \left[(1-\pi) V^N(b_0, s_0) + \pi V^L(b_0, s_0)\right]$$

subject to the constraints. The Lagrangian is

$$\begin{aligned}\mathcal{L}_0 = &\log(c_0) + \beta \left[(1-\pi) V^N(b_0, s_0) + \pi V^L(b_0, s_0)\right] \\ &- \lambda_0 \left[c_0 + \frac{b_0}{R^b} + \frac{s_0}{R^s}\right] \\ &+ \nu_0 \frac{s_0}{R^s}\end{aligned}$$

The first-order conditions are given as follows:

$$c_0 : \frac{1}{c_0} = \lambda_0 \tag{A32}$$

$$b_0 : \lambda_0 = \beta R^b \left[(1-\pi)V_b^N(b_0, s_0) + \pi V_b^L(b_0, s_0)\right] \tag{A33}$$

$$s_0 : \lambda_0 - \nu_0 = \beta R^s \left[(1-\pi)V_s^N(b_0, s_0) + \pi V_s^L(b_0, s_0)\right] \tag{A34}$$

G.5 Decentralized Equilibrium

Plugging the envelope conditions (A27), (A28), (A30), and (A31) into the Euler equations (A33) and (A34), we obtain the explicit expressions for the Euler equations regarding debt and reserves:

$$\frac{1}{c_0} = \beta R^b \left[(1-\pi) + \pi \left\{1 + \psi_1^L \theta\right\}\right] \tag{A35}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s \left[(1-\pi) + \pi \left\{1 + \psi_1^L\right\}\right] \tag{A36}$$

and ψ_1^L is given by

$$\psi_1^L = \begin{cases} \frac{A}{q_1} - 1 & \text{if } -\theta b_0 - s_0 > 0 \text{ and } a_1^\ell > 0. \\ 0 & \text{if } -\theta b_0 - s_0 \leq 0 \text{ and } a_1^\ell = 0. \end{cases} \tag{A37}$$

If $a_1^\ell > 0$, it is implicitly given by

$$q_1(a_1^\ell) a_1^\ell = -\theta b_0 - s_0 \tag{A38}$$

which is the binding liquidity constraint. Combining the two Euler equations (A35) and (A36),

$$\beta(R^b - R^s) = \pi\beta\psi_1^L(R^s - \theta R^b) + \nu_0 \tag{A39}$$

which is a simplified version of the key equation (16) in the main text. It is straightforward to prove the three propositions in Section 3.2. In particular, Proposition 3 says that households never hold enough reserves to cover the entire early repayment, given the opportunity cost of holding reserves $R^b > R^s$. This means that ψ_1^L takes a positive value in (A37). As explained in the main text, ψ_1^L is the *private* value of one unit of liquidity when there is a liquidity shortage. Henceforth, we denote ψ_1^L as ψ_1^{DE} to distinguish it from the *social* value of liquidity ψ_1^{SP}. Namely,

$$\psi_1^{DE} = \frac{A}{q_1} - 1 \tag{A40}$$

The decentralized equilibrium of this model is the six variables $\{c_0, b_0, s_0, a_1^\ell, \psi_1^{DE}, \nu_0\}$

that satisfy the six equations (A23), (A35), (A36), (A38), (A40), and $\nu_0 s_0/R^s = 0$ with $\nu_0 \geq 0$. c_1^N and c_1^L are given by the resource constraint (A24).

G.6 Relation between a_1^ℓ and θ

We now show that the equilibrium size of liquidation a_1^ℓ increases as θ becomes higher. Plugging ψ_1^{DE} in (A40) into the key equation (A39) leads to the following equation:

$$\beta(R^b - R^s) = \pi\beta\left(\frac{A}{q_1} - 1\right)(R^s - \theta R^b) + \nu_0 \tag{A41}$$

If the parameter values are such that $s_0 > 0$ and $\nu_0 = 0$ in the equilibrium, (A41) with $\nu_0 = 0$ alone pins down a_1^ℓ. In this case, we can compute how a_1^ℓ changes as θ increases by the implicit function theorem. Formally,

$$\frac{\partial a_1^\ell}{\partial \theta} = -\frac{\pi\beta R^b\left(\frac{A}{q_1} - 1\right)}{-\pi\beta(R^s - \theta R^b)\frac{-Aq_1'}{q_1^2}} > 0$$

It shows that when $s_0 > 0$, the size of liquidation a_1^ℓ increases as θ becomes higher.

If, instead, the parameter values are such that $s_0 = 0$ and $\nu_0 > 0$ in the equilibrium, the liquidation equation (A38) reduces to

$$q_1(a_1^\ell)a_1^\ell = -\theta b_0 \tag{A42}$$

Because of the assumption that the liquidation proceeds $q_1 a_1^\ell$ increase in a_1^ℓ, a_1^ℓ and b_0 have a one-to-one relation. Plugging this equation into the Euler equation regarding debt (A35) with $c_0 = -b_0/R^b$, we obtain

$$\frac{\theta R^b}{q_1(a_1^\ell)a_1^\ell} = \beta R^b\left[(1-\pi) + \pi\left\{1 + \theta\left(\frac{A}{q_1} - 1\right)\right\}\right] \tag{A43}$$

This equation pins down a_1^ℓ. Applying the implicit function theorem,

$$\frac{\partial a_1^\ell}{\partial \theta} = -\frac{\frac{R^b}{q_1 a_1^\ell} - \beta R^b \pi\left(\frac{A}{q_1} - 1\right)}{-\theta R^b \frac{1}{(q_1 a_1^\ell)^2}\frac{\partial(q_1 a_1^\ell)}{\partial a_1^\ell} - \beta R^b \pi\theta\left(-\frac{Aq_1'}{q_1^2}\right)} > 0$$

The denominator is negative given $\partial(q_1 a_1^\ell)/\partial a_1^\ell > 0$ and $q_1' < 0$. The numerator is positive because multiplying θ gives $\theta R^b/q_1 a_1^\ell - \beta R^b \pi\theta(A/q_1 - 1)$ and this is equal to $\beta R^b > 0$ from (A43). Therefore, we have $\partial a_1^\ell/\partial\theta > 0$ when $s_0 = 0$. It follows that whether $s_0 > 0$ or $s_0 = 0$ in the equilibrium, the size of liquidation a_1^ℓ increases as θ becomes higher. This relation will be used later when we analyze the relation between the optimal tax on debt τ_b and θ.

G.7 Social Planner's Allocation

The only difference between the decentralized equilibrium and the social planner's allocation is that the planner internalizes that the liquidation price q_1 is decreasing in the amount of liquidation a_1^ℓ, i.e. $q_1' < 0$. This affects the problem of period 1 with a liquidity shock. The setup is the same as above, but the first-order condition with respect to a_1^ℓ leads to the following equation:

$$\psi_1^{SP} + \varphi_1^{SP} = \frac{A}{q_1 + q_1' a_1^\ell} - 1$$

Accordingly, the planner's Euler equations are given as follows:

$$\frac{1}{c_0} = \beta R^b \left[(1-\pi)1 + \pi\left\{1 + \psi_1^{SP}\theta\right\}\right] \tag{A44}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s \left[(1-\pi)1 + \pi\left\{1 + \psi_1^{SP}\right\}\right] \tag{A45}$$

and ψ_1^{SP} in these Euler equations is given by

$$\psi_1^{SP} = \frac{A}{q_1 + q_1' a_1^\ell} - 1 \tag{A46}$$

$q_1' < 0$ implies $\psi_1^{SP} > \psi_1^{DE}$ given the same a_1^ℓ.

Comparing the Euler equations under the decentralized equilibrium (A35) and (A36) and those under the planner's allocation (A44) and (A45), the optimal tax on debt and subsidy on reserves are characterized as follows:

$$\frac{1}{c_0} = \beta R^b (1+\tau_b)\left[1 + \pi\psi_1^{DE}\theta\right] \tag{A47}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s (1+\tau_s)\left[1 + \pi\psi_1^{DE}\right] \tag{A48}$$

with the tax and subsidy given by

$$1 + \tau_b = \frac{1 + \pi\psi_1^{SP}\theta}{1 + \pi\psi_1^{DE}\theta} \tag{A49}$$

$$1 + \tau_s = \frac{1 + \pi\psi_1^{SP}}{1 + \pi\psi_1^{DE}} \tag{A50}$$

and ψ_1^{DE} given by (A40) and ψ_1^{SP} given by (A46). Because $\psi_1^{SP} > \psi_1^{DE}$, both τ_b and τ_s are positive, implying a tax on debt and a subsidy on reserves.

G.8 Relation between τ_b and θ

We now show how τ_b changes as θ increases. Because τ_b in (A49) is a function of only a_1^ℓ given θ, we can directly take the derivative of τ_b with respect to θ, taking into account the positive effect of θ on a_1^ℓ shown above:

$$\frac{\partial \tau_b}{\partial \theta} = \frac{(\pi\psi_1^{SP} + \pi\theta\frac{\partial\psi_1^{SP}}{\partial\theta})(1+\pi\theta\psi_1^{DE}) - (\pi\psi_1^{DE} + \pi\theta\frac{\partial\psi_1^{DE}}{\partial\theta})(1+\pi\theta\psi_1^{SP})}{(1+\pi\psi_1^{DE}\theta)^2} \tag{A51}$$

The sign of this expression is given by the sign of the numerator. First, it is useful to examine the terms without partial derivatives, because it shows how θ affects τ_b given the size of liquidation a_1^ℓ fixed. Collecting the terms without derivatives,

$$\pi\psi_1^{SP}(1+\pi\theta\psi_1^{DE}) - \pi\psi_1^{DE}(1+\pi\theta\psi_1^{SP}) = \pi(\psi_1^{SP} - \psi_1^{DE}) > 0$$

This result is straightforward from (A49). With ψ_1^{SP} and ψ_1^{DE} being fixed and given $\psi_1^{SP} > \psi_1^{DE}$, an increase in θ increases the numerator proportionally more than the denominator, increasing the optimal tax rate τ_b.

Another effect of θ on τ_b works through its impacts on a_1^ℓ, which are captured by the partial derivatives. Collecting the terms with derivatives in the numerator of (A51),

$$\pi\theta\left[\frac{\partial\psi_1^{SP}}{\partial\theta}(1+\pi\theta\psi_1^{DE}) - \frac{\partial\psi_1^{DE}}{\partial\theta}(1+\pi\theta\psi_1^{SP})\right]$$

Using (A40) and (A46), we can rewrite the bracketed terms explicitly as follows:

$$\begin{aligned}&\left(-\frac{A}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}\frac{\partial a_1^\ell}{\partial\theta}\right)\left[1+\pi\theta\left\{\frac{A}{q_1}-1\right\}\right]\\ -&\left(-\frac{A}{(q_1)^2}q_1'\frac{\partial a_1^\ell}{\partial\theta}\right)\left[1+\pi\theta\left\{\frac{A}{q_1+q_1'a_1^\ell}-1\right\}\right]\end{aligned} \tag{A52}$$

To obtain further analytical results, we introduce an explicit functional form for the liquidation price $q_1(a_1^\ell)$. We assume essentially the same function as in the main text, $q_1(a_1^\ell) = C(a_1^\ell)^{-\zeta}$ with a constant $C > 0$ and $0 < \zeta < 1$. Then we obtain the following equations:

$$\begin{aligned}q_1' &= -\zeta C(a_1^\ell)^{-\zeta-1} < 0\\ \frac{\partial(q_1a_1^\ell)}{\partial a_1^\ell} &= q_1 + q_1'a_1^\ell = (1-\zeta)C(a_1^\ell)^{-\zeta} = (1-\zeta)q_1 > 0\\ \frac{\partial^2(q_1a_1^\ell)}{\partial(a_1^\ell)^2} &= \frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell} = -\zeta(1-\zeta)C(a_1^\ell)^{-\zeta-1} = (1-\zeta)q_1' < 0\end{aligned}$$

Using these expressions, we compute the gap between the two terms in parentheses in (A52),

which is actually $\partial\psi_1^{SP}/\partial\theta - \partial\psi_1^{DE}/\partial\theta$:

$$\begin{aligned} A\frac{\partial a_1^\ell}{\partial\theta}\left(-\frac{1}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}+\frac{1}{(q_1)^2}q_1'\right) &= A\frac{\partial a_1^\ell}{\partial\theta}\left(-\frac{q_1'}{(1-\zeta)(q_1)^2}+\frac{q_1'}{(q_1)^2}\right) \\ &= A\frac{\partial a_1^\ell}{\partial\theta}\frac{-\zeta q_1'}{(1-\zeta)(q_1)^2} > 0 \end{aligned}$$

The sign comes from $\partial a_1^\ell/\partial\theta > 0$ shown above and $q_1' < 0$. This result shows that both ψ_1^{SP} and ψ_1^{DE} increase as θ becomes higher, but ψ_1^{SP} increases more than ψ_1^{DE} does. Intuitively, as θ becomes higher, both the private and social values of liquidity increase, but the social value increases more as the planner internalizes that higher θ induces larger liquidation a_1^ℓ and decreases the liquidation price more. This is the second effect of θ on τ_b.

Lastly, we examine the remaining terms in (A52), which are the terms in parentheses times the fractions in the curly brackets. The sign of these terms is given by

$$\begin{aligned} &\left(-\frac{1}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}\right)\frac{A}{q_1} - \left(-\frac{1}{(q_1)^2}q_1'\right)\frac{A}{q_1+q_1'a_1^\ell} \\ =& -\frac{q_1'}{(1-\zeta)(q_1)^2}\frac{A}{q_1}+\frac{q_1'}{(q_1)^2}\frac{A}{(1-\zeta)q_1} = 0 \end{aligned}$$

These terms cancel each other out. Therefore, the expression in (A52) is positive and the entire term in (A51) is positive. This completes the proof that $\partial\tau_b/\partial\theta > 0$.

To summarize, an increase in θ leads to a higher tax rate τ_b through two channels. The first is the direct channel, which is that the right-hand side of the Euler equation increases more in the planner's allocation than in the decentralized equilibrium given $\psi_1^{SP} > \psi_1^{DE}$. The second channel is indirect through the effect on a_1^ℓ. As θ increases, the size of liquidation a_1^ℓ also increases, and both the private and social values of liquidity ψ_1^{DE} and ψ_1^{SP} increase. But ψ_1^{SP} increases more than ψ_1^{DE}, because the planner internalizes that larger liquidation causes a larger drop in the fire-sale price, and additional liquidity becomes more valuable.

H Micro-foundation for Financial Development

This section provides a micro-foundation for financial development θ in our baseline model, rationalizing why only a fraction of total external debt is subject to a liquidity shock.

Assume that domestic agents borrow from the rest of the world only through domestic banks. As in our baseline model, in each period t, they borrow $-b_t$ at the price $p_t \equiv \frac{1}{R_t}$. At the beginning of $t+1$, a fraction θ of $-b_t$ must be repaid early if a liquidity shock occurs. We need a framework where θ is determined by the domestic intermediation process.

Consider a continuum of domestic banks indexed by $i \in [0,1]$. Each bank i can obtain a foreign loan x_t^i at an exogenous price p_s and lend it to domestic agents at price p_t. However, there is an intermediation cost given by $\frac{\phi}{2}(x_t^i)^2$, where $\phi > 0$ is a parameter.

As assumed in our baseline model, a liquidity shock affects external borrowing. During the intermediation process, bank i earns an intermediation profit of $(p_s - p_t)x_t^i$ if no liquidity shock occurs. However, if a liquidity shock hits, the bank must repay the external debt x_t^i to foreign lenders, requiring domestic agents to pay early. This situation reduces expected profit to $\delta(p_s - p_t)x_t^i$ with $\delta \in (0,1)$, capturing the reputation cost for domestic banks.

We also assume that domestic banks can hedge against external liquidity shocks by purchasing derivatives at a cost of $\kappa i(p_s - p_t)x_t^i$, where κ reflects the degree of domestic financial market development in providing derivatives, and $i \in [0,1]$ represents the idiosyncratic cost for banks. As demonstrated below, the endogenous hedging decision influences the fraction of loans subject to liquidity shocks. Furthermore, domestic financial development κ affects both the hedging decision and the *effective* size of liquidity shocks throughout the economy.

In this setting, banks endogenously determine both the volume of loans to intermediate and their hedging decisions, taking p_t as given.

$$\pi_i^{\text{Hedging}} = (p_s - p_t)x_t^i - \frac{\phi}{2}(x_t^i)^2 - \kappa i(p_s - p_t)x_t^i \tag{A53}$$

$$\pi_i^{\text{No Hedging}} = \delta(p_s - p_t)x_t^i - \frac{\phi}{2}(x_t^i)^2 \tag{A54}$$

If banks hedge, the optimality condition and profit are:

$$x_t^{i,*,\text{Hedging}} = \frac{1 - \kappa i}{\phi}(p_s - p_t) \tag{A55}$$

$$\pi_t^{i,*,\text{Hedging}} = \frac{(p_s - p_t)^2(1 - \kappa i)^2}{2\phi} \tag{A56}$$

If banks do not hedge, the optimality condition and profit are:

$$x_t^{i,*,\text{Not Hedging}} = \frac{\delta}{\phi}(p_s - p_t) \tag{A57}$$

$$\pi_t^{i,*,\text{Not Hedging}} = \frac{(p_s - p_t)^2\delta^2}{2\phi} \tag{A58}$$

Banks choose to hedge if the profit with hedging exceeds that without hedging.

$$1-\kappa i>\delta, \quad \text{or} \quad i<\frac{1-\delta}{\kappa} \tag{A59}$$

In equilibrium, the price p_t clears the market:

$$-b_t=\int_0^{\frac{1-\delta}{\kappa}}\frac{1-\kappa i}{\phi}(p_s-p_t)di+\int_{\frac{1-\delta}{\kappa}}^1\frac{\delta}{\phi}(p_s-p_t)di \tag{A60}$$

This implies:

$$p_s-p_t=\frac{\phi}{\delta+\frac{(1-\delta)^2}{2\kappa}}(-b_t) \tag{A61}$$

The share of loans subject to liquidity shocks is:

$$\theta\equiv\frac{\int_{\frac{1-\delta}{\kappa}}^1\frac{\delta}{\phi}(p_s-p_t)di}{\int_0^{\frac{1-\delta}{\kappa}}\frac{1-\kappa i}{\phi}(p_s-p_t)di+\int_{\frac{1-\delta}{\kappa}}^1\frac{\delta}{\phi}(p_s-p_t)di}=\frac{\delta\left(1-\frac{1-\delta}{\kappa}\right)}{\delta+\frac{(1-\delta)^2}{2\kappa}}\in[0,1] \tag{A62}$$

It can be shown that $\frac{d\theta}{d\kappa}>0$.

As $p_t\equiv\frac{1}{R_t}$ is endogenously determined, we select p_s such that equation (A61) aligns with the interest rate dynamics described in equation (4) of our baseline model. In this framework, a higher level of financial development, represented by a lower κ, is associated with a lower θ, indicating a lower share of loans subject to liquidity shocks.

I Model of Endogenous Debt Maturity

This section extends the baseline model in the main text by endogenizing θ as a share of short-term debt. In this model, we assume that households can issue long-term bonds, b_t^L, and short-term bonds, b_t^S. Short-term bonds are subject to a liquidity shock, which requires all of b_t^S to be repaid at the beginning of period $t+1$ before new borrowing. Long-term bonds are not subject to this risk and are standard non-contingent bonds. The total amount of foreign debt is $b_t = b_t^L + b_t^S$, and we denote the share of short-term debt by θ_t:

$$\theta_t = \frac{b_t^S}{b_t^L + b_t^S} \tag{A63}$$

This implies that a θ_t fraction of total debt b_t is subject to a liquidity shock, as is the case in the baseline model. The interest rate on short-term debt is the same as R_t in the main text given by (4), which consists of a stochastic component and the debt-elastic component. The interest rate on long-term debt, R_t^L, is R_t plus the term premium, which is assumed to increase in the share of long-term debt $1-\theta_t$:

$$R_t^L = R_t + \Gamma(1-\theta_t)^2 \tag{A64}$$

where $\Gamma > 0$ governs the sensitivity of R_t^L to the share of long-term debt, $1-\theta_t$. This equation can be micro-founded using the model of financial intermediaries developed in Gabaix and Maggiori (2015). Broner et al. (2013b) show that the term premium for long-term debt is higher in emerging economies than in developed countries and argue that this is one reason why emerging economies rely more on short-term borrowing. Following their discussion, we interpret Γ as a measure of financial development, where high Γ corresponds to low financial development, making the term premium on long-term debt higher and more sensitive to maturity composition.

The households' budget constraint is now given by

$$c_t + \frac{b_t^S}{R_t} + \frac{b_t^L}{R_t^L} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^\ell \tag{A65}$$

There are three Euler equations, regarding b_t^S, b_t^L, and s_t respectively:

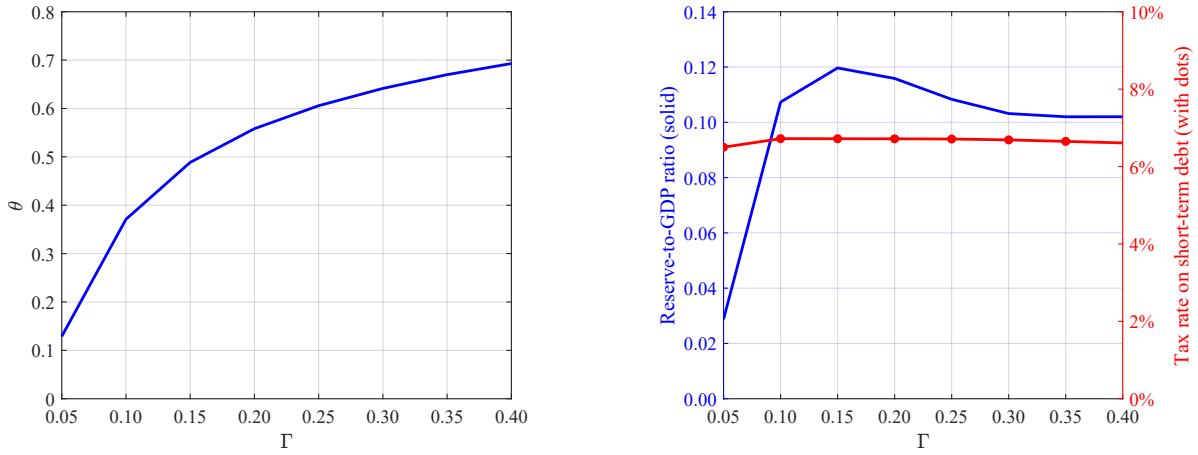
$$u'(c_t)\left[\frac{1}{R_t} - \frac{b_t^L}{(R_t^L)^2}\frac{\partial R_t^L}{\partial b_t^S} + \psi_b \exp\left(-b_t - \bar{b}\right)\frac{b_t^S}{R_t^2}\right] = \beta \mathbb{E}_t\left[u'(c_{t+1}) + \psi_{t+1}\right] \tag{A66}$$

$$u'(c_t)\left[\frac{1}{R_t^L} - \frac{b_t^L}{(R_t^L)^2}\frac{\partial R_t^L}{\partial b_t^L} + \psi_b \exp\left(-b_t - \bar{b}\right)\frac{b_t^S}{R_t^2}\right] = \beta \mathbb{E}_t\left[u'(c_{t+1})\right] \tag{A67}$$

$$u'(c_t) - \nu_t = \beta R^S \mathbb{E}_t\left[u'(c_{t+1}) + \psi_{t+1}\right] \tag{A68}$$

In the Euler equation for b_t^S, (A66), the second term in brackets is positive because $\partial R_t^L / \partial b_t^S < 0$. This term implies that increasing short-term debt will reduce the share of

Figure I1 Share of short-term debt, reserves, and capital control across Γ



Note. The horizontal axis is Γ in both panels. The left panel plots the stochastic simulation mean of the share of short-term debt θ_t across different Γ. The right panel plots the stochastic simulation mean of the reserve-to-GDP ratio (left axis) and the short-term debt tax rate (right axis).

long-term debt $1-\theta_t$, thereby lowering the long-term interest rate R_t^L. Because the right-hand side of (A66) and (A68) coincide, households borrow short term until the inverse of the bracketed term in (A66) becomes equal to R^S. If this term did not exist, then the inverse of the bracketed term would always be greater than R^S, and households would not hold reserves.

The source of inefficiency in the decentralized equilibrium is the same as the baseline model: the fire-sale externality captured by the gap between ψ_t and ψ_t^{SP}. Because ψ_{t+1} appears in (A66) and (A68), decentralized households over-borrow short term and accumulate an insufficient amount of reserves. However, long-term debt b_t^L is not distorted because it is not subject to liquidity shocks. Therefore, the planner uses capital controls on short-term debt and official reserves to achieve the socially optimal allocation.

We solve the planner's allocation across different Γ, and compute the simulation mean of the share of short-term debt θ_t, the tax rate on short-term debt τ_t^b, and the amount of official reserves. Figure I1 plots the results. The left panel shows that as the value of Γ increases from 0.05 to 0.40, the share of short-term debt θ_t increases monotonically from 0.1 to 0.7. This is because higher Γ makes the interest rate on long-term debt more sensitive to maturity composition, discouraging households from borrowing in the long term.

The right panel plots the simulation mean of the reserve-to-GDP ratio and the tax rate on short-term debt across different values of Γ. The reserve-to-GDP ratio is non-monotonic in Γ because it is non-monotonic in θ as shown in the main text. A clear contrast to the baseline model in the main text is that the tax rate is constant across Γ, which implies that it is also constant across θ. This difference comes from the right-hand side of the Euler equation for debt, (10) in the main text and (A66) here. In (10) in the baseline model, the value of liquidity ψ_{t+1} is multiplied by θ. Since the gap between ψ_{t+1} and ψ_{t+1}^{SP} is the source

of the externality, higher θ results in a larger gap and externality, requiring a higher tax rate to correct the externality. In contrast, ψ_{t+1} in (A66) is not multiplied by θ, leading to a stable tax rate across Γ.

However, this does not necessarily imply that the optimal degree of capital controls is constant across Γ. While the optimal tax rate is constant across Γ, the coverage of capital controls, namely, the share of short-term debt θ_t, increases as Γ becomes higher. Therefore, this model is still consistent with the cross-country pattern that countries with higher financial development use capital controls less actively.

J Numerical Solution

J.1 Stationarized Competitive Equilibrium

We first stationarize the equilibrium conditions. Below is the complete list of the competitive equilibrium conditions:

$$a_t^* = (1+\bar{g})a_{t-1}^* \tag{A69}$$

$$\psi_t \left[q_t a_t^\ell + \theta_t b_{t-1} + s_{t-1}\right] = 0,\ \psi_t \geq 0 \tag{A70}$$

$$q_t = (1-\zeta)\left(\frac{a_t^*}{a_t^\ell}\right)^\zeta \tag{A71}$$

$$\varphi_t q_t a_t^\ell = 0,\ \varphi_t \geq 0 \tag{A72}$$

$$a_t = a_{t-1} + \eta(z_{t-1})^\gamma (a_{t-1} + \kappa a_{t-1}^*)^{1-\gamma} - a_t^\ell \tag{A73}$$

$$\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t) \tag{A74}$$

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^\ell \tag{A75}$$

$$\frac{1}{c_t} = \beta \tilde{R}_t \mathbb{E}_t \left[\frac{1}{c_{t+1}} + \psi_{t+1}\theta_{t+1}\right] \tag{A76}$$

$$\frac{1}{c_t} - \nu_t = \beta R^s \mathbb{E}_t \left[\frac{1}{c_{t+1}} + \psi_{t+1}\right] \tag{A77}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t \left[\xi_{t+1}\eta\gamma \left(\frac{z_t}{a_t + \kappa a_t^*}\right)^{\gamma-1}\right] \tag{A78}$$

$$\nu_t s_t = 0,\ \nu_t \geq 0 \tag{A79}$$

$$\begin{aligned}\xi_t = u'(c_t)\left[1 + \left(\frac{b_t/a_t}{R_t}\right)^2 \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right)\right] \\ + \beta \mathbb{E}_t \left[\xi_{t+1}\left\{1 + \eta(1-\gamma)\left(\frac{z_t}{a_t + \kappa a_t^*}\right)^\gamma\right\}\right]\end{aligned} \tag{A80}$$

$$R_t = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\frac{b_t}{a_t} - \bar{b}\right) - 1\right] \tag{A81}$$

We divide thirteen endogenous variables into two groups. The first group contains six variables determined automatically given a liquidity shock θ_t; the second contains seven variables determined by households' endogenous choices.

First, a_t^* is determined by (A69). Given θ_t and the state variables b_{t-1} and s_{t-1}, the liquidity constraint (A70) indicates whether there is a liquidity shortage and $a_t^\ell > 0$ or $a_t^\ell = 0$.[A1]

[A1] Technically, agents may liquidate assets more than necessary or even when there is no liquidity shock,

- In the case of $a_t^\ell > 0$, the binding liquidity constraint (A70) and (A71) pin down q_t and a_t^ℓ. $\varphi_t = 0$ by (A72), and a_t is determined by (A73). ψ_t is computed by (A74) after obtaining c_t and ξ_t.

- In the case of $a_t^\ell = 0$, $\psi_t = 0$ by (A70), q_t and (A71) are irrelevant, and $\varphi_t > 0$ by (A72). a_t is determined by (A73). φ_t is computed by (A74) after obtaining c_t and ξ_t.

Therefore, values for a_t^ℓ, q_t, a_t^*, a_t, and signs for ψ_t and φ_t are automatically determined at the beginning of a period when θ_t is realized, independent of endogenous choices in this period. These six variables are in the first group. The remaining seven variables, c_t, b_t, s_t, z_t, ξ_t, ν_t, and R_t are jointly determined to satisfy seven equations (A75)-(A81). In these seven equations, the state variables are only b_{t-1} and s_{t-1} in (A75), and $q_t a_t^\ell$ is already determined. This means that when agents make decisions to determine seven variables in the second group, the relevant endogenous state variable is $b_{t-1} + s_{t-1} + q_t a_t^\ell$ in (A75), and there is no need to know b_{t-1}, s_{t-1}, and z_{t-1} individually.

We utilize this fact to simplify our numerical solution. In particular, we denote $w_t = b_{t-1} + s_{t-1} + q_t a_t^\ell$, and solve the model as if agents first choose endogenous variables determining c_t, b_t, s_t, z_t, ξ_t, ν_t, and R_t given the state variables (w_t, a_t^*, θ_t, ε_t^R); then a liquidity shock θ_{t+1} is realized and a_{t+1}^ℓ, q_{t+1}, a_{t+1}^*, a_{t+1}, ψ_{t+1}, φ_{t+1}, and w_{t+1} are automatically determined.

We stationarize the equilibrium conditions by dividing by the productivity level. $\hat{c}_t$, $\hat{b}_t$, $\hat{s}_t$, $\hat{z}_t$, $\hat{w}_t$, $\hat{a}_t^\ell$, and $\hat{a}_t^*$ denote each variable divided by a_t. $\hat{\psi}_t$, $\hat{\varphi}_t$, $\hat{\nu}_t$, and $\hat{\xi}_t$ denote each variable multiplied by a_t, because these variables shrink over time at the same rate as the marginal utility $1/c_t$. R_t and q_t are already stationary. g_t denotes the growth rate of the productivity

and $a_t^\ell > 0$ and $\psi_t = 0$ may hold at the same time. In our calibrated model, however, $\psi_t > 0$ if and only if $a_t^\ell > 0$, implying that agents never liquidate more than necessary to cover the liquidity shortage.

level $a_t/a_{t-1}-1$. Below is the list of the stationarized equilibrium conditions:

$$R_t = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\hat{b}_t - \bar{b}\right) - 1\right] \tag{A82}$$

$$\hat{c}_t + \frac{\hat{b}_t}{R_t} + \frac{\hat{s}_t}{R^s} + \hat{z}_t = 1 + \hat{w}_t \tag{A83}$$

$$\frac{1}{\hat{c}_t} = \beta \tilde{R}_t \mathbb{E}_t \left[\frac{1}{\hat{c}_{t+1}(1+g_{t+1})} + \frac{\hat{\psi}_{t+1}}{1+g_{t+1}}\theta_{t+1}\right] \tag{A84}$$

$$\frac{1}{\hat{c}_t} - \hat{\nu}_t = \beta R^s \mathbb{E}_t \left[\frac{1}{\hat{c}_{t+1}(1+g_{t+1})} + \frac{\hat{\psi}_{t+1}}{1+g_{t+1}}\right] \tag{A85}$$

$$\frac{1}{\hat{c}_t} = \beta \mathbb{E}_t \left[\frac{\hat{\xi}_{t+1}}{1+g_{t+1}} \eta\gamma \left(\frac{\hat{z}_t}{1+\kappa\hat{a}_t^*}\right)^{\gamma-1}\right] \tag{A86}$$

$$\hat{\nu}_t \hat{s}_t = 0 \tag{A87}$$

$$\begin{aligned}\hat{\xi}_t = {} & \frac{1}{\hat{c}_t}\left[1 + \left(\frac{\hat{b}_t}{R_t}\right)^2 \psi^b \exp\left(-\hat{b}_t - \bar{b}\right)\right] \\ & + \beta \mathbb{E}_t \left[\frac{\hat{\xi}_{t+1}}{1+g_{t+1}}\left\{1 + \eta(1-\gamma)\left(\frac{\hat{z}_t}{1+\kappa\hat{a}_t^*}\right)^{\gamma}\right\}\right]\end{aligned} \tag{A88}$$

$$\hat{\psi}_{t+1}\left[q_{t+1}\hat{a}_{t+1}^{\ell}(1+g_{t+1}) + \theta_{t+1}\hat{b}_t + \hat{s}_t\right] = 0 \tag{A89}$$

$$q_{t+1} = (1-\zeta)\left(\frac{\hat{a}_{t+1}^*}{\hat{a}_{t+1}^{\ell}}\right)^{\zeta} \tag{A90}$$

$$\hat{\varphi}_{t+1} q_{t+1} \hat{a}_{t+1}^{\ell} = 0 \tag{A91}$$

$$1 + g_{t+1} = 1 + \eta(\hat{z}_t)^{\gamma}(1+\kappa\hat{a}_t^*)^{1-\gamma} - \hat{a}_{t+1}^{\ell}(1+g_{t+1}) \tag{A92}$$

$$\hat{a}_{t+1}^* = \frac{1+\bar{g}}{1+g_{t+1}}\hat{a}_t^* \tag{A93}$$

$$\hat{w}_{t+1} = \frac{\hat{b}_t}{1+g_{t+1}} + \frac{\hat{s}_t}{1+g_{t+1}} + q_{t+1}\hat{a}_{t+1}^{\ell} \tag{A94}$$

$$\hat{\psi}_{t+1} + \hat{\varphi}_{t+1} = \frac{\hat{\xi}_{t+1}}{q_{t+1}} - \frac{1}{\hat{c}_{t+1}} \tag{A95}$$

The state variables at the beginning of a period are ($\hat{w}_t$, $\hat{a}_t^*$, θ_t, ε_t^R).

J.2 Numerical Solution Algorithm

To reduce the number of state variables, we define the net wealth of households after liquidation, $w_t = b_{t-1} + s_{t-1} + q_t a_t^{\ell}$, as a state variable instead of keeping track of b_{t-1} and s_{t-1}

individually. This is possible because b_{t-1} and s_{t-1} determine the amount of asset liquidation a_t^ℓ and its price q_t through the binding liquidity constraint (5) and the asset price equation (15), while net wealth summarizes their role in subsequent decisions.

We solve this stationarized model numerically by policy-function iteration.

1. We create 101 equally spaced grid points for $\hat{w}_t$ and 11 grid points for $\hat{a}_t^*$. For each grid point, we create initial guesses for $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (A85), and next-period liquidation $\hat{a}_{t+1}^\ell$ if a liquidity shock hits at $t+1$.

2. For each grid point, we derive five equations for five unknowns $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (A85), and $\hat{a}_{t+1}^\ell$. We then solve for these unknowns using a non-linear solver. Below are the steps:

   (a) Given the five unknowns, (A82) gives R_t.

   (b) We assume $\hat{s}_t > 0$ and $\hat{\nu}_t = 0$. Then (A85) gives $\hat{c}_t$.

   (c) We plug $\hat{c}_t$, $\hat{b}_t$, and $\hat{z}_t$ into (A83) to check whether $\hat{s}_t > 0$. If $\hat{s}_t < 0$, then we set $\hat{s}_t = 0$ and compute $\hat{c}_t$ from (A83) and $\hat{\nu}_t$ from (A85). Now we have $\hat{c}_t$, $\hat{b}_t$, $\hat{s}_t$, $\hat{z}_t$, R_t, $\hat{\xi}_t$, and $\hat{\nu}_t$.

   (d) Next-period shocks $(\theta_{t+1}, \varepsilon_{t+1}^R)$ are realized. For each realization,

   - If no liquidity shock occurs, then $\hat{a}_{t+1}^\ell = 0$, and q_{t+1} and (A90) are irrelevant. (A89) implies $\hat{\psi}_{t+1} = 0$, (A91) implies $\hat{\varphi}_{t+1} > 0$. Compute g_{t+1} from (A92), $\hat{a}_{t+1}^*$ from (A93), and $\hat{w}_{t+1}$ from (A94).
   - If a liquidity shock hits, then (A89) implies $\hat{\psi}_{t+1} > 0$, (A91) implies $\hat{\varphi}_{t+1} = 0$. We use a guess for $\hat{a}_{t+1}^\ell$ and compute g_{t+1} from (A92), $\hat{a}_{t+1}^*$ from (A93), q_{t+1} from (A90), and $\hat{w}_{t+1}$ from (A94).
   - Given the next-period state variables $(\hat{w}_{t+1}, \hat{a}_{t+1}^*, \theta_{t+1}, \varepsilon_{t+1}^R)$, we compute $\hat{b}_{t+1}$, $\hat{z}_{t+1}$, and the right-hand side of (A85) at $t+1$ by linearly interpolating the initial guesses. Then we compute next-period endogenous variables $\hat{c}_{t+1}$, $\hat{b}_{t+1}$, $\hat{s}_{t+1}$, $\hat{z}_{t+1}$, R_{t+1}, and $\hat{\xi}_{t+1}$ following the same steps as above. Also compute $\hat{\psi}_{t+1}$ using (A95).

   (e) At this point, unused equations are (A84), (A86), (A88), and a binding liquidity constraint (A89) when $\hat{a}_{t+1}^\ell > 0$. We also compute the right-hand side of (A85) by explicitly taking the expectations over shocks. Then we have five equations for five unknowns $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (A85), and $\hat{a}_{t+1}^\ell$. We use a nonlinear solver to obtain the decision rules from these five simultaneous equations.

3. After obtaining the decision rules for $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (A85), and $\hat{a}_{t+1}^\ell$ for all grid points, we compare them with the initial guesses. If the gap is sufficiently small, we stop. Otherwise, we update the guesses using these decision rules and repeat step 2 until convergence.

We solve the social planner's problem using the same steps, replacing q_{t+1} with $q_{t+1} - \zeta q_{t+1}$ in (A95).

K Comparison with Existing Models

In this section, we compare our model with Bianchi (2011), Arce, Bengui, and Bianchi (2025) (ABB), and Lutz and Zessner-Spitzenberg (2023) (LZS). We proceed in two steps. First, we examine whether interpreting the collateral constraint parameter as financial development can generate hump-shaped reserves and declining capital controls as financial development improves. Second, we examine LZS, which allows the joint use of the two policies, by varying either collateral pledgeability or access to trade financing.

K.1 Collateral Pledgeability in Bianchi (2011) and ABB

In models with collateral borrowing constraints, such as Bianchi (2011) and Arce, Bengui, and Bianchi (2025), a natural measure of financial development is collateral pledgeability, κ, which determines how much income can be pledged to foreign lenders. Higher κ therefore represents greater financial development. We examine whether this interpretation can account for the joint empirical pattern.

In these models, the price of nontradable goods determines collateral values and borrowing capacity. Households do not internalize how their borrowing affects this price and other households' borrowing capacity, creating a pecuniary externality. The strength of this externality, and hence the required policy intervention, depends on κ. A thought experiment is that the externality vanishes as κ approaches zero or becomes sufficiently large that the borrowing constraint never binds. This suggests a hump-shaped policy response. There are, however, two issues with using this argument to explain the empirical pattern.

First, it does not establish the joint behavior of reserves and capital controls. Bianchi's capital control tax and ABB's public reserve policy provide alternative implementations of the planner's allocation; they do not jointly determine the two instruments. Even when considered separately, a hump-shaped capital control tax would not match the declining relationship in the data.

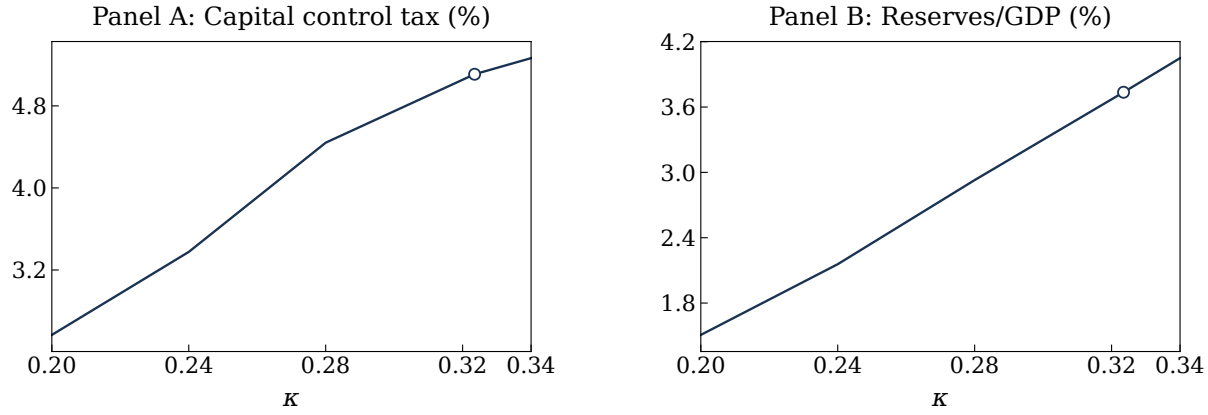
Second, extending κ to very high values poses a computational challenge because multiple equilibria can arise. We therefore examine the policy responses over a range for which we can solve the model under Bianchi's calibration, holding the remaining parameters fixed. Figure K1 reports the capital control tax and the reserves implied by ABB's implementation formula. Over $\kappa \in [0.20, 0.34]$, the average borrowing tax rises from 2.57% to 5.27%, and reserves/GDP rises from 1.51% to 4.05%. Capital controls therefore move opposite to the data, and reserves do not exhibit a hump over this range.

This exercise shows that greater collateral pledgeability does not readily generate the joint empirical pattern that our model explains.

K.2 Collateral Pledgeability and Trade Financing in Lutz and Zessner-Spitzenberg (2023)

Lutz and Zessner-Spitzenberg (2023) build on a collateral borrowing constraint and add

Figure K1 Comparative Statics with Respect to Collateral Pledgeability in Bianchi (2011) and ABB



Note. Simulated means under Bianchi's planner allocation. Panel A reports the capital control tax; Panel B reports reserves from the alternative implementation in Arce, Bengui, and Bianchi (2025), $\kappa GDP_t + b_{t+1}^{SP}$, where $b_{t+1}^{SP} < 0$ denotes net debt and GDP_t is output valued in tradable goods. Both measures are zero when the planner's borrowing constraint binds. Units are percent; circles mark the benchmark $\kappa = 0.3235$.

a liquidity requirement for intermediate inputs, providing a rationale for both a capital control tax and reserve intervention. Their model therefore allows us to examine the two policies jointly and compare its predictions with the empirical relationships that motivate our model. We consider two ways to capture financial development: varying κ and introducing a parameter into the liquidity constraint. We use their calibration and vary one parameter at a time, holding the remaining parameters fixed.

We first vary collateral pledgeability over $\kappa \in [0.15, 0.33]$, holding the remaining parameters fixed at their LZS values. Panel A of Figure K2 shows that reserves/GDP and the capital control tax both rise with κ. As in the previous exercise, greater pledgeability produces increasing capital controls and no reserve hump over this range. Thus, over the ranges examined, varying κ does not match the empirical pattern, whether the policies are considered separately in Bianchi and ABB or jointly in LZS.

In addition to the pecuniary externality, the liquidity constraint is another important difference. We use the repayment fraction θ in our liquidity constraint to measure financial development; LZS do not conduct a corresponding exercise. To examine whether their model can generate a similar pattern, we introduce a parameter into their liquidity constraint that allows financial development to improve access to trade financing. The LZS liquidity constraint is $S_t p_v v_t \leq a_t$, where $S_t \in \{0, 1\}$ indicates a liquidity shock, $p_v v_t$ is expenditure on intermediate inputs, and a_t denotes liquid reserves. We introduce ε into this constraint:

$$\varepsilon S_t p_v v_t \leq a_t.$$

A lower ε means that a smaller fraction of input expenditure requires liquid financing during

a disruption and therefore represents greater financial development. Panel B of Figure K2 varies ε over $[0.001, 1]$, keeping the other parameters unchanged. As ε falls, reserves/GDP declines monotonically and the borrowing tax rises slightly. Better trade financing reduces liquidity needs and reserve demand. Weaker precautionary saving also raises the optimal tax used to correct the collateral externality. This extension therefore produces neither the reserve hump nor declining capital controls.

Our model generates a different pattern through its liquidity constraint and fire-sale externality. Liquidity needs arise from early repayment of outstanding debt. During a liquidity shock, households must satisfy

$$-\theta b_{t-1} - s_{t-1} \leq q_t a_t^{\ell},$$

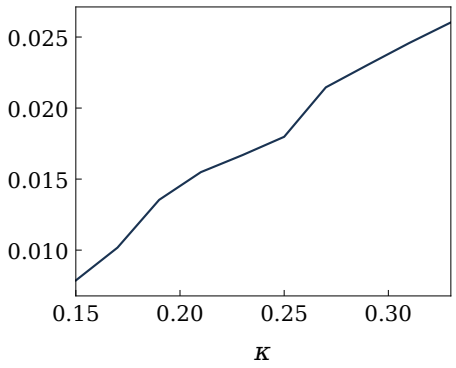
where $-b_{t-1}$ is outstanding debt, θ is the fraction repaid early, s_{t-1} is liquid reserves, and $q_t a_t^{\ell}$ is revenue from liquidating productive assets. Higher θ represents lower financial development. It raises repayment needs for given debt, but also induces precautionary reductions in borrowing. At high θ, this debt response reduces the amount exposed to the shock and hence reserve demand. At low θ, little debt requires early repayment. Together, these forces make reserves highest at intermediate levels of financial development in our quantitative model.

The externality also differs: asset sales lower the liquidation price received by other households, rather than tightening a collateral borrowing constraint. Capital controls address this fire-sale externality associated with early debt repayment. In our quantitative model, capital controls increase with θ and therefore decline as financial development improves. Together with the reserve hump, this result gives the joint policy pattern shown in Section 5.1.

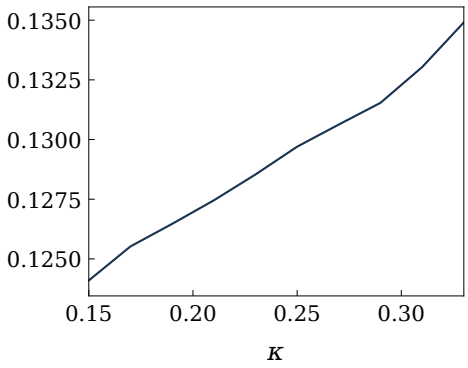
Figure K2 Comparative Statics in LZS

Panel A: Collateral pledgeability (κ)

Capital control tax

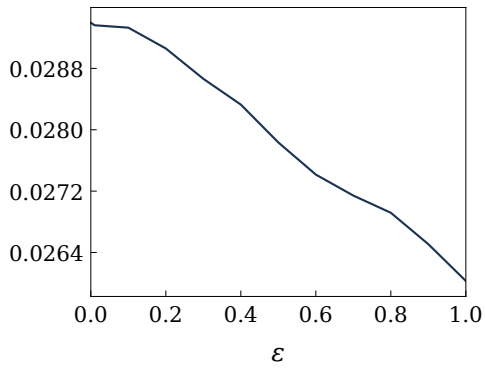


Reserves/GDP

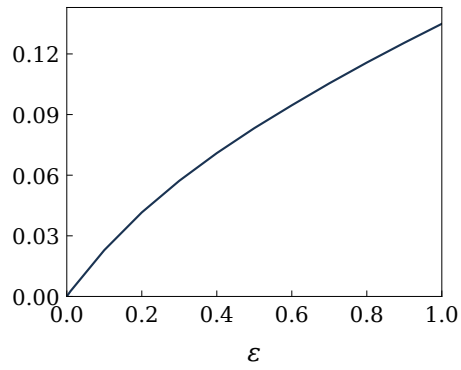


Panel B: Trade financing (ε)

Capital control tax



Reserves/GDP



Note. Simulated means of the capital control tax and reserves/GDP under the planner allocation. Panel A varies collateral parameters on tradable and nontradable income together, $\kappa_T = \kappa_N = \kappa$; higher κ means greater financial development. Panel B varies ε in the liquidity constraint $\varepsilon S_t p_v v_t \leq a_t$; lower ε means better access to trade financing and greater financial development. In each exercise, other parameters retain their LZS values. Both measures are expressed as fractions.

L Growth and Welfare

This appendix provides the detailed analysis underlying Section 5.2. We first decompose welfare and compare the productivity effects of policy at two levels of liquidity-shock severity. We then consider exogenous growth and no growth, and examine sensitivity to the fire-sale price parameter ζ.

L.1 Welfare Decomposition and Liquidity-Shock Severity

With log utility, consumption $c_t = \hat{c}_t a_t$ separates into stationary consumption $\hat{c}_t = c_t/a_t$ and productivity a_t. For a given initial state, define their consumption-equivalent gains by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln\big((1+\gamma_c)\hat{c}_t^{DE}\big) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln \hat{c}_t^{SP},$$
$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln\big((1+\gamma_g)a_t^{DE}\big) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln a_t^{SP}.$$

Here, β is the discount factor, and the superscripts denote the decentralized equilibrium and planner's allocation. If γ is the total gain at the same initial state, then

$$1+\gamma = (1+\gamma_c)(1+\gamma_g).$$

As in Section 5.2, we average the reported gains across initial states using the DE ergodic distribution. Figure 7 in the main text reports the total gain and its decomposition.

The stationary-consumption component is positive, as in Ma (2020). The productivity component is also positive at the benchmark $\theta = 0.45$, whereas it is negative in the baseline model of Ma (2020). Our accumulation equation helps explain the difference:

$$1 + g_{t+1} = \frac{1 + \eta \hat{z}_t^{\gamma} (1 + \kappa \hat{a}_t^*)^{1-\gamma}}{1 + \hat{a}_{t+1}^{\ell}}. \tag{A96}$$

Here, g_{t+1} is productivity growth, $\hat{z}_t = z_t/a_t$ is investment relative to domestic productive assets, $\hat{a}_t^* = a_t^*/a_t$ is relative foreign productivity, and $\hat{a}_{t+1}^{\ell} = a_{t+1}^{\ell}/a_{t+1}$ is normalized liquidation. The parameters η and γ govern investment technology, while κ governs foreign technology spillovers. Holding investment and the current state fixed, greater liquidation directly lowers productivity growth through the denominator.

In Ma (2020), crises affect growth through the investment Euler equation by changing current marginal utility relative to future marginal utility. Although the planner seeks to influence growth to mitigate the pecuniary externality, the effect on growth is second order and quantitatively small. Our model instead features a direct loss of productive assets through liquidation. By mitigating the fire-sale externality and reducing liquidation, policy can have a larger effect on growth. The magnitude of this effect depends on liquidity-shock severity, θ.

Figure L1 compares DE and SP paths at $\theta = 0.45$ and $\theta = 0.1$. For each value of θ, the two allocations start from the same state. The interest rate is low and there is no liquidity shock in periods -5 to -1. A liquidity shock occurs at period 0, followed by a low interest rate and no further liquidity shocks. In the benchmark, SP productivity is about 1% below DE before the shock, but becomes more than 2.5% higher at the shock because the planner holds more reserves and liquidates fewer assets. The productivity advantage persists for almost 20 periods. Policy therefore entails a trade-off: it lowers growth in normal times but raises it during crises. At $\theta = 0.45$, the crisis-time benefit is sufficient to generate a positive productivity component of welfare.

When $\theta = 0.1$, the smaller liquidity shock limits the benefit from reducing liquidation. The productivity gap becomes negative around period 10, and the crisis-time benefit does not offset the productivity cost in normal times. The productivity component is negative and total welfare gains are about 0.1%, close to 0.06% in the baseline of Ma (2020). The two cases share the same endogenous-growth mechanism; their different welfare implications reflect the scope for policy to prevent productivity losses.

L.2 Exogenous Growth and No Growth

To examine the contribution of endogenous growth, we remove investment and let output grow at an exogenous rate $\bar{g}$. We also modify the fire-sale specification: allowing liquidation to reduce future productive assets would be inconsistent with the prescribed output path. Households instead sell consumption goods at a fire-sale price when a liquidity shock occurs. This retains the liquidity constraint and price externality while removing the effect of liquidation on future productivity. The budget constraint and output process are

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} = a_t - a_t^\ell + b_{t-1} + s_{t-1} + q_t a_t^\ell,$$
$$a_t = (1 + \bar{g}) a_{t-1}.$$

Here, a_t^ℓ denotes consumption goods sold to foreign buyers at the fire-sale price

$$q_t = \kappa_q (1 - \zeta) \left(\frac{1}{a_t^\ell} \right)^\zeta.$$

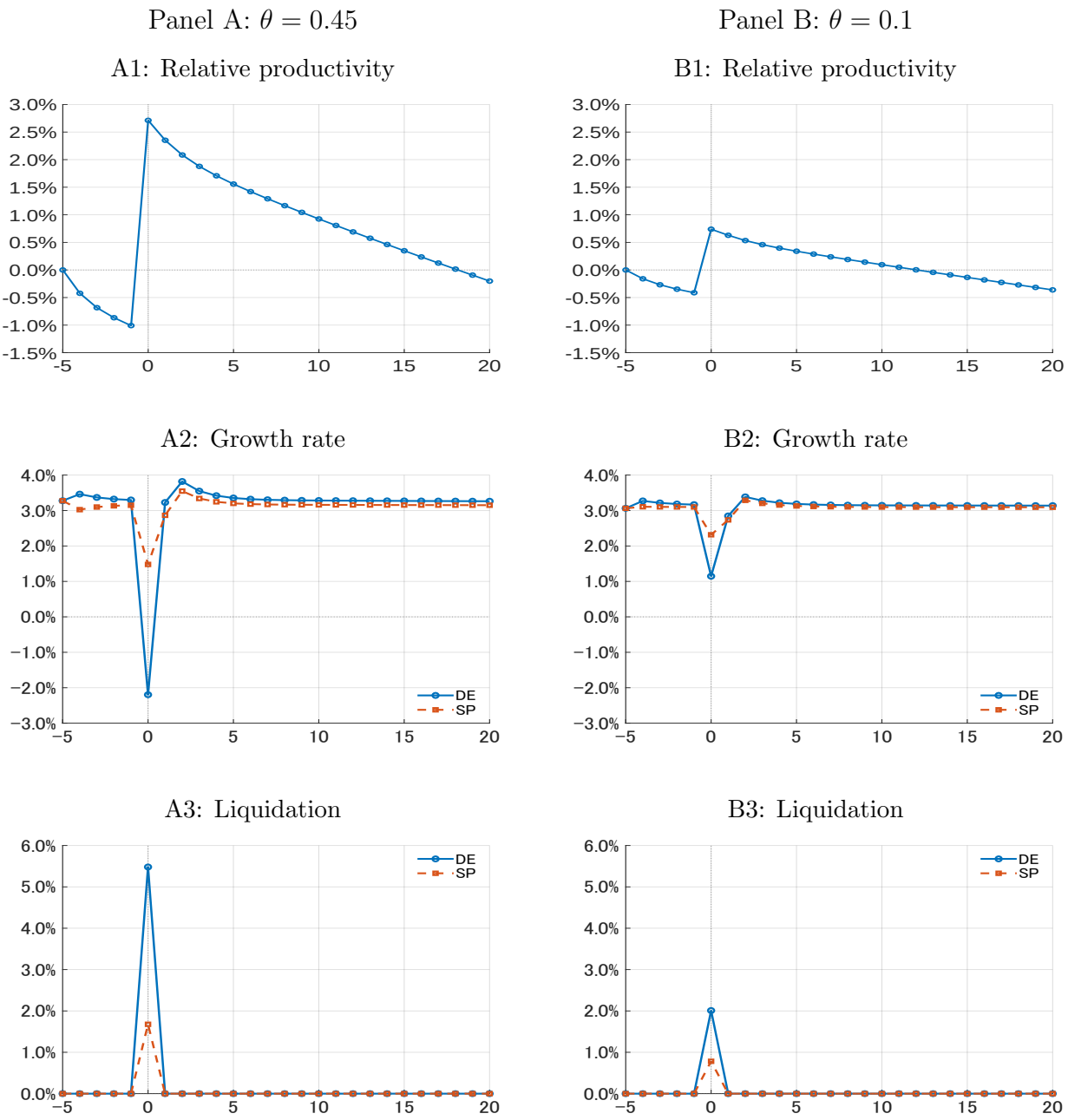
The remaining elements of the benchmark are retained, including the liquidity constraint

$$q_t a_t^\ell \geq -\theta b_{t-1} - s_{t-1}.$$

For this exercise, we set the price-scale parameter $\kappa_q = 0.37$ by targeting the benchmark reserve-to-GDP ratio. We solve the model for $\bar{g} = 0.0295$, the benchmark annual growth rate of 2.95%, and for $\bar{g} = 0$, with no growth.

Figure L2 reports the results across θ. In both specifications, reserves remain non-monotonic and the capital control tax increases with θ. The joint qualitative policy

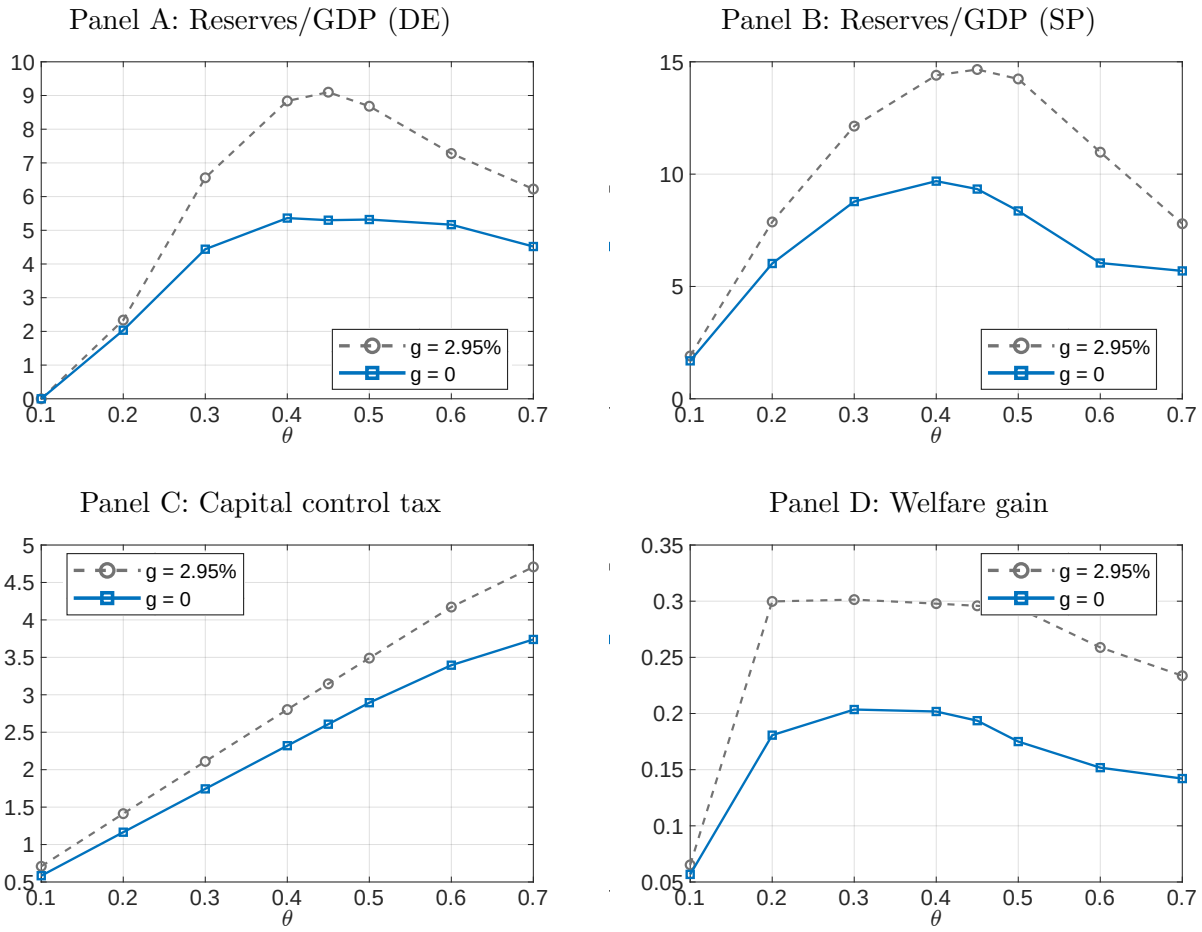
Figure L1 Productivity, Growth, and Liquidation under DE and SP



Note. Panel A uses $\theta = 0.45$ and Panel B uses $\theta = 0.1$. Each column compares the decentralized equilibrium (DE) and social planner's allocation (SP) from the same initial state. The horizontal axes show periods relative to the liquidity shock at $t = 0$; all other periods shown have a low interest rate and no liquidity shock. Panels A1 and B1 report relative productivity as the percentage deviation from DE, $100(a_t^{SP}/a_t^{DE} - 1)$. Panels A2 and B2 report productivity growth, $100g_t$. Panels A3 and B3 report liquidation as a percentage of previous-period productivity, $100a_t^{\ell}/a_{t-1}$. Blue solid lines denote DE and red dashed lines denote SP in the growth and liquidation panels.

pattern therefore does not depend on endogenous growth. Welfare gains are of a similar order of magnitude to the stationary-consumption component in the benchmark. Together with the decomposition, this exercise shows the importance of the additional productivity benefit for benchmark welfare gains.

Figure L2 Policies and Welfare with Exogenous Growth



Note. Panels A and B report mean reserves/GDP under DE and SP, respectively; Panel C reports the mean capital control tax. These quantities are expressed in percent. Panel D reports the welfare gain from SP relative to DE as a permanent percentage increase in consumption. Dashed lines use $\bar{g} = 0.0295$; solid lines use $\bar{g} = 0$. Both specifications use $\kappa_q = 0.37$.

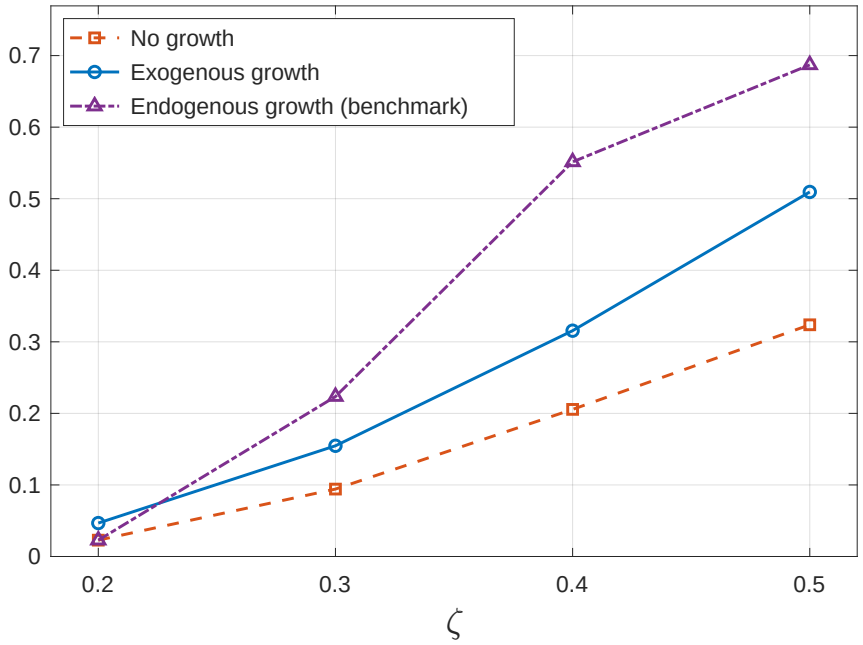
L.3 Sensitivity to the Fire-Sale Price

While θ determines early repayment needs, ζ governs the sensitivity of the fire-sale price to liquidation. The price equation implies $\partial \ln q_t / \partial \ln a_t^\ell = -\zeta$: a higher ζ means a larger percentage price decline for a given percentage increase in liquidation. This provides another

way to examine how the strength of the externality affects welfare gains, analogous to the role of the elasticity of substitution between tradable and nontradable goods in Bianchi (2011).

Figure L3 compares endogenous growth, exogenous growth at $\bar{g} = 0.0295$, and no growth at $\theta = 0.45$. Within each specification, welfare gains rise with ζ. These results show that the sensitivity of the fire-sale price matters for welfare gains whether growth is endogenous, exogenous, or absent. The additional productivity channel contributes to the benchmark gain, but its contribution depends on the liquidity shock and the underlying externality.

Figure L3 Welfare Gains across Growth Specifications



Note. Welfare gains from SP relative to DE are measured as a permanent percentage increase in consumption. All specifications use $\theta = 0.45$. Circles denote exogenous growth at $\bar{g} = 0.0295$; squares denote no growth; triangles denote endogenous growth. The exogenous-growth specifications use $\kappa_q = 0.37$.

M The Role of DIR

This appendix examines the role of the debt-elastic interest rate (DIR) introduced in Section 3.1. We first explain why we use it, then vary its strength, and finally replace it with borrowing constraints. The exercises show that DIR affects the levels of debt and reserves, but is not essential for the joint qualitative policy patterns.

M.1 Why We Use DIR

Our calibration has $\beta R^b < 1$, where β is the discount factor and R^b is the baseline gross interest rate on foreign debt. This creates an incentive to borrow. Liquidity risk provides a precautionary motive to limit debt, since households may need to repay part of their debt before new borrowing is available. This motive weakens as θ, the fraction of debt subject to early repayment during a liquidity shock, falls. A lower θ represents higher financial development.

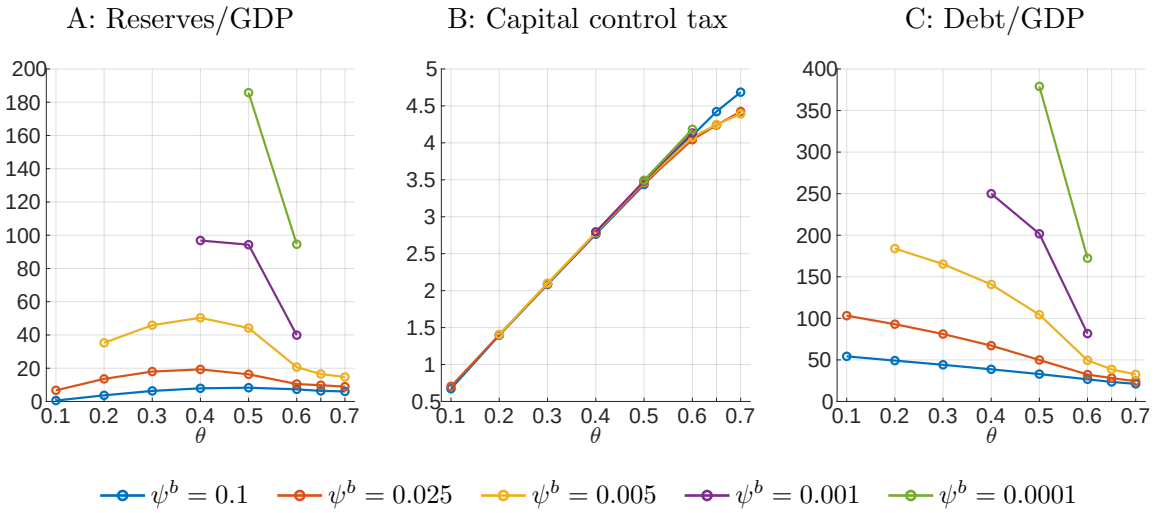
In an unreported exercise, we obtain a stationary solution without DIR at the benchmark $\theta = 0.45$, where liquidity risk is sufficient to discipline borrowing. As we lower θ, however, debt can become very large and the numerical solution may fail to converge. DIR limits borrowing smoothly by raising the borrowing rate as debt increases. Households internalize how their choices affect this rate, so DIR does not introduce an additional externality. A borrowing constraint is an alternative, but a fixed constraint can bind throughout the simulation at low θ and directly restrict the comparative statics. We therefore retain DIR in the baseline and examine both approaches below.

M.2 Sensitivity to the Strength of DIR

Figure M1 varies ψ^b, which governs the sensitivity of the borrowing rate to debt, and reports simulated means under the social planner's allocation. Reserve holdings remain nonmonotonic in θ for $\psi^b = 0.1$, 0.025, and 0.005, while the capital control tax rises with θ across the computed cases. For $\psi^b = 0.001$ and 0.0001, we obtain solutions only at relatively high values of θ because of convergence difficulties at lower values. These curves therefore cover only part of the reserve profile.

Debt becomes substantially larger as ψ^b falls and as θ declines. Thus, debt and reserve levels are sensitive to DIR. However, the qualitative policy patterns are robust.

Figure M1 Reserves, Capital Controls, and Debt under Different ψ^b



Note. The panels report simulated means under the social planner's allocation. All ratios and tax rates are expressed in percent. The horizontal axis is θ, the fraction of debt subject to early repayment during a liquidity shock; lower θ represents higher financial development. Each line corresponds to a value of ψ^b, which governs the sensitivity of the borrowing rate to debt.

M.3 Replacing DIR with Borrowing Constraints

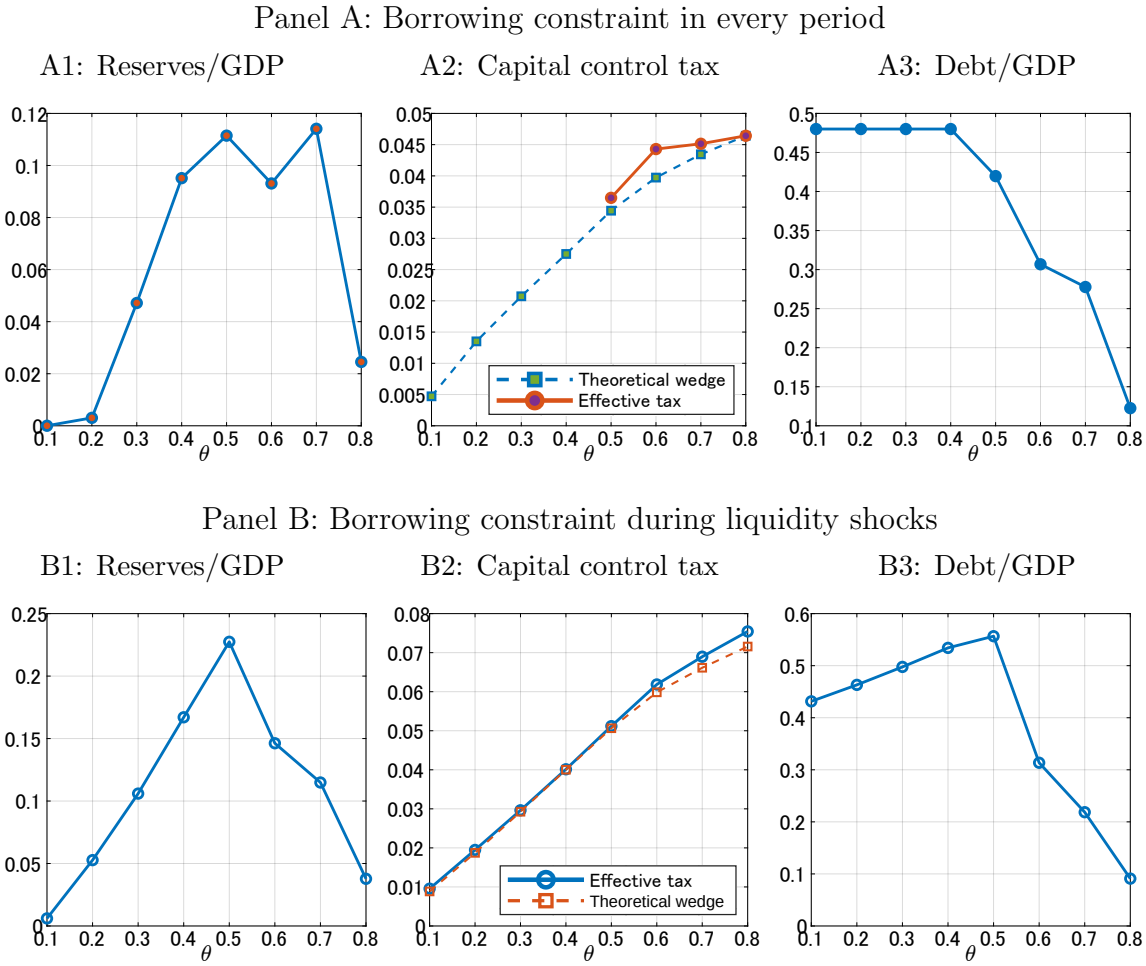
Figure M2 replaces DIR with two alternative borrowing constraints: one that applies in every state (Panel A), and one that applies only during a liquidity shock (Panel B). Both generate non-monotonic reserves and capital control taxes that increase with θ.

In Panel A, the fixed borrowing constraint binds throughout the simulation for $\theta \leq 0.4$, as reflected in the flat debt profile. In this region, the constraint already limits borrowing, so a separate debt tax is not needed to reduce borrowing at the margin and the effective tax is zero. Panel A2 distinguishes the theoretical tax wedge from the effective tax, which is plotted only where the constraint is not always binding. Both rise with θ over their respective plotted ranges. Reserves are non-monotonic, although the profile has local fluctuations.

In Panel B, the borrowing constraint applies only when a liquidity shock occurs. The possibility that it will bind in the next period limits borrowing ex ante. In this specification, reserves are hump-shaped and the capital control tax rises with θ.

These exercises support the use of DIR as a tractable way to limit debt in the baseline. Its strength affects quantitative levels, but replacing it with borrowing constraints preserves the joint qualitative policy patterns.

Figure M2 Alternative Borrowing Constraints



Note. The panels report simulated means under the social planner's allocation when a borrowing constraint replaces DIR. Panel A imposes the constraint in every period; Panel B imposes it only during a liquidity shock. The horizontal axis is the early-repayment fraction θ. All plotted ratios and tax rates are expressed as fractions. The middle plots distinguish the theoretical tax wedge from the effective tax. In Panel A, no effective tax is plotted for $\theta \leq 0.4$, where the constraint binds throughout the simulation and the effective tax is zero.

N Financial Development and Economic Development

Financial and economic development are related but capture different aspects of an economy. This appendix first examines whether output levels and growth account for the empirical policy relationships associated with financial development. It then allows financial development to evolve with relative productive assets and examines policy along the resulting development path.

N.1 Empirical Distinction

Financial development and economic development are closely related but conceptually distinct, as discussed in the survey by Levine (2005). Financial development reflects the depth, access, and efficiency of financial institutions and markets. These characteristics shape an economy's ability to absorb a liquidity shock. Economic development concerns the level of output, either in absolute terms or relative to the world frontier, while economic growth measures how output changes over time.

We use the IMF Financial Development Index because it captures multiple dimensions of financial institutions and markets. To distinguish financial development from economic development, we consider four output measures: log GDP per capita, log GDP per capita relative to the United States, real GDP per capita growth, and aggregate real GDP growth. The first measure is included as a control in the baseline regressions. We use these measures in three exercises.

Are financial and economic development related? Panel A of Table N1 reports cross-country relationships, and Panel B reports panel relationships with country and year fixed effects. Financial development is strongly and positively associated with both measures of output level. Its relationship with growth is less stable: the association with GDP per capita growth changes sign between the cross-section and the panel, while the association with aggregate GDP growth is negative in both. Thus, the positive association with the level of economic development does not imply the same association with economic growth.

Does controlling for economic development weaken the reserve hump? Table N2 adds each output measure and its square to the reserve regressions. The linear financial-development coefficient remains positive and significant, and its squared term remains negative and significant in all four specifications. Economic development therefore does not absorb the inverse-U relationship between reserves and financial development.

Can economic development replace financial development in explaining the policy patterns? Panels A and B of Table N3 replace financial development with each output measure in the reserve and capital-control regressions. The output-level measures have negative and significant squared terms in the reserve regressions, but their linear terms are insignificant, and neither has a significant relationship with capital controls. The growth measures have positive linear reserve coefficients but insignificant squared terms, and their capital-control coefficients are positive rather than negative. Taken together, the exercises show that financial development adds information beyond output levels and growth.

Table N1 Financial and economic development: Descriptive relationships

Panel A: Cross-country relationships

	(1) Log GDP per capita	(2) Log relative GDP per capita	(3) GDP per capita growth	(4) Aggregate GDP growth
Financial development	6.406***	6.481***	1.356**	-1.468**
	(0.364)	(0.358)	(0.657)	(0.729)
Countries	85	85	85	85
R^2	0.731	0.750	0.041	0.044

Panel B: Panel relationships with country and year fixed effects

	(1) Log GDP per capita	(2) Log relative GDP per capita	(3) GDP per capita growth	(4) Aggregate GDP growth
Financial development	0.930***	0.930***	-7.518***	-6.812***
	(0.219)	(0.219)	(1.034)	(1.101)
Countries	85	85	85	85
Observations	3159	3159	3159	3159
Within R^2	0.061	0.061	0.023	0.018

Note. Panel A reports cross-country OLS regressions for 85 countries. For each country, financial development and the four economic-development measures are averaged over the same nonmissing country-years within 1980–2019. Heteroskedasticity-robust standard errors are in parentheses. Panel B reports annual panel regressions using the same 85 countries. All specifications include country and year fixed effects; standard errors are clustered by country. Relative output is the log of current-dollar GDP per capita relative to the United States. GDP per capita growth and aggregate GDP growth are real growth rates. These estimates are descriptive associations. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

N.2 Financial Development along the Development Path

The positive empirical association between financial development and output levels motivates a reduced-form extension of the baseline model. Financial development improves as domestic productive assets catch up with the foreign frontier. This exercise complements the baseline comparisons across fixed levels of financial development.

In our model, a_t denotes domestic productive assets, which also determine output, and a_t^* denotes foreign productive assets, which grow exogenously. Because their levels are nonstationary, we link financial development to relative productive assets. This parallels our empirical use of GDP per capita relative to the United States to measure economic development: domestic catch-up corresponds to a decline in the foreign-to-domestic asset ratio a_t^*/a_t. Specifically, the fraction of debt that must be repaid early is

$$\theta_t \equiv \mathbb{I}_t \theta, \qquad \theta = \frac{1}{1 + \exp\left[-\left(\chi_0 + \chi_1 \ln(a_{t-1}^*/a_{t-1})\right)\right]}, \qquad \chi_1 > 0,$$

where $\mathbb{I}_t$ equals one when a liquidity shock occurs and zero otherwise. Here, θ is the repayment share conditional on a shock and varies with relative assets. The logistic function keeps this share between zero and one. As domestic assets catch up with the fron-

Table N2 Financial and economic development in reserve regressions

	(1)	(2)	(3)	(4)
Financial development	1.068***	1.048***	0.904***	0.897***
	(0.263)	(0.273)	(0.258)	(0.257)
Financial development^2	-0.193**	-0.238***	-0.218**	-0.218**
	(0.079)	(0.078)	(0.084)	(0.085)
Log GDP per capita	-0.359			
	(0.351)			
Log GDP per capita^2	-0.289***			
	(0.092)			
Log relative GDP per capita		-0.379		
		(0.358)		
Log relative GDP per capita^2		-0.337***		
		(0.117)		
Real GDP per capita growth			0.081**	
			(0.035)	
Real GDP per capita growth^2			0.002	
			(0.006)	
Real GDP growth				0.072*
				(0.035)
Real GDP growth^2				-0.001
				(0.006)
Observations	1853	1853	1853	1853
R^2	0.711	0.708	0.703	0.702

Note. This table reports reserve panel regressions in standardized units. Financial development enters linearly and quadratically in every column. Column (1) includes log GDP per capita and its square. Column (2) includes log GDP per capita relative to the United States and its square. Columns (3) and (4) include, respectively, real GDP per capita growth and aggregate real GDP growth and their squared terms. Other baseline controls are trade relative to GDP, institutional quality, the exchange-rate regime, and the current account relative to GDP. All specifications include country and year fixed effects. Standard errors are two-way clustered by country and year. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table N3 Financial development versus economic development measures

Panel A: Reserves/GDP

	(1)	(2)	(3)	(4)	(5)
Financial development	0.910***				
	(0.256)				
Financial development^2	-0.233***				
	(0.082)				
Log GDP per capita		-0.058			
		(0.353)			
Log GDP per capita^2		-0.205**			
		(0.083)			
Log relative GDP per capita			-0.060		
			(0.350)		
Log relative GDP per capita^2			-0.243**		
			(0.114)		
Real GDP per capita growth				0.070**	
				(0.033)	
Real GDP per capita growth^2				0.002	
				(0.006)	
Real GDP growth					0.064*
					(0.033)
Real GDP growth^2					-0.000
					(0.006)
Log GDP per capita control	Yes	No	No	Yes	Yes
Observations	1853	1853	1853	1853	1853
R^2	0.700	0.694	0.692	0.690	0.690

Panel B: Capital control index

	(1)	(2)	(3)	(4)	(5)
Financial development	-0.239**				
	(0.115)				
Log GDP per capita		-0.302			
		(0.262)			
Log relative GDP per capita			-0.288		
			(0.250)		
Real GDP per capita growth				0.035**	
				(0.016)	
Real GDP growth					0.032**
					(0.015)
Log GDP per capita control	Yes	No	No	Yes	Yes
Observations	1852	1852	1852	1852	1852
R^2	0.877	0.875	0.875	0.876	0.876

Note. All variables are standardized. Reserve regressions include linear and quadratic development terms; capital-control regressions include only the linear term. When an output-level measure is used, the separate log GDP per capita control is omitted. Other baseline controls and country and year fixed effects are included. Standard errors are two-way clustered by country and year. Columns within each panel use a common sample. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

tier, a^*_{t-1}/a_{t-1} falls and θ declines. A lower θ represents better financial development and less severe liquidity risk.

Because θ now depends on aggregate relative assets, private asset accumulation affects future financial development. If households take aggregate relative assets, and hence θ, as given, they do not internalize this effect. This introduces a second externality alongside the fire-sale externality. We compare three allocations: (1) *Decentralized equilibrium (DE):* Households take liquidation prices and aggregate relative assets, and hence θ, as given, so neither externality is internalized. (2) *Intermediate allocation:* Asset-accumulation decisions internalize their effect on future θ through the reduced-form mapping, while liquidation prices continue to be taken as given. This counterfactual internalizes only the financial-development channel; the fire-sale externality remains. It is labelled "θ-internalized DE" in the figure. (3) *Social planner (SP):* The planner internalizes both the effect of asset accumulation on future θ and the effect of aggregate liquidation on the fire-sale price.

To examine whether our key results hold in this extension, we calibrate the reduced-form mapping using two outcomes from the baseline model with exogenous θ. Mean relative productive assets a^*/a equal 5.2 at $\theta = 0.6$ and 2.5 at $\theta = 0.1$. Matching these two points gives approximately $\chi_0 = -5.43$ and $\chi_1 = 3.53$. These parameters anchor the mapping to the baseline model's outcomes; they are not estimated from the empirical relationship between financial development and income. We then compare the three allocations, each starting from $a^*/a = 5.2$. Figure N1 reports the mean paths across 1,000 stochastic simulations, retaining the other benchmark parameters. Two findings emerge from allowing financial development to evolve.

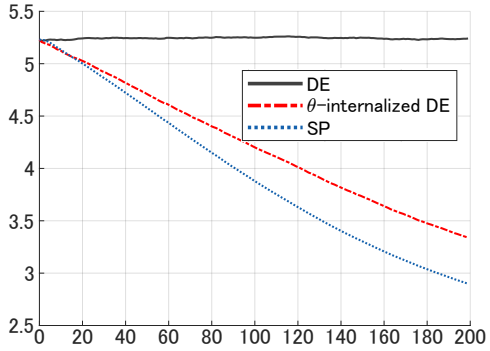
Catch-up and reserve holdings. In DE, relative assets and θ change little over 200 periods. Both the intermediate allocation and SP generate substantially faster catch-up and declining θ. The intermediate allocation's reserve-to-GDP ratio nevertheless remains much closer to DE than to SP. These comparisons indicate that internalizing the financial-development channel is important for catch-up, while also correcting the fire-sale externality leads to substantially higher reserve holdings. The faster SP transition therefore reflects the internalization of both channels.

The slow movement of θ in DE supports treating it as approximately fixed when describing the decentralized economy in the baseline calibration. This approximation need not apply to the planner's transition, along which financial development changes substantially.

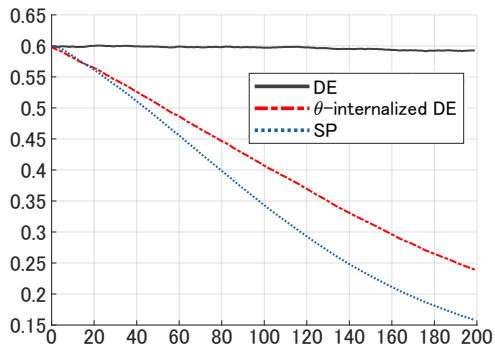
Policy patterns along the transition. Along the planner's transition, the reserve-to-GDP ratio first rises and then falls as θ declines and financial development improves. After the initial adjustment, the capital control tax also declines. Thus, the reserve hump and declining capital controls arise along the development path of a single economy, complementing the cross-country interpretation of the baseline model.

Figure N1 Transition Paths with Endogenous Financial Development

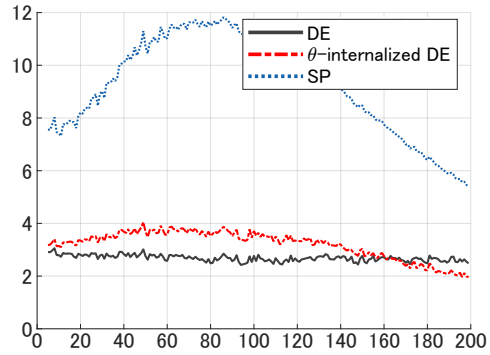
Panel A: Relative productive assets



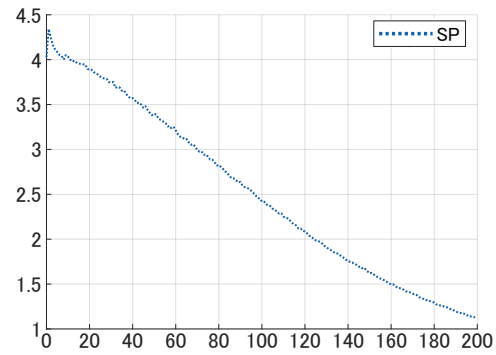
Panel B: Repayment share θ



Panel C: Reserves/GDP



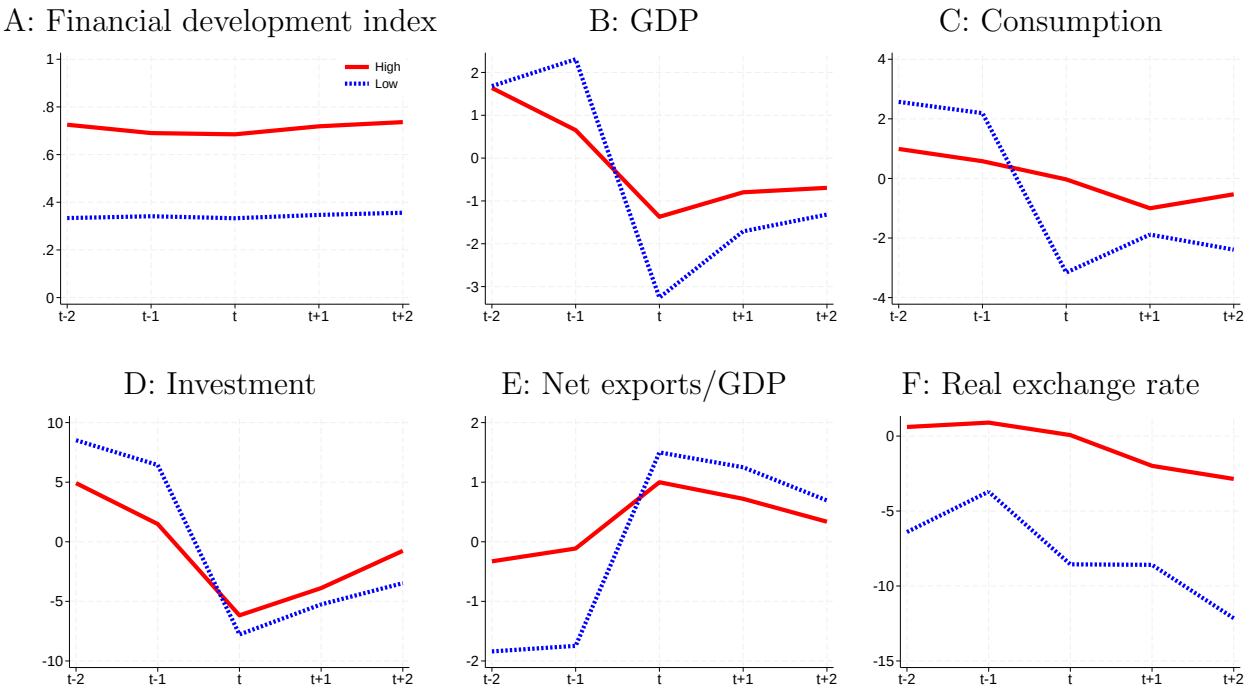
Panel D: Capital control tax



Note. Mean transition paths across 1,000 stochastic simulations starting from $a^*/a = 5.2$, with the other parameters at their benchmark values. Panel A plots foreign relative to domestic productive assets, a^*/a; Panel B plots the repayment share conditional on a liquidity shock; Panel C plots reserves as a percentage of domestic output; and Panel D plots the capital control tax rate in percent. The horizontal axis measures periods. Solid black lines denote the decentralized equilibrium (DE); dashed red lines denote the intermediate allocation, labelled "θ-internalized DE," which internalizes only the effect of asset accumulation on future θ; dotted blue lines denote the social planner (SP), who internalizes both the fire-sale externality and the effect of aggregate asset accumulation on financial development. Panel D reports only SP, since DE and θ-internalized DE have no capital control tax.

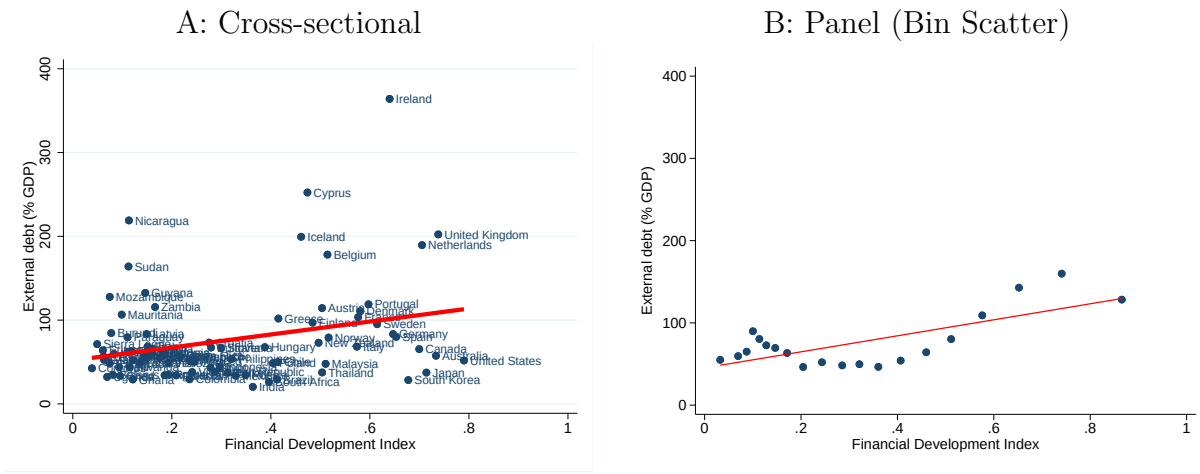
O Additional Figures

Figure O1 Financial Development and Sudden-Stop Dynamics: Medians



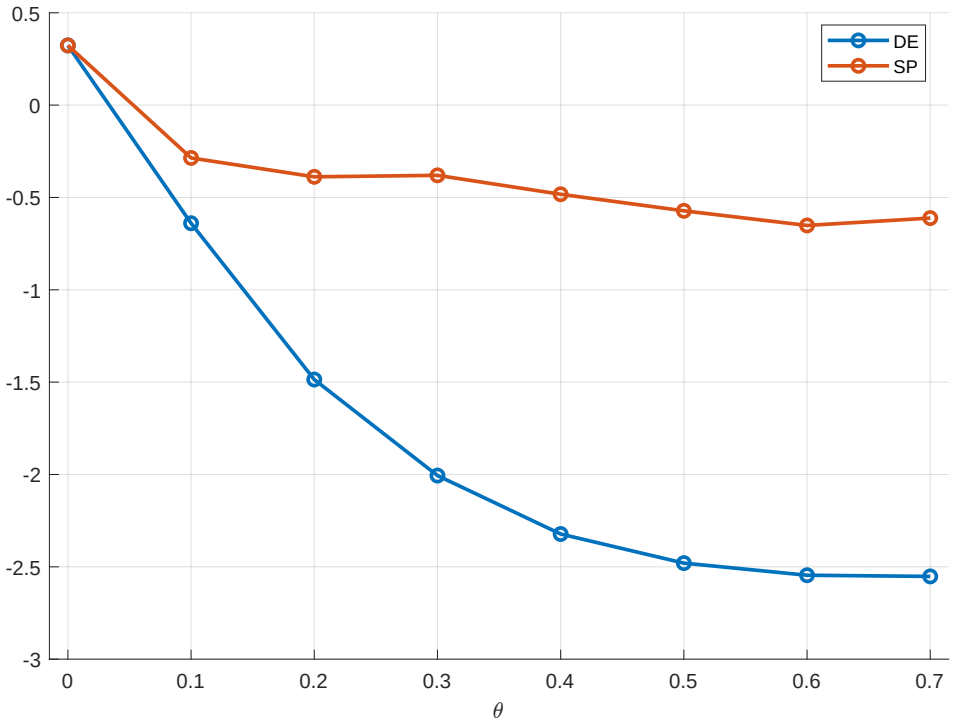
Note. Sudden-stop episodes and macroeconomic data are from Korinek and Mendoza (2014). The 56 episodes are split into 28 low- and 28 high-financial-development episodes using the financial development index two years before the crisis ($t-2$). Lines report group medians at each event time. GDP, consumption, and investment are real per-capita series expressed as 100 times their log deviations from HP trends. Net exports/GDP is expressed as a percentage-point deviation from its HP trend. The real exchange rate is expressed as a percentage deviation from its country sample mean; a decline indicates depreciation. The financial development index is shown in levels.

Figure O2 External Debt and Financial Development



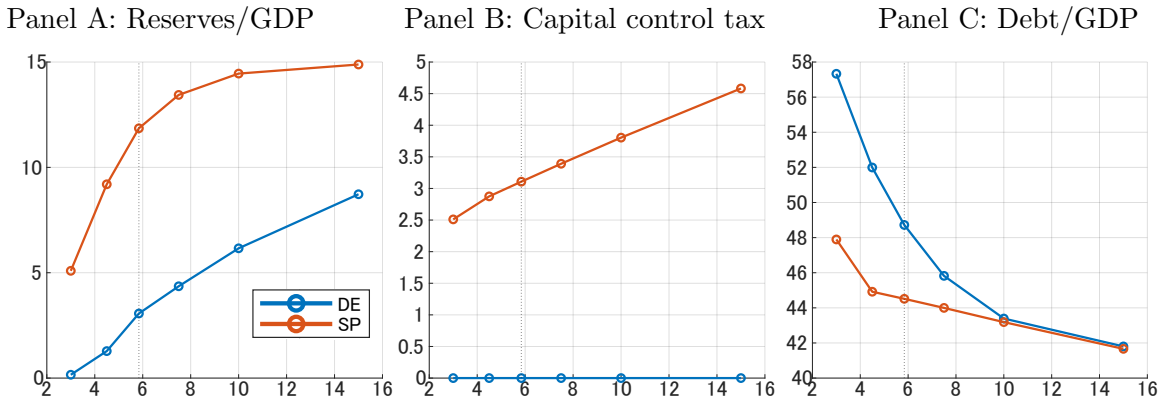
Note. The figure presents the relationship between external debt liabilities (% GDP) and the financial development index for a panel of 85 economies in 1980–2019. The Financial Development Index is taken from the IMF and summarizes the development of financial institutions and financial markets in terms of depth, access, and efficiency (Svirydzenka 2016). Panel A plots the cross-sectional relationship, where each data point is a country's time average over the available years for each series. Panel B is a bin scatter plot of 3,202 country-year observations grouped into 20 equal-frequency bins. The lines are OLS fits to the country averages and the underlying country-year observations, respectively.

Figure O3 Crisis Output Deviations across θ



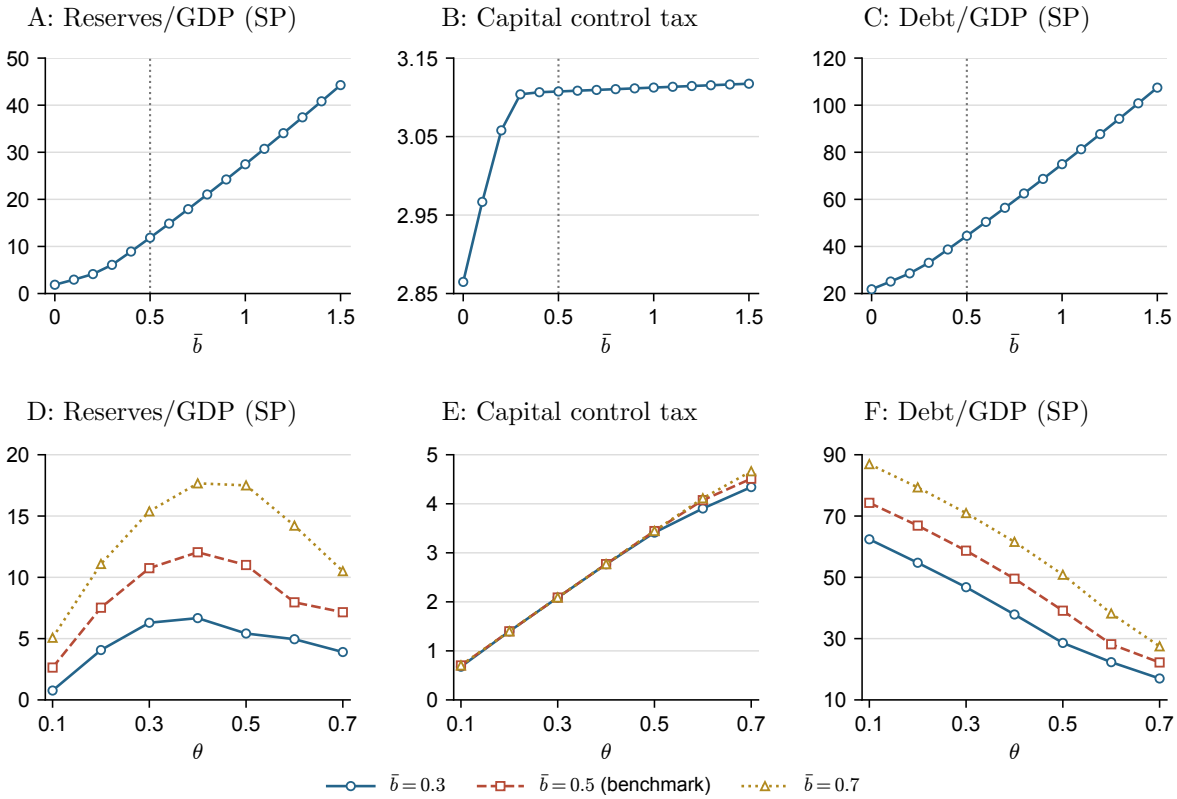
Note. The horizontal axis shows θ, the fraction of outstanding foreign debt that must be repaid early during a liquidity shock. The vertical axis reports the simulated mean percentage deviation of output from its HP-filtered trend at the sudden-stop episode. DE denotes the decentralized equilibrium without policy, and SP denotes the social planner's allocation.

Figure O4 Comparative statics with respect to liquidity shock probability



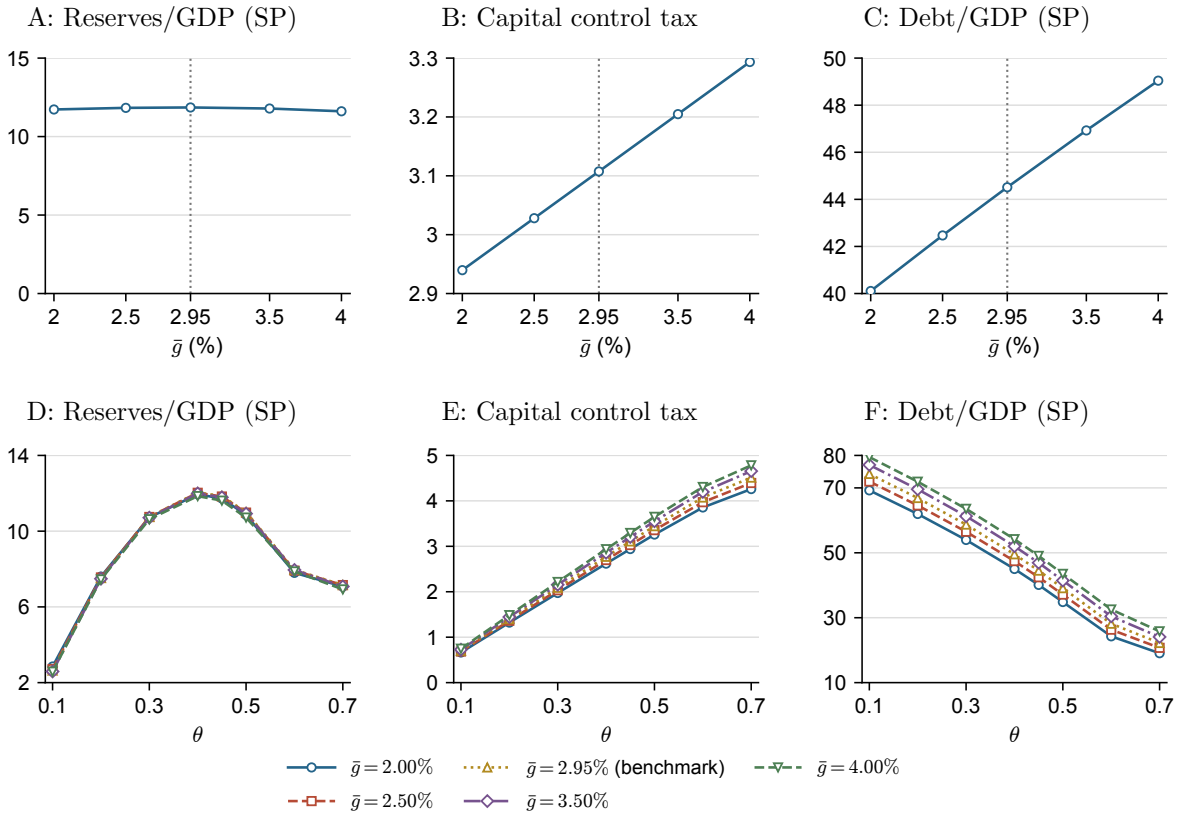
Note. The horizontal axis shows the liquidity shock probability, π, in percent. The panels report simulated mean reserve-to-GDP ratios, capital control tax rates, and debt-to-GDP ratios, respectively, all in percent. DE denotes the decentralized equilibrium without policy, and SP denotes the social planner's allocation. The capital control tax is zero in DE. The vertical dotted line marks the benchmark probability, $\pi = 5.84\%$.

Figure O5 Alternative reference debt ratios



Note. All panels report simulated means under the social planner (SP); ratios and tax rates are in percent. Panels A–C vary the reference debt-to-GDP ratio, $\bar{b}$, at the benchmark $\theta = 0.45$. The vertical dotted lines mark $\bar{b} = 0.5$. Panels D–F vary θ for $\bar{b} = 0.3$, 0.5, and 0.7, as indicated by the common legend. Other parameters remain at their benchmark values. Lower θ represents higher financial development.

Figure O6 Alternative growth calibrations



Note. All panels report simulated means under the social planner (SP); ratios and tax rates are in percent. Panels A–C vary $\bar{g}$ and its associated investment-efficiency parameter, η, at the benchmark $\theta = 0.45$. The vertical dotted lines mark $\bar{g} = 2.95\%$. Panels D–F vary θ under each of the five growth calibrations indicated by the common legend. Investment curvature, γ, and the remaining parameters stay at their benchmark values. Lower θ represents higher financial development.

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