# Welfare Gains from Market Insurance: The Case of Mexican Oil Price Risk *

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**Abstract**

Mexico has a long-standing practice of hedging oil price risk through the purchase of put options. We examine the resulting welfare gains using a standard sovereign default model calibrated to Mexican data. We show that hedging increases welfare by reducing income volatility and reducing risk spreads on sovereign debt. We find welfare gains equivalent to a permanent increase in consumption of 0.44 percent with 90 percent of these gains stemming from lower risk spreads.

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**JEL Classification:** F3; F4; G1

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# 1 Introduction

During the past two decades, oil prices have exhibited large swings: The rise and fall around the global financial crisis; the collapse during 2014-2016; hitting all-time lows and even negative levels early in the pandemic; to then surpass US$ 100 a barrel only two years later, following the Russian Invasion of Ukraine.

Confronted with this volatility, an important and recurrent question in policymakers' minds is how to best manage the fiscal risk of oil price movements. And the search for an answer often starts with Mexico. Following a long-standing practice, the Mexican Ministry of Finance typically purchases put options every year to hedge the price risk of a fraction of Mexico's oil production. The strike price approximates the oil price assumed in the fiscal budget with the government exercising the options in 2009, 2015, 2016, and 2020.

Historically, sharp changes in oil prices have coincided with substantial fluctuations in economic activity and inflation (Kilian and Murphy 2014, Husain et al. 2015, Fernández, Schmitt-Grohé, and Uribe 2020). For net oil exporters, the negative consequences of oil price declines are often amplified by rising risk spreads on sovereign debt (Baffes et al., 2015). Therefore, designing policies to manage risks emerging from the exposure to commodity-price swings remains an important policy and research question, particularly for commodity exporters (Borensztein, Jeanne, and Sandri 2013, Araya, Figueroa, Rosso, and Wagner 2020). Drawing on Mexico's experience, we assess the benefits and costs of using market insurance to hedge commodity price risk and enhance macroeconomic resilience. To this end, we augment a standard sovereign default model with access to put options—calibrated to Mexican data—to determine the size and main channels of welfare gains relative to a counterfactual scenario without put options.

Our main contribution is to analyze how the availability of hedging instruments affects default incentives and welfare. Rather than centering the attention on the optimal policy, we analyze a policy that is implementable, leveraging the Mexican experience. This approach preempts concerns that welfare gains may only be theoretical if the optimal hedging policy faces significant implementation challenges (e.g. political economy constraints).

Our benchmark small-open economy is exposed to the price risk of its exporting commodity (such as oil) and can borrow through one-period defaultable debt from risk-neutral foreign investors. The model follows a willingness-to-pay framework à la Eaton and Gersovitz (1981) and Arellano (2008). Since the country can default whenever optimal, bond prices fluctuate with the default risk. The country can also purchase put options from foreign investors to lock in a minimum commodity price in the subsequent period, which reduces the income risk and increases welfare. However, put options require an upfront cost which reduces current consumption. The tradeoff thus reflects the classical risk-sharing mechanism between the economy and foreign investors. In this framework, there is an additional effect from hedging as default incentives might change with the access to put options. When hedging reduces the incentive to default, it is also beneficial because the default is costly and incurs a pure deadweight loss. In a simplified two-period model, we establish the aforementioned effects of hedging analytically.

Our quantitative exercise calibrates the benchmark model to Mexico, a country that has used put options to hedge oil-price risk for more than two decades. We then analyze the effects of hedging by comparing the benchmark economy to a counterfactual one without hedging. We find that using put options yields welfare gains equivalent to a permanent 0.44 percent increase in consumption.[1] We then decompose the gains into two parts. The first emerges from the lower default incentives and thus borrowing cost. The second is similar to Lucas (1987), where a lower income risk translates into smoother consumption.[2] We find that about 90 percent of gains stem

[1]Welfare gains are the improvement in the present discounted value of the utility.

[2]The welfare gains from the income smoothing part reach 0.04 percent of consumption, which are very similar to what comes out of applying Lucas (1987)'s methodology to Mexican consumption series during 1996-2016.

from lower borrowing costs. Compared to the economy without hedging, risk spreads on debt are 19 basis points lower in the hedging economy.

Welfare gains from hedging are very robust. Our benchmark economy assumes that put options are priced actuarially fair. We conduct a robustness exercise by including a cost premium above the actuarially fair price to purchase put options. We find that only a sizable cost premium would reduce hedging gains to zero. We also find that welfare gains increase with the strike price of put options, the hedged volume of oil, the volatility of oil prices, and with risk aversion of foreign investors. Finally, we find that selling oil forward can generate larger gains than put options due to no upfront insurance cost. However, political economy may become more important especially as forwards mean giving up potential revenue windfalls if oil prices rise.[3]

Our paper contributes to the literature on welfare gains from market insurance with contributions including Caballero and Panageas (2008), which focus on optimal hedging strategies in countries facing risks of sudden stops in capital flows; and Borensztein et al. (2013) who examine the welfare gains from hedging through options and forwards in the presence of non-defaultable debt. Our paper differs from these studies by exploring synergies between hedging instruments and defaultable debt in increasing welfare. Furthermore, our paper is also related to studies examining the welfare gains from contingent debt, such as Hatchondo and Martinez (2012), which focus on GDP-indexed bonds, and Borensztein et al. (2017), which focus on catastrophic bonds. Our paper also contributes to the literature on oil prices and default risks such as Lopez-Martin et al. (2019) and Hamann et al. (2023). Lopez-Martin et al. (2019) analyzed the interaction of endogenous fiscal policy and commodity prices in a sovereign default model. They also analyze the welfare effects of alternative hedging instruments in lowering the volatility of macroeconomic

[3]Ecuador serves as an example. According to Daniel (2001), Ecuador's government conducted several hedging transactions through options and oil swaps in early 1993 that led to significant losses, ultimately triggering heavy criticism and even the appointment of a special committee to investigate allegations of corruption against the monetary authorities.

variables and their correlation with commodity revenues through fiscal adjustment. Different from their work, we focus on the tradeoff and welfare gains from macro hedging. We also conduct an in-depth analysis investigating the source of gains, especially through the relaxation of endogenous borrowing constraints imposed by sovereign default. Our analysis of macro-hedging for oil price shock is through the put options, which is different from Hamann et al. (2023) who study the effect of hedging through oil reserves management in a sovereign default model with oil extraction. They find that oil reserves can change the default incentive, which resembles our finding that macro-hedging can change the default incentives. Finally, our paper is also related to the literature on quantitative models of sovereign default such as Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), Mendoza and Yue (2012), although our focus is on the welfare gains from relying on hedging instruments as a complement to defaultable debt.

The paper is organized as follows: Section 2 describes Mexico's oil hedging program; Section 3 presents a two-period model to understand hedging; Section 4 presents the benchmark model; Section 5 presents quantitative results; Section 6 presents two extensions, and Section 7 concludes.

# 2 Mexico's Oil Hedging Program

Mexico's government has systematically hedged oil-price risk for more than twenty years through a hedging program that is known to be the largest in the world (Blass, 2017). The program, as it is known today, was set up in 2001 (Duclaud and Garcia, 2012), although Mexico used market hedging instruments as early as 1990 (Daniel 2001, Potts and Lippman 1991); however, details about those earlier operations are scarce. Note that Mexico is not the only country that has used financial derivatives to hedge against oil-price volatility. Oil-importing countries have also made use of these instruments to manage oil price risk. Panama has hedged the price risk of its imports of petroleum derivatives under the 2009 National Strategy for Hydrocarbons Risk Hedging (Consejo de Gabinete 2009). Uruguay did it in 2016 with technical assistance from the World Bank to hedge about 6 million barrels of oil (Navarro-Martin 2016). Other examples of oil-importing countries that have used financial derivatives to hedge oil-price risk include Morocco, Jamaica, and Ghana. UNCTAD (2019) notes that Ghana used call options during 2010-2012 to hedge the price risk of its oil imports and then also hedged its oil exports once it became an oil exporter in 2011/2012.

According to the U.S. International Energy Administration, as of 2022, Mexico was the world's $11^{th}$ largest crude oil producer. The oil sector is controlled by the fully state-owned company, Petroleos Mexicanos (PEMEX). Therefore, oil-related risks directly affect Mexico's public finances, which explains why the Mexican treasury conducts the hedging. On average, from 2000-2016,[4] oil-related revenues represented 32 percent of total fiscal revenues, of which, 47 percent corresponded to oil exports, and the remainder to net domestic sales of petroleum products. Over the same period, oil exports averaged 11 percent of total exports. While the importance of oil for the economy and Mexico's public finances has declined since the mid-2000s, oil revenues still represented about 22 percent of total fiscal revenues and 8 percent of total exports in 2022.[5] Moreover, there is a high negative correlation between risk spreads on external sovereign debt and oil prices, with a correlation coefficient of $-0.59$ over the past twenty years (Figure 1).[6] A 2013 constitutional reform opened the oil sector to private investment but the private oil sector remains in its infancy with PEMEX still concentrating more than 90 % of oil production in 2022.[7]

The Mexican treasury conducts hedging with the main objective of reducing the risk of fiscal revenue shortfalls during any given fiscal year. Specifically, the treasury includes in its annual

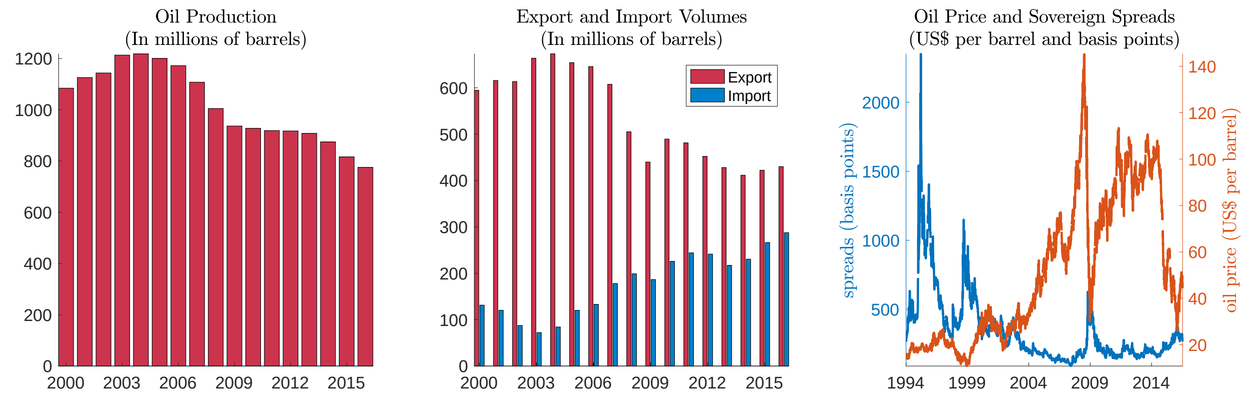
[4] We focus on this period because, in subsequent years, Mexico stopped publishing details about the hedging program to avoid influencing the cost of the options.

[5] The decline has been mainly the result of falling oil production due to aging oil fields and tax reforms that increased non-oil tax revenues.

[6] In a similar vein, Donders et al. (2018) find that corporate debt also responds to commodity prices.

[7] A description of the reform that opened the energy sector to private investment can be found in IMF (2014).

**Figure 1** Oil Production, Oil Prices, and Sovereign Spreads



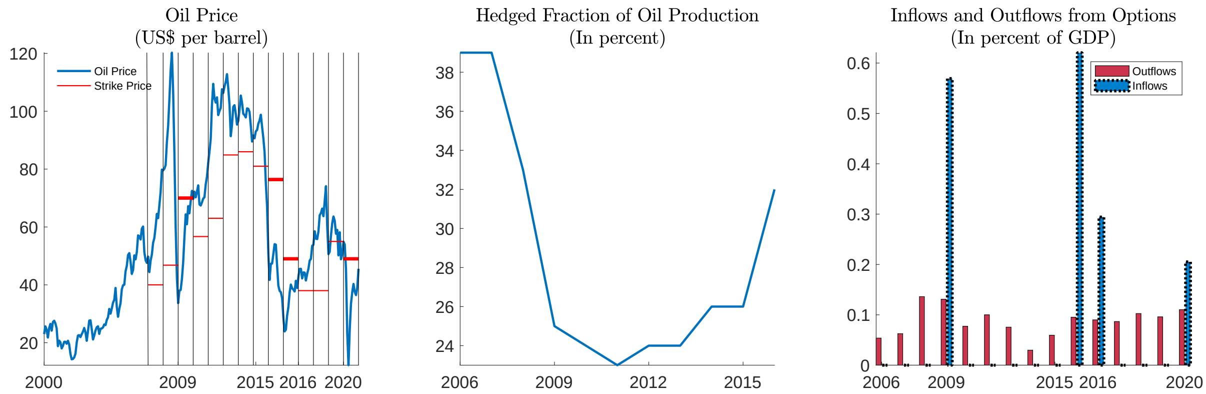

Source: INEGI.

budget an assumption on the export price of oil for the subsequent fiscal year, computed as the weighted average between historical prices and futures. To reduce the risk of a decline in oil-related revenues, the treasury purchases Asian put options with a strike price equal (close) to the oil price assumed in the budget. The use of Asian options allows the treasury to lock in a minimum price for the whole fiscal year.[8] The program is executed through several contracts with foreign banks as counterparts. Most of the contracts include Maya oil, a type of Mexican heavy crude oil, as an underlying asset, but a small fraction of contracts use Brent as the underlying asset. Maya oil dominates because it represents about 80 percent of Mexico's oil export volumes.

While on average, Mexico produced 1 billion barrels annually from 2000-2016, of which it exported roughly half, Mexico also imported about 178 million barrels of petroleum products annually, over the same period. The domestic sale of imported petroleum products at regulated prices, which did not move one-for-one with international prices, compensated losses (or gains) in

[8] An American or European put option is exercised if the spot price on a particular day is lower than the strike price. In contrast, an Asian put option is exercised if the average spot price for a pre-determined period, which in the case of Mexico is one year, is lower than the strike price. In this way, Mexico guarantees a minimum average price of oil for the whole fiscal year.

**Figure 2** Mexico's Oil Hedging Program




Note. Data is from INEGI. Mexican government stopped publishing the number of barrels hedged after 2016.

crude oil export revenues that resulted from fluctuations in international oil prices.[9] After taking these offsetting factors into account, the Mexican treasury hedged, on average, 29 percent of total production over 2006-2016.

The cost of the options has averaged 0.1 percent of GDP per year and they have been exercised only on four occasions: in 2009, 2015, 2016, and 2020 with payoffs reaching 0.5, 0.6, 0.3, and 0.2 percent of GDP respectively (Figure 2).

# 3 Hedging in a Two-period Model

We start with a simple two-period model, $t \in \{0,1\}$ to illustrate analytically the benefits and costs of hedging and its relationship with defaultable debt. Consumers choose in period 0 how many bonds $d$ to issue at a price $q$ to maximize the present discounted value of utility derived from consumption, with discount factor $\beta < 1$. Income is given by $y$ in period 0 while it can take values of $y^H$ or $y^L < y^H$ in period 1, with probabilities $p$ and $1-p$, respectively. After income uncertainty

[9] A process of liberalization of domestic fuel prices began in 2016 and was completed by end-2017.

is realized in period 1, consumers can default on their bonds, in which case income equals $y^{def} > 0$. The maximization problem is summarized by

$$\begin{aligned} U_0 &= \max_d \log c_0 + \beta E_0 \log c_1 \\ \text{s.t.} & \quad c_0 = qd + y \\ & \quad c_1^i = \max\left\{y^i - d, y^{def}\right\}, i \in \{H, L\} \end{aligned}$$

where for simplicity we assume $u(c) = \log c$. Assuming that the risk-free rate, $r^*$, equals zero, risk-neutral foreign investors price the bonds satisfying

$$q = \begin{cases} 1, & \text{if } y^L - d \geq y^{def}; \\ p, & \text{if } y^L - d < y^{def} \leq y^H - d; \\ 0, & \text{if } y^H - d < y^{def}, \end{cases}$$

where the first condition implies that risk spreads are zero because in those circumstances default is never optimal. The second condition states that consumers always default under a bad realization of income in period 1, in which case $q = p < 1$. Finally, the third condition implies that the bond is worthless since the consumers would default with probability 1 as it is always optimal to do so. We now introduce hedging in this framework. Suppose that the consumer buys insurance in period 0 that guarantees a level of income of at least $\bar{y}$ in period 1 at a cost $\xi$ that satisfies

$$\xi = \begin{cases} p(\bar{y} - y^H) + (1-p)(\bar{y} - y^L), & \text{if } \bar{y} \geq y^H; \\ (1-p)(\bar{y} - y^L), & \text{if } y^L < \bar{y} < y^H; \\ 0, & \text{if } \bar{y} \leq y^L. \end{cases}$$

Given the structure of put options, the problem for the economy becomes

$$\begin{aligned} U_0^{hedge} &= \max_d \log c_0 + \beta E_0 \log c_1 \\ \text{s.t.} \quad & c_0 = qd + y - \xi \\ & c_1^i = \max\left\{\max\{\bar{y}, y^i\} - d, y^{def}\right\}, i \in \{H, L\} \end{aligned}$$

**Role of Hedging.** Let us first assume that the insured level of income, $\bar{y}$, equals the unconditional mean of period-1 income, that is $\bar{y} = py^H + (1-p)y^L$. Hedging plays first an income-smoothing role by reducing income fluctuations in period 1 since $y^L < \bar{y} < y^H$ and with hedging, period-1 income is either $\bar{y}$ or $y^H$. Second, hedging can alter default and borrowing incentives, but not necessarily in an unambiguous way. In the following propositions, we demonstrate various implications of hedging for default incentives and welfare. We also want to stress that this exercise is to compare an economy with hedging and an economy without hedging. In particular, we want to understand whether buying nothing is better or worse than buying a specific quantity of put options with a specific strike price.

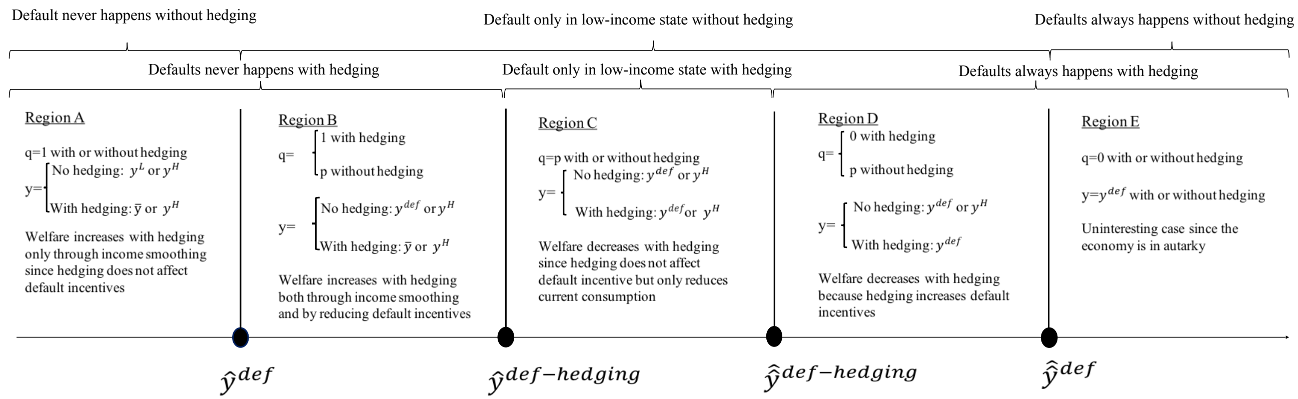
**Proposition 1. Default Incentives and Hedging**

*Consider an economy with no hedging in which $\hat{y}^{def}$ and $\hat{\hat{y}}^{def}$ are such that consumers never default if the income loss from default is too large, i.e. $y^{def} < \hat{y}^{def}$; they always default if the income loss from default is too small, i.e. $y^{def} > \hat{\hat{y}}^{def}$; and they only default after a low realization of income if the income loss from default is neither too large nor too small, $y^{def} \in (\hat{y}^{def}, \hat{\hat{y}}^{def})$. Introducing hedging in this economy increases $\hat{y}^{def}$ and reduces $\hat{\hat{y}}^{def}$.*

*Proof.* See Appendix B.1. □

Intuitively, Proposition 1 states that hedging can change default thresholds, by either increasing or decreasing default incentives. The direction in which those incentives change depends on how

**Figure 3** Two-period Model



costly it is to default. When default is so costly that it never happens (i.e. the farthest left region in Figure 3, $y^{def} < \hat{y}^{def}$), hedging does not affect default incentives. Reduce default costs a bit and we enter the middle region, i.e. $y^{def} \in (\hat{y}^{def}, \hat{\hat{y}}^{def})$, where changes in the default thresholds can lead the economy to never default or to always default. In the former case, the result follows from the fact that hedging helps secure a minimum income—above the default level—and therefore reduces incentives to default and the cost of debt. In the latter case, the income under default is higher, and therefore default is less costly. Because hedging requires increasing borrowing to pay for the upfront cost of insurance, it may worsen default incentives given that it is not so costly to default. The farthest right region in the Figure is for completeness only. In this region, the cost of default is so small that the economy would always default. Therefore, it is not an interesting case since no creditor would lend to consumers who would default with probability 1. In the following propositions, we analyze the implications for welfare under all cases except for the last one.

**Proposition 2. No Default in Equilibrium**

*When default is too costly, such that the economy does not default in equilibrium, introducing hedging increases social welfare and the country borrows more.*

*Proof.* See Appendix B.2. □

In this case, hedging is beneficial. Income becomes smoother and the economy can afford to borrow more. This insight is similar to the work by Borensztein et al. (2013) who derived welfare implications of hedging in a world with non-defaultable debt.

**Proposition 3. Default Only When Income is Low**

*When the economy defaults only when $y = y^L$, whether hedging increases or decreases welfare depends on its impact on default incentives:*

1. *If hedging reduces default incentives, hedging increases welfare, but borrowing might increase or decrease.*

2. *If hedging does not change default incentives, it reduces welfare and increases borrowing.*

3. *If hedging increases default incentives, both social welfare and borrowing decline.*

*Proof.* See Appendix B.3. □

In case 1, both the income-smoothing and borrowing cost channels imply a welfare gain of hedging despite the upfront insurance cost. However, the impact on borrowing is ambiguous: On the one hand, more borrowing is desirable to pay the insurance cost; on the other hand, more borrowing increases the default incentive and hence the borrowing cost.

In case 2, hedging ensures higher income in the low state of the world than in the absence of hedging, only if there is no default; however, if the economy defaults when $y = y^L$, hedging does not change default incentives, nor the level of income since default implies the same level of income under default, $y^{def}$, as the no-hedging economy. In this case, consumers borrow more in period 0 to purchase insurance, but income in period 1 is the same with or without hedging. As a result, hedging only lowers current disposable income and reduces welfare.

In case 3, if hedging increases default incentives it reduces welfare since it would imply that the economy moves from the region where it only defaults in the bad state to the region where it always defaults.

Figure 3 summarizes key insights from the above propositions. The left regions in the figure correspond to areas where the cost of default is high. In these regions, hedging is always desirable either because both, the borrowing costs channel and the income smoothing channel are at work, which is the case when hedging reduces default risk, or because only the income smoothing channel is at work, which is the case when there is no default in equilibrium. The model also includes regions where hedging reduces welfare because the costs of default are small. However, the fact that defaults are rare events—Mexico has defaulted only 8 times since 1821—and the empirical literature documents significant output losses following sovereign defaults, the left regions in Figure 3 are likely to be the more empirically relevant cases. We resort now to our quantitative analysis to shed light on the size of welfare gains from hedging.

# 4 Model Economy

Our quantitative exercise is based on a standard sovereign default model as in Aguiar and Gopinath (2006) and Arellano (2008). The only departure is to introduce hedging as in Borensztein et al. (2013, 2017) who did so in models with non-defaultable debt. In the economy, a country can issue one-period bonds in international markets on which the country can default when it finds it beneficial to do so. But the default is costly, including losing access to international markets, although not permanently, and a lower income. There is only one source of risk: oil prices. In addition to issuing defaultable debt, the country can purchase put options to hedge oil price risk. The quantitative assessment of the welfare gains from using put options will be conducted by comparing economies with and without put options.

## 4.1 Benchmark Model with Defaultable Debt and Put Options

The economy is populated by infinitely-living, risk-averse representative agents who make decisions to maximize the expected present discounted value—with discount factor $\beta$—of the utility derived from consumption:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

where the preferences of the consumer are represented by a standard constant relative risk aversion (CRRA) utility function with a coefficient of risk aversion $\gamma$.

Total income ($Y_t$) has two components: non-oil income ($F_t$) and oil income ($X_t$),

$$Y_t = F_t + X_t \equiv F_t + p_t Q_t \tag{2}$$

where $p_t$ and $Q_t$ are the price and quantity of oil production respectively. The only source of risk is from the price of oil, $p_t$, which is assumed to follow an autoregressive stochastic process, to be defined momentarily. We assume that non-oil income, $F_t$, grows deterministically at a constant rate $G$ in every period. We normalize all variables by $F_t$ and denote them with lower letters:

$$y_t \equiv \frac{Y_t}{F_t} = 1 + p_t \frac{Q_t}{F_t} \equiv 1 + p_t Q \tag{3}$$

$$E_0 \left[ \sum_{t=0}^{\infty} (\beta G^{1-\gamma})^t \frac{c_t^{1-\gamma}}{1-\gamma} \right] \tag{4}$$

where $Q = \frac{Q_t}{F_t}$ and $c_t = \frac{C_t}{F_t}$ denote normalized oil production and consumption. To further simplify the exposition, we assume that $Q$ is constant, which as we will discuss in the calibration section, is not an inaccurate representation of the data. From now on we will focus on the normalized problem knowing that the original problem can always be recovered by multiplying normalized variables by $F_t$ (See Appendix A for details).

In every period, agents (or consumers) have an initial level of wealth, $w_t$, composed of income, $y_t$, and bonds, $b_t$, acquired in the previous period: $w_t = y_t + b_t$. Consumers allocate this wealth among consumption, $c_t$; zero-coupon one-period bonds,[10] $b_{t+1}$, with a price $q_t$; and put options

[10] We focus on one-period bonds rather than long-term bonds for simplicity. An important limitation of the one-period bond model with sovereign default is a seemingly low value of discount factor $\beta$, which is an important parameter for the welfare analysis. However, solving sovereign default models with long-term bonds is computationally challenging as it is well known that the value function iteration methods do not always ensure convergence. To miti-

acquired from international financial markets at a unit price $\xi(\bar{p}_t)$, which entitles them to sell a fraction $\alpha Q$ of oil production in period $t+1$ at a pre-determined strike price $\bar{p}_t$.[11][12]

$$c_t + q_t G b_{t+1} + \alpha Q G \xi(\bar{p}_t) = w_t \tag{5}$$

where $b_{t+1}$ can take positive or negative values reflecting whether the country lends or borrows in international markets. The consumer arrives to the next period, $t+1$, with wealth $w_{t+1}$, given by

$$w_{t+1} = y_{t+1} + \alpha Q \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1} \tag{6}$$

where $\alpha Q \max\{\bar{p}_t - p_{t+1}, 0\}$ reflects the fact that put options locked in a minimum price, $\bar{p}_t$, for the hedged fraction of production. The optimization problem, under no default, is given by

$$\begin{aligned} V^c(w_t, p_t) \quad &= \quad \max_{c_t, b_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} E_t \left[ V(w_{t+1}, p_{t+1}) \right] \\ \text{s.t.} \quad & \quad c_t + q_t G b_{t+1} + \alpha Q G \xi(\bar{p}_t) = w_t \\ & \quad w_{t+1} = y_{t+1} + \alpha Q \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1} \end{aligned} \tag{7}$$

where $V^c(w_t, p_t)$ denotes the value function under continuation or no default, with the state of the economy summarized by two state variables, $\{w_t, p_t\}$.

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gate concerns, we conduct a robustness analysis on $\beta$ and include a related discussion later. Most importantly, the key economic insights of our exercise do not change qualitatively if we were to introduce long-term bonds. The reason for this conjecture is that long-term bond prices would incorporate the risk of default in all future periods making it more costly to borrow than under one-period bonds. Therefore, hedging can lead to higher gains than in our one-period bond model if hedging further relaxes the borrowing constraint. Our baseline calculation with one-period bonds can be interpreted as a lower bound for the hedging gains.

[11] $\alpha \in (0, 1)$ captures the fact that Mexico hedges only part of the production, as discussed in the previous section.

[12] The oil put options in our economy are equivalent to GDP put options since the non-oil income is non-stochastic.

**Default Decision.** In every period, consumers can default on their debt, in which case the economy gets excluded from international financial markets. In default, consumers cannot borrow nor purchase put options, and the economy resorts to financial autarky. Furthermore, consumers cannot exercise the put options and then default. These restrictions on exercising and purchasing put options can be motivated by sanctions in the state of default. Besides the exclusion from international financial markets, default implies an income loss $h(y_t)$ in every period, reflecting the assumption that credit plays an essential role in the economy. This assumption can be justified by the existence of a minimum scale for some investment projects that would not be reached without external financing, preventing those investments from being carried out. Alternatively, the exclusion from international financial markets may obstruct the normal conduct of business of companies operating with non-residents by reducing or eliminating access to financial services that may be essential to their activity, such as trade finance. The assumption ultimately aims at capturing output losses often linked to sovereign default episodes (e.g. Laeven and Valencia, 2013 and Gornemann, 2014). A default status does not imply permanent exclusion from financial markets. It is assumed that in any given period, there is a probability $\lambda \in (0,1)$ that the economy is "redeemed" and re-enters international credit markets with zero net assets.

Denoting the value function under default as $V_t^d(p_t)$, the problem under default is given by

$$\begin{aligned} V^d(p_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma}\left[\lambda E_t V(w_{t+1}, p_{t+1}) + (1-\lambda) E_t V^d(p_{t+1})\right] \\ \text{s.t.} \quad & c_t = y_t - h(y_t) \\ & w_{t+1} = y_{t+1} \end{aligned} \tag{8}$$

where $V(w_t, p_t) = \max\left(V^c(w_t, p_t), V^d(p_t)\right)$. Default happens if and only if $V^d(p_t) > V^c(w_t, p_t)$.

**Risk-Neutral Foreign Investors.** We assume that there is a continuum of risk-neutral foreign investors who can purchase bonds or sell put options to consumers. If default happens, foreign investors do not recover any value from the bonds and renege to honor the put options. However, for simplicity, any value foreign investors recover by reneging to honor the options is assumed to be consumed in transaction costs or legal fees. Consequently, recovery values are assumed at zero in the pricing of the bonds. Note that this assumption may ultimately understate the welfare gains from hedging as these recoveries if not zero, could lead to lower risk spreads on debt.

Denoting $r^*$ the world risk-free rate and $D(w_{t+1}, p_{t+1})$ an indicator default function which equals one if default happens and zero otherwise, no-arbitrage conditions require that

$$
\begin{aligned}
q(b_{t+1}, p_t) &= \frac{E_t\left[1 - D(\underbrace{y_{t+1} + \alpha Q \max\{\bar{p}_t - p_{t+1}, 0\} + b_{t+1}}_{w_{t+1}}, p_{t+1})\right]}{1 + r^*} \\
\xi(\bar{p}_t) &= \frac{E_t[\max\{\bar{p}_t - p_{t+1}, 0\}]}{1 + r^*}
\end{aligned}
\tag{9}
$$

The above equations imply that the expected return to the foreign investor from holding bonds or being the counterpart of a put option is equalized and given by the risk-free return. Hedging income appears in the default function, affecting the price of bonds and the risk spreads.

## 4.2 An Economy without Put Options

The benchmark economy includes the availability of put options because the model will be calibrated to Mexican data over a period where Mexico hedged oil price risk through these instruments. To quantify gains from hedging, we set up a counterfactual economy with no access to put options. To differentiate these two economies, we use a tilde over variables that correspond to the no-hedging economy. State variables $\{w_t, p_t\}$ are defined in the same way as before, and value functions are given by

$$\tilde{V}(w_t, p_t) = \max\left(\tilde{V}^c(w_t, p_t), \tilde{V}^d(p_t)\right)$$

where $\tilde{V}^c(w_t, p_t)$ and $\tilde{V}^d(p_t)$ denote the value functions of continuation and default respectively.

As before, the problem under no default or continuation is given by

$$\begin{aligned} \tilde{V}^c(w_t, p_t) &= \max_{c_t, b_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma} E_t\left[\tilde{V}(w_{t+1}, p_{t+1})\right] \\ \text{s.t.} &\quad c_t + \tilde{q}_t G b_{t+1} = w_t, \\ &\quad w_{t+1} = y_{t+1} + b_{t+1}, \end{aligned} \tag{10}$$

where $\tilde{q}_t$ is the bond price in the no-hedging economy. Note also the absence of terms related to the purchase and exercise of put options in the budget constraint. Under default, we have

$$\tilde{V}^d(p_t) = \frac{[y_t - h(y_t)]^{1-\gamma}}{1-\gamma} + \beta G^{1-\gamma}\left[\lambda E_t \tilde{V}(w_{t+1}, p_{t+1}) + (1-\lambda) E_t \tilde{V}^d(p_{t+1})\right]. \tag{11}$$

The pricing of bonds follows the same structure as before:

$$\tilde{q}(b_{t+1}, p_t) = \frac{E_t\left[1 - \tilde{D}(y_{t+1} + b_{t+1}, p_{t+1})\right]}{1 + r^*} \tag{12}$$

## 4.3 Recursive Equilibrium

As standard in the sovereign default literature, we solve the problem from the perspective of a benevolent government, which makes the decision on behalf of private agents in the economy. In what follows we define the recursive equilibrium in this economy.

**Definition.** *Markov Perfect Equilibrium*

1. *The Markov Perfect Equilibrium of our benchmark economy is characterized by a set of*

*value functions* $\{V(w_t, p_t), V^c(w_t, p_t), V^d(w_t, p_t)\}$*, default function* $D(w_t, p_t)$*, consumption function* $c_t$*, next period bond holding* $b_{t+1}$*, and bond price* $q_t$ *such that given the state variables* $\{w_t, p_t\}$*, the cost of put options* $\xi(\bar{p}_t)$*, and the strike price* $\bar{p}_t$*, they solve the optimization problems (7) and (8). Furthermore,* $V(w_t, p_t) = \max\left(V^c(w_t, p_t), V^d(p_t)\right)$ *and the price* $q_t$ *satisfies equation (9).*

2. *The Markov Perfect Equilibrium of the economy without put options is characterized by a set of value functions* $\{\tilde{V}(w_t, p_t), \tilde{V}^c(w_t, p_t), \tilde{V}^d(w_t, p_t)\}$*, default function* $\tilde{D}(w_t, p_t)$*, consumption function* $\tilde{c}_t$*, next period bond holding* $\tilde{b}_{t+1}$*, and bond price* $\tilde{q}_t$ *such that given the state variables* $\{w_t, p_t\}$*, they solve the optimization problems (10) and (11). Furthermore,* $\tilde{V}(w_t, p_t) = \max\left(\tilde{V}^c(w_t, p_t), \tilde{V}^d(p_t)\right)$ *and the price* $\tilde{q}_t$ *satisfies equation (12).*

# 5 Quantitative Analysis

## 5.1 Calibration

We calibrate the benchmark model to Mexican data over 1996-2016, a period during which Mexico used put options to hedge oil price risk and published details on the cost of the options. While Mexico continued the hedging program in subsequent years, it discontinued the publication of the unit cost (per barrel) of the options to avoid influencing the market conditions (CEFP, 2020). The benchmark model has 13 parameters, which we split into three groups before assigning values.

The first group of parameters, comprising $\{r^*, \gamma, \lambda, p, \rho, \sigma, Q, \alpha, G\}$, are directly taken from the literature or data. The real risk-free interest rate, $r^*$, equals the average over 1996-2016 of the nominal yield on 1-year U.S. treasury bills, converted to real terms using the U.S. GDP deflator, resulting in a value of 0.64 percent. The risk aversion parameter, $\gamma$, is set at 2, a standard value in the literature. The probability of returning to international financial markets after default, $\lambda$, is calibrated to match the duration of default episodes for Mexico. To get this number, we examine a much longer period, covering 1821-2016, over which Mexico defaulted 8 times. On average, the duration of default episodes is 9.38 years, which suggests a value of $\lambda$ equals 0.11.[13] The parameters of the oil-price process, $p, \rho, \sigma$, are obtained from estimating a log AR(1) process

$$\log p_t = (1-\rho)\left[\log(p) - \frac{1}{2}\frac{\sigma^2}{1-\rho^2}\right] + \rho \log p_{t-1} + \varepsilon_t \tag{13}$$

where the unconditional mean $p$, the persistence parameter $\rho$, and volatility $\sigma$ are estimated using a Maximum Likelihood Estimator (MLE) over the period 1996-2016. The oil price dynamics $\{\rho, \sigma\}$ are important for our welfare analysis. We calibrate the oil price as follows. We use an *implied* oil price faced by Mexico, i.e. $p_t = \frac{X_t}{Q_t}$ with $X_t$ and $Q_t$ being the oil-related GDP (in real term) and oil production. The implied oil price better captures the actual price risk faced by Mexico. As discussed in Section 2, Mexico produces oil but only exports half of it. Moreover, Mexico also imports petroleum derivatives to satisfy domestic demand. In this situation, simply using the Mexican oil price for calibration masks the true external risk faced by Mexicans. For this reason, we used the implied oil price dynamics as our benchmark result to calculate welfare gains, which yields an estimation of $\{(\rho, \sigma) = (0.71, 0.25)\}$. We also conduct a robustness test on these parameters later. To complete the calibration of the income process, we use actual quantities of Mexican oil production, which over 1996-2016 averaged 1.03 billion barrels per year. Non-oil income, $F_t$, is approximated by non-oil GDP—measured as total Mexican GDP after subtracting oil and gas extraction. Since the model is written in terms of one tradable good, we convert $F_t$ to U.S. dollars using market exchange rates, and then to real terms using the U.S. GDP deflator. The

[13]The default data are taken from Carmen Reinhart's website. See http://www.carmenreinhart.com/data/browse-by-topic/topics/7/.

deterministic annual real growth rate, $G$, of non-oil income is computed as the average growth of non-oil income over 1996-2016, resulting in a value of 3.13 percent. We calculate the normalized value of oil production, $Q$, by dividing oil GDP by non-oil income and the oil price. In the model, we are assuming that this ratio is constant, equal to 0.1 percent, which is not a significant departure from the data. This ratio was fairly stable over 1996-2016. The fraction of oil production hedged, $\alpha$, is set at 29 percent, which corresponds to the average fraction of production hedged by Mexico from 2006-2016. We consider this range only because of the lack of publicly available data on the actual fraction hedged before 2006.

The second group of parameters, comprising $\{\beta, y^*\}$, is chosen to match relevant moments in the data. In selecting the output loss from default, we follow Arellano (2008) which adopts an asymmetric output cost function that matches default rates and spreads within the range seen in the data.[14] To this end, the output loss function is given by

$$h(y_t) = \begin{cases} y_t - y^*, & \text{if } y_t \geq y^*, \\ 0, & \text{if } y_t < y^*. \end{cases}$$

We choose $\beta$ and $y^*$ to match two empirical moments: (1) the Mexican government's gross financing needs—defined as the overall fiscal deficit in any given year plus debt rollover needs—to non-oil fiscal revenue ratio over 2006-2016, of 11.90 percent, a definition of debt that most closely matches the definition of debt in the model; (2) the average risk spreads on sovereign debt over

[14] We conduct robustness analysis using an alternative default cost function, i.e. $h(y) = \max\{0, \lambda_0 y + \lambda_1 y^2\}$ following Chatterjee and Eyigungor (2012). We use $\{\beta, \lambda_0, \lambda_1\} = \{0.72, 1.04, -0.98\}$ to target the debt ratio (11.90%), sovereign spreads (1.48%), and default probability (2%). Notice that we did not target the default probability in our baseline calibration because the default cost function in Arellano (2008) only has one parameter $y^*$. Quantitatively, all the results are robust. Notice that this cost function features a value of $\beta$ at 0.72, comparable to our baseline calibration of 0.76 but also low. This suggests that a different default cost function alone cannot generate a high discount factor. This is consistent with Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2017), who also find that sovereign default models with one-period bonds cannot match the debt levels and sovereign spreads simultaneously with a high $\beta$.

2000-2016, 1.48 percent. Risk spreads are calculated as the difference between the yield in dollars on Mexico's 1-year government bonds and the yield on U.S. 1-year treasury bills. We compute the average over 2000-2016 to avoid distortions from the sharp increase in spreads around the Tequila crisis of 1995.

The last group of parameters includes the cost and strike price of the options, $\{\xi(\bar{p}_t), \bar{p}_t\}$. As discussed in Section 4 the price is determined by a risk-neutral pricing condition and therefore emerges endogenously once other parameters in the model have been determined. The exact implementation of the pricing function is given in Appendix C. The strike price is chosen to match the conditional mean of the oil price. While we have data for the actual strike price chosen by Mexico, the sample is too short to estimate a robust empirical relationship between the strike price and the actual oil price. Instead, we proceed as follows. First, we assume that $\bar{p}_t = \mu E_t[p_{t+1}|p_t]$, to capture Mexico's actual choice for the strike price, which intends to be close to the oil price assumed in the budget for the subsequent year. This budget oil price is in turn determined by a weighted average between past and future prices implied by forward contracts, which aims at capturing the long-run price of oil, given current market conditions. Second, we choose a value of $\mu$ such that the simulated long-run probability of exercising the options is 18.75 percent, consistent with the fact that between 2001 and 2016, the Mexican government exercised the options only 3 times. The approach yields a value of $\mu$ equal to 0.77. To cross-check that this approach does not result in a number significantly different from the one implied by the data, we compute $\mu$ directly from the data by dividing the actual strike price by the average oil price in the year when the options were purchased (Table 1). This alternative approach returns an average value for $\mu$ of 0.85, close to the value of 0.77 used in the baseline calibration.

Finally, it is important to note that a period in this model corresponds to one year, and all values are expressed in 2009 constant U.S. dollar terms. All parameter values are reported in Table

**Table 1** Actual Strike Prices from Options

| Years | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | Average |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Strike price ($) | 40.00 | 46.80 | 70.00 | 56.69 | 63.00 | 84.90 | 86.00 | 81.00 | 76.40 | 49.00 | 38.00 | 62.89 |
| Conditional mean ($) | 51.75 | 53.98 | 88.86 | 54.82 | 69.93 | 91.86 | 103.87 | 99.76 | 93.80 | 55.29 | 51.69 | 74.15 |
| Ratio ($\mu$) | 0.77 | 0.87 | 0.79 | 1.03 | 0.90 | 0.92 | 0.83 | 0.81 | 0.81 | 0.89 | 0.74 | 0.85 |

Source: Auditoria Superior Federal and authors' calculations.

2. The bottom part of the table shows that the model-simulated moments are very close to their data counterparts. It is worth noting also that while our discount factor, at 0.76, appears low for an annual frequency, values in this range are found in the literature, for example, Yue (2010) chooses a discount rate at 0.72. It is well-known that sovereign default models with one-period bonds have difficulty in matching both default spreads and debt ratios simultaneously. To achieve both goals, we have to pick a lower value for the discount factor.[15][16]

We solve the model numerically using value function iteration. We use the Rouwenhorst method as in Kopecky and Suen (2010) to determine the grid for oil prices. Specifically, we use 21 and 500 grids to approximate oil prices and bond holdings respectively.

## 5.2 Welfare Gains from Hedging

We measure welfare gains from hedging by comparing the utility derived from the stream of consumption under the benchmark economy, and the one in the no-hedging economy. We follow the

[15] One rationale from a low $\beta$ is from a political economy interpretation, which reflects a short-sighted government.

[16] It is important to note that judging whether a discount factor is too low or too high is not simple. There is ample evidence of substantial heterogeneity in preferences across countries. For example, Falk et al. (2018) developed an experimentally validated survey dataset (the Global Preference Survey, GPS) of time preference, risk preference, positive and negative reciprocity, altruism, and trust from 80,000 people in 76 countries. They document substantial heterogeneity in preferences, including patience, with variation across countries explained by factors ranging from per-capita income to entrepreneurial activities, to the frequency of armed conflicts. Other studies with qualitatively similar conclusions, pointing to significant differences in patience across countries, include Wang et al. (2016) and Nieminen (2022). All these papers present evidence consistent with a country like Mexico, given its economic and social development, exhibiting much more impatient than a more developed counterpart.

**Table 2** Calibration

| Parameters | Value | Source |
|---|---|---|
| Risk-free rate | $r^* = 0.64\%$ | U.S. real interest rate |
| Risk aversion | $\gamma = 2$ | Standard value |
| Probability of redemption | $\lambda = 0.11$ | Average years in default |
| Growth rate | $G = 1.0313$ | Data |
| Unconditional mean | $p = 54.60$ | Data |
| Persistence | $\rho = 0.71$ | Data |
| Volatility | $\sigma = 0.25$ | Data |
| Oil to non-oil GDP ratio | $pQ = 6\%$ | Data |
| Strike price | $\bar{p}_t = \mu E_t[p_{t+1}\|p_t] = 0.77E_t[p_{t+1}\|p_t]$ | Prob of exercising options |
| Hedging share | $\alpha = 0.29$ | Data |
| Hedging cost | $\xi(\bar{p}_t)$ | Risk-neutral pricing |
| Discount rate | $\beta = 0.76$ | Match debt ratio |
| Output loss function | $y^* = 0.98E[y] = 1.03$ | Match spreads |
| **Target Moments** | **Data** | **Model Simulation** |
| Debt-GDP ratio | 11.90 % | 11.97 % |
| Sovereign spreads | 1.48% | 1.40 % |

Source: INEGI, Federal Reserve Board of Governors, and authors' calculations.

standard convention in the literature of expressing welfare gains in terms of a permanent increase in annual consumption. Formally, the definition is given in equation (14).

$$\Delta(w_t, p_t) = 100 * \left[ \left( \frac{V(w_t, p_t)}{\tilde{V}(w_t, p_t)} \right)^{\frac{1}{1-\gamma}} - 1 \right] \tag{14}$$

Under this definition, welfare gains are conditional on the values of the state variables, $\{w_t, p_t\}$; therefore, we refer to $\Delta(w_t, p_t)$ as conditional welfare gains.[17] Furthermore, we also define unconditional welfare gains by $E[\Delta(w_t, p_t)]$, where the expectation is taken with respect to the state variables using their ergodic distribution under the benchmark economy.[18] To compute the welfare gains, we first run 100 Monte Carlo simulations of 2,000 periods each for the benchmark economy. We draw oil prices from the estimated stochastic process, given some initial price. This initial condition, together with one for wealth, and the optimal solutions for consumption and borrowing

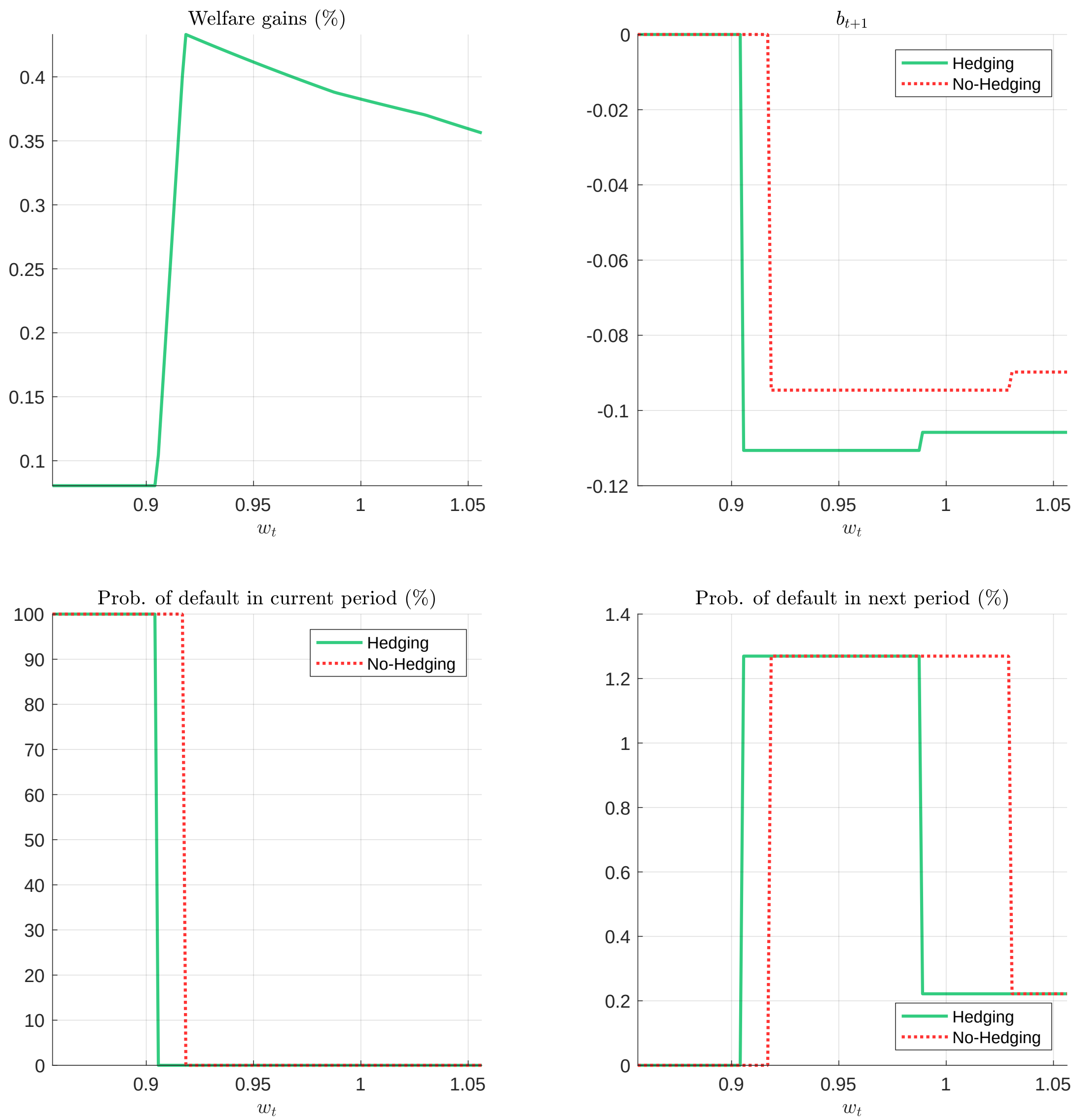
[17]Note that $\Delta(w_t, p_t)$ does not depend on any particular time $t$. We keep the time script $t$ for consistency of notation.

[18]Using instead the ergodic distribution under the economy without hedging yields similar outcomes.

determine the optimal value of these variables for the current period. We then check if a default is optimal or not, to then proceed to use the law of motion for wealth and oil prices to determine the value of the state variables for the subsequent period and so on. We repeat this process until we reach 2,000 periods. We throw away the initial 500 periods and approximate welfare—or the value function—by computing the present discounted value of the utility derived from the simulated path for consumption. We construct the counterpart value function for the economy without hedging, using the same procedure and initial conditions for $w_t$ and $p_t$.

In Figure 4 we show the conditional welfare gains $\Delta(w_t, p_t)$; bond purchase/sale $b_{t+1}$; the probability of default in the current period $t$; and the probability of default in the next period $t+1$, given by $E[D(w_{t+1}, p_{t+1})|p_t]$; for different values of state variable $w_t$, after setting the price of oil, $p_t$, to its unconditional mean. These conditional welfare gains vary from 0 to a 0.45 percent permanent increase in consumption. When the economy has less wealth to start, default incentives are strong, the probability of default in the current period $t$ is high, and welfare gains from hedging are small. In this region, hedging does little to improve welfare since default happens regardless, analogous to the result in the farthest right region of Figure 3. When the economy is less indebted, default incentives weaken, and the probability of default declines, but more quickly for the economy with hedging than for the one without it. For values of wealth $w_t$ between 0.91 and 0.92, the economy without hedging defaults in the current period, but the economy with hedging does not. This region is analogous to region B in Figure 3. In this region, welfare jumps from close to 0 to 0.43 percent. As wealth increases, welfare gains decline as default becomes less and less relevant. At some point, even the no-hedging economy does not default in the current period and it has the same default probability in the next period as the hedging economy. Welfare declines further since hedging is costly and its benefit through a reduction in borrowing costs is much lower. This result is analogous to the result depicted in the left regions of Figure 3.

**Figure 4** Welfare Gains, Borrowing, and Probability of Default




Note: Conditional welfare gains, $\Delta(w_t, p_t)$; borrowing, $b_{t+1}$; the probability of default in the current period and in the next period are plotted against values of wealth, $w_t$ with $p_t$ set equal to its unconditional mean.

We also find unconditional welfare gains equivalent to a permanent increase in annual consumption of 0.44 percent. These gains are within the range found by related studies. Borensztein et al. (2017) finds that the unconditional gains of catastrophe (CAT) bonds, in the presence of defaultable debt, are typically small: less than 0.12 percent. They rationalize their results by claiming that the CAT bonds do not change the default threshold. Hatchondo and Martinez (2012) explore the welfare gains from issuing GDP-indexed bonds in a model with defaultable debt. They find that GDP-indexed bonds could change the default threshold and find welfare gains equivalent to a permanent increase in consumption of 0.46 percent.

We also compare our gains with a default-free debt model under an exogenous borrowing limit as in Borensztein et al. (2013). We find that welfare gains of put options increase to 1.21%, which is comparable to the calculation of 1-4% in section 5.1 of Borensztein et al. (2013) for welfare gains from put options.

Gains are higher under natural borrowing limits and default-free debt than gains (0.44%) in our baseline model with endogenous borrowing limits from defaultable debts.[19] This is because defaultable debts act as a form of insurance, that is, the country strategically defaults whenever it is optimal to do so. Therefore, options and defaultable debts serve as substitute mechanisms for consumption smoothing. As a result, gains are smaller in the model with defaultable debts (and endogenous borrowing limit) than default-free debts (and exogenous natural borrowing limit).

The quantitative results of lower welfare gains from hedging in a model with defaultable debts than a model with default-free debts are consistent with Borensztein et al. (2013). In section 5.2 of their paper, they calculate the welfare gains from using forward contracts in the presence of defaultable debt to obtain gains of around 0.8%, lower than their baseline calculation with default-free debt of more than 1%.

**Source of Welfare Gains.** As discussed in the context of the 2-period model, we explore two

[19]Note that comparing hedging gains in a model with default vs. non-default is unfair because defaults are costly and affect the sovereign risk premium. Yet, this exercise is still useful for understanding the source of gains and facilitates comparison with existing literature such as Borensztein et al. (2013).

**Table 3** Stochastic Steady State in the Hedging and No-hedging Economies

| Economy | Debt Ratio | Default Spreads | Default Probability |
|---|---|---|---|
| Hedging | 11.97 % | 1.40 % | 1.27 % |
| No-Hedging | 10.50 % | 1.59 % | 1.41 % |

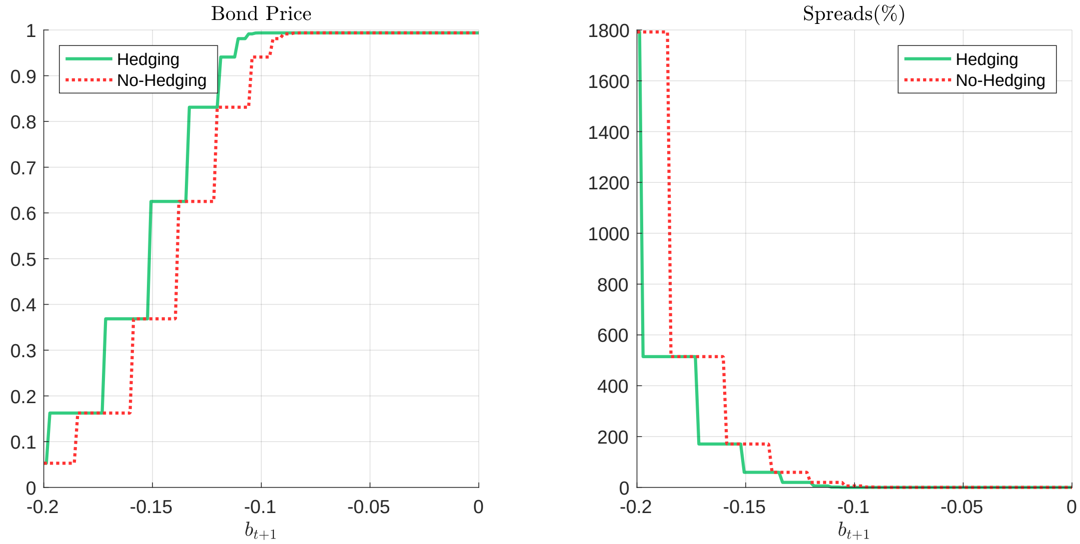
Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, conditional on no default for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods.

channels, one through income smoothing and the other through default incentives, which ultimately affect borrowing costs. The latter channel can already be seen in Table 3, where we compare the stochastic steady state—defined as the average value of the corresponding variables in the long-run simulations—of the benchmark model with the one from the model without hedging. We find that the probability of default is higher in the model without hedging, 1.41 percent versus 1.27 percent, default spreads are also higher, 1.59 percent versus 1.4 percent, and the debt level is lower, 10.50 percent versus 11.97 percent. Recall that proposition 3 implied that the impact of hedging on the debt level was ambiguous; however, our quantitative results suggest that debt increases with hedging. It increases due to a lower borrowing cost and also a stronger incentive to borrow because of the additional borrowing needed to purchase the put options.

Figure 5 shows bond prices and spreads and highlights that the model with hedging has systematically higher prices, except in the region where default risk is zero in which case bond prices equal 1 for both the benchmark and the no-hedging model. Such a higher bond price from hedging contributes to the calculated welfare gains as the borrowing cost is lower.

To decompose more explicitly the borrowing cost and income-smoothing channels of welfare gains, we solve the model without hedging after imposing the same bond price that emerges in the economy with hedging. Note that now we have two versions of the no-hedging economy. One that is solved as if bond prices were the same as if hedging was present, and the standard one

**Figure 5** Bond Price and Sovereign Spreads




Note: Bond prices, $q_t$, and spreads are plotted as a function of $w_t$, with $p_t$ set equal to its unconditional mean. Spreads are computed as the difference between bond yields and the international risk-free rate, $100\left(\frac{1}{q_t} - 1 - r^*\right)$.

where bond prices correspond to the no-hedging world. Since the only difference between these two models is the borrowing cost, the resulting welfare gains stem entirely from the borrowing costs channel.[20] Our simulation suggests that the gains for the no-hedging economy to issue bonds under the price in the hedging economy are equivalent to a permanent increase in consumption of 0.40 percent, that is, 90 percent of the total welfare gains from hedging.

To gain further intuition, we examine the dynamic behavior of key variables around default episodes in Figure 6. Specifically, we construct an 11,000-period simulation for the hedging and no-hedging economies. After dropping the first 1,000 periods, we identify default episodes in the no-hedging economy in the remaining 10,000 periods. We construct a 20-period window centered

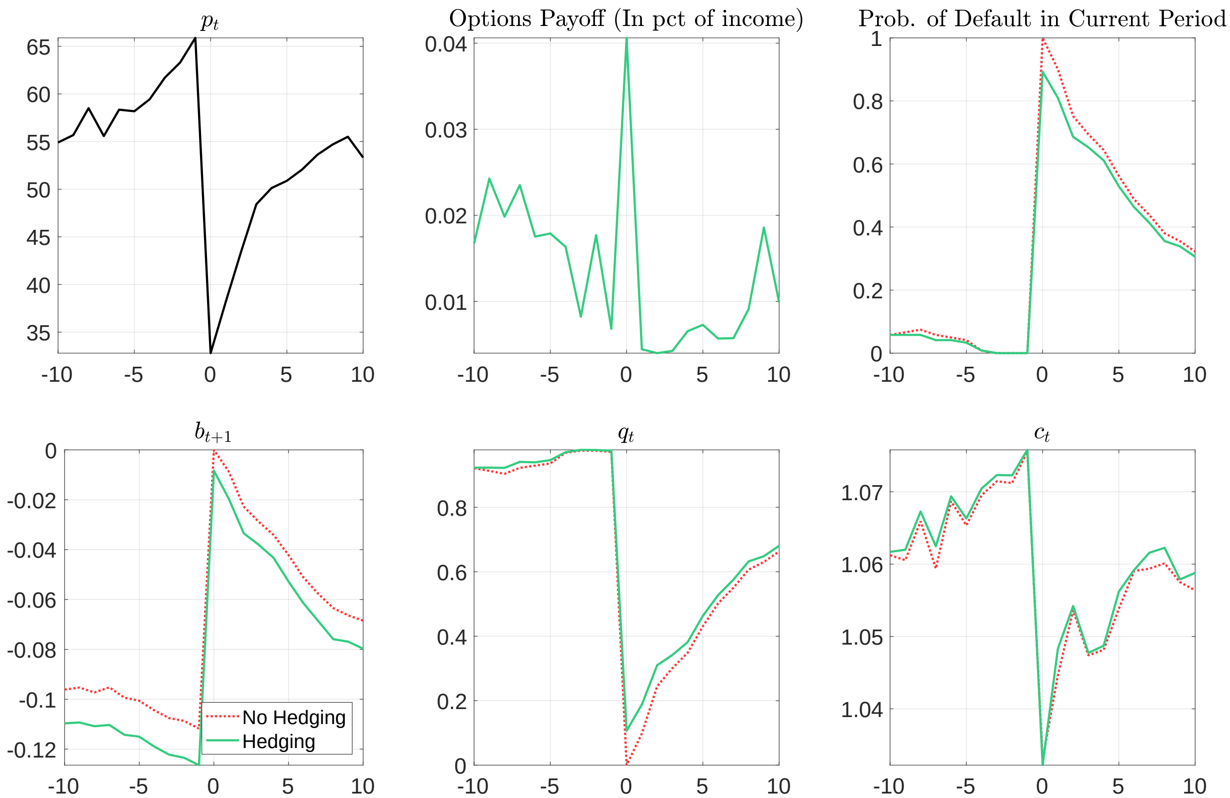
[20]The remaining channel should include gains from income smoothing, net of the cost of hedging because the above exercise does not take into account the cost of hedging.

on the default period and then examine the dynamics of key variables. Figure 6 plots the average path of the corresponding variable for the hedging and no-hedging economy, keeping in mind that the hedging economy may not have defaulted.

A sharp decline in oil prices, $p_t$, at time 0, triggers a payoff from the options which compensates for the income fall in the hedging economy. The no-hedging economy defaults, which reduces the stock of debt to zero, but the probability of default rises sharply even in the hedging economy, peaking at 89 percent. The high persistence in the oil price process keeps income prospects weak for some time in both economies, with borrowing being restored gradually, more so in the no-hedging economy than in the hedging economy. This result is the consequence of the temporary exclusion from financial markets and the higher borrowing costs for the no-hedging economy. Finally, the hedging economy can sustain higher levels of consumption than the no-hedging economy, despite the cost of the options, because of the lower cost of debt.

It is interesting to compare the output dynamics with and without hedging (panel A of Figure 7). Conceptually, hedging improves output dynamics in two ways. First, it covers the low realization of oil price shocks. When the country does not default (i.e. in periods $t \neq 0$), the output is marginally higher with hedging, captured by the difference between the oil price and strike price multiplied by the hedged production. This is through exercising put options when the oil price falls below the strike price. A large output difference between hedging and no-hedging is shown in period 0 when the non-hedging economy defaults. The hedging economy, however, may not necessarily default with the help of put options, which avoids the high default cost on output. This can be seen in panel B of Figure 7 which displays the minimum oil price $p^*$ under which the country defaults. We find that put options lower $p^*$ for every $b_t$, i.e. $p^{*,\text{hedging}} < p^{*,\text{no hedging}}$. In other words, put options allow the country to default at a lower oil price. As defaults are costly, this increases output.

**Figure 6** Event Windows

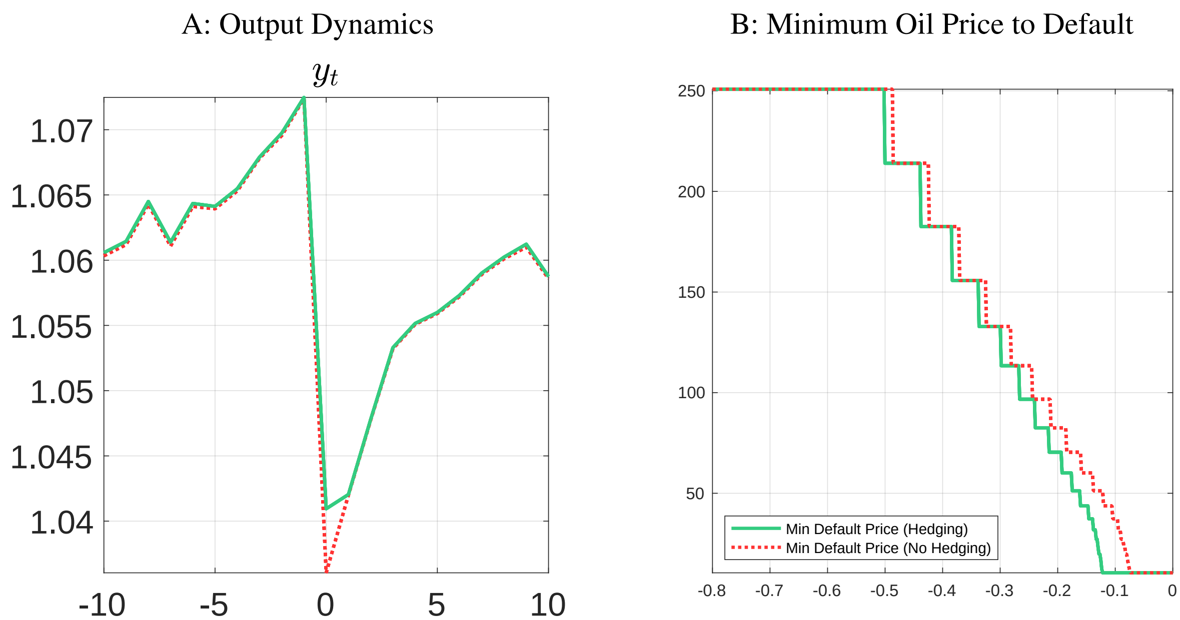



Note: The event windows are selected by first running an 11,000-period simulation for the hedging and no-hedging economies. After dropping the first 1,000 periods, we identify all default episodes by the no-hedging economy in the remaining 10,000 periods and compute the average evolution of the selected variables depicted in the charts.

## 5.3 Robustness Check

**Cost of Put Options.** Our baseline calculation assumes an actuarially fair price for put options. In practice, the actual price can include a premium above the actuarially fair price. This premium may stem from non-competitive behavior, regulatory constraints, risk aversion, and market illiquidity. In the case of Mexico, the use of over-the-counter options with Maya oil as the underlying asset could lead to a cost premium given that such instruments are not as liquid as options on the Brent

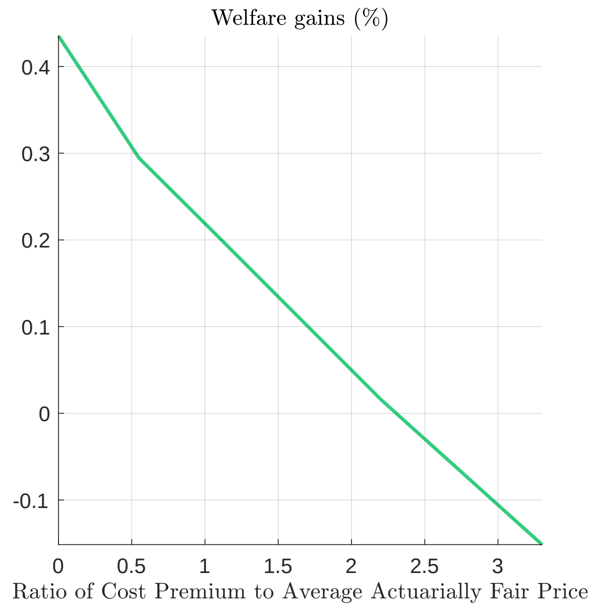
**Figure 7** Comparing Output Dynamics with and without Hedging




Note: Panel A presents the output dynamics using an event study analysis. The event windows are selected by first running an 11,000-period simulation for the hedging and no-hedging economies. After dropping the first 1,000 periods, we identify all default episodes by the no-hedging economy in the remaining 10,000 periods and compute the average evolution of output dynamics. Panel B presents the minimum oil price under which the economy defaults as a function of bond $b_t$. The greed solid line is for the hedging economy while the red dashed line is for the non-hedging economy.

or the West Texas Intermediate (WTI).[21] To examine the implications of such a cost premium, we now assume that there is an additional cost of $x$ per barrel of oil, above the actuarially fair price. In Figure 8, we plot the gains from hedging against various levels of the cost premium $x$, expressed as a ratio to the actuarially fair price. Not surprisingly, the welfare gains decline with $x$; however, reducing the welfare gains to zero in this model would require a sizable premium, in the order of 2.3 times the actuarially fair price.[22] The reason for the decline in welfare gains is that the options become relatively more expensive than debt, assuming that debt remains fairly priced. Naturally, if a cost premium also affects the price of debt, then the impact on welfare gains would depend on

**Figure 8** Welfare Gains under Different Cost Premiums




Note: Unconditional welfare gains as a function of the cost premium $x$, expressed as a ratio to the actuarially fair price.

the relative size of the distortions in debt and option prices.

**Strike Prices.** In our benchmark analysis, the strike price is a fraction $\mu$ of the expected oil price for next year conditional on the current period's price. We noted in the calibration section that we could compute $\mu$ directly in the data, although for only a handful of years for which there was publicly available information. The data suggested a range for $\mu$ between 0.72 and 1.14, as shown in Table 1. We arbitrarily chose values for $\mu$ of 0.74 and 1.03 and solved the model again to compute the welfare gains. We found that welfare gains increase with the strike price. Moreover, as welfare gains increase, the cost premium computed above also becomes larger, suggesting that

[21]Mexico's decision to use Maya oil as the underlying asset is justified on the grounds of avoiding base risk, defined as unexpected movements in Maya oil price not explained by movements in the price of Brent or WTI oil.

[22]The cost premium at which welfare gains are zero, expressed in 2009 constant dollars, is equivalent to US$2.1 per barrel. From 2006-2016, Mexico paid on average US$3.5 per barrel to purchase the put options, which is an alternative way to corroborate that the cost premium has to be sizable to reduce welfare gains to zero.

the gains become less sensitive to the presence of a cost premium in the price of the options. These results, and those described in the remainder of this section, are reported in Table 4.

**Oil Price Process.** Three parameters govern the oil price process, i.e. persistence $\rho$, volatility $\sigma$ and the unconditional mean $p$. Increasing the unconditional mean is inconsequential to our results if volatility and persistence remain the same. This is intuitive since the exercise leaves risk intact. Increasing the persistence reduces welfare gains because, given the one-year horizon of the options, hedging would compensate for a smaller fraction of the cumulative income loss relative to a scenario where oil prices recover more quickly. In turn, the gains increase with the volatility of oil prices, which is associated with a higher risk of default, because the borrowing costs channel strengthens. Note that when welfare gains become larger, the cost premium at which gains vanish becomes larger, suggesting as before that the gains become more robust.

Given the importance of $\rho$ and $\sigma$, we did three additional robustness tests in calibrating these two parameter values. First, we use a different set of parameter values $(\rho, \sigma) = (0.85, 0.28)$ estimated on the price of Maya oil, the type of Mexican heavy crude oil. Second, we use the parameter values $(\rho, \sigma) = (0.90, 0.20)$ used by Borensztein et al. (2013) for the average commodity prices. Third, we use the parameter values $(\rho, \sigma) = (0.94, 0.23)$ reported by Borensztein et al. (2013) for the estimation of the petroleum price. Consistent with the message in Table 4, welfare gains are lower with these three sets of parameter values. The gains from hedging are 0.08%, 0.01%, and -0.01% respectively, lower than our baseline welfare gains of 0.44% with the parameter value of $(\rho, \sigma) = (0.71, 0.25)$.

This robustness exercise again underscores the importance of the underlying risk process in the welfare analysis. Welfare gains are lower, and can even turn negative due to a higher cost to hedge, as oil prices become more persistent. However, compared to the alternative calibrations presented in these robustness exercises, our baseline calibration captures the setting of Mexico (and

**Table 4** Sensitivity Analysis

| | Welfare Gains (%) | Debt (%) | | Default Spreads (%) | | Default Prob. (%) | | Cost Premium |
|---|---|---|---|---|---|---|---|---|
| | Overall | Hedging | No Hedging | Hedging | No Hedging | Hedging | No Hedging | Hedging |
| baseline | 0.44 | 11.97 | 10.50 | 1.40 | 1.59 | 1.27 | 1.41 | 2.10 |
| $\mu = 0.74$ | 0.30 | 11.71 | 10.50 | 1.41 | 1.59 | 1.28 | 1.41 | 1.74 |
| $\mu = 1.03$ | 0.75 | 13.47 | 10.50 | 0.94 | 1.59 | 0.90 | 1.41 | 3.74 |
| $\alpha = 0.23$ | 0.31 | 11.52 | 10.50 | 1.41 | 1.59 | 1.27 | 1.41 | 2.10 |
| $\alpha = 0.39$ | 0.65 | 12.29 | 10.50 | 1.01 | 1.59 | 0.96 | 1.41 | 2.13 |
| $\rho = 0.8$ | 0.21 | 6.85 | 6.23 | 3.37 | 3.75 | 2.54 | 2.75 | 1.32 |
| $\rho = 0.9$ | 0.03 | 2.91 | 3.69 | 7.29 | 16.01 | 3.54 | 5.81 | 0.48 |
| $\sigma = 0.1$ | -0.00 | 18.46 | 18.46 | 0.80 | 0.80 | 0.79 | 0.79 | N.A. |
| $\sigma = 0.4$ | 0.37 | 4.28 | 2.20 | 0.12 | 2.71 | 0.15 | 2.07 | 3.03 |
| $p = 50$ | 0.44 | 11.97 | 10.50 | 1.40 | 1.59 | 1.27 | 1.41 | 1.93 |
| $p = 70$ | 0.44 | 11.97 | 10.50 | 1.40 | 1.59 | 1.27 | 1.41 | 2.92 |
| $r^* = 2\%$ | 0.29 | 9.69 | 8.81 | 1.43 | 1.97 | 1.28 | 1.66 | 2.13 |
| $r^* = 4\%$ | 0.17 | 7.92 | 7.37 | 1.50 | 2.26 | 1.31 | 1.82 | 1.75 |
| $\gamma = 3$ | 0.55 | 11.64 | 10.15 | 1.24 | 1.71 | 1.13 | 1.51 | 2.24 |
| $\gamma = 4$ | 0.74 | 11.70 | 9.55 | 1.24 | 1.75 | 1.14 | 1.53 | 2.42 |
| $\lambda = 0.2$ | 0.17 | 5.30 | 5.36 | 2.62 | 4.85 | 2.29 | 3.80 | 2.10 |
| $\lambda = 0.3$ | 0.04 | 3.58 | 4.10 | 4.54 | 10.18 | 3.79 | 6.93 | 0.54 |
| $y^* = 0.97E[y]$ | 0.69 | 27.97 | 25.95 | 0.72 | 0.95 | 0.72 | 0.92 | 2.21 |
| $y^* = 0.99E[y]$ | 0.35 | 5.26 | 4.05 | 0.76 | 2.17 | 0.73 | 1.83 | 2.01 |
| $y^* = 1.05E[y]$ | -0.00 | 0.30 | 0.00 | 283.55 | 4.29 | 10.53 | 1.14 | N.A. |
| $G = 1$ | 0.22 | 8.22 | 7.59 | 1.47 | 1.94 | 1.33 | 1.64 | 1.93 |
| $G = 1.04$ | 0.67 | 14.03 | 12.20 | 0.90 | 1.46 | 0.87 | 1.32 | 2.13 |
| $\beta = 0.9$ | 0.23 | 15.56 | 14.33 | 0.36 | 0.48 | 0.40 | 0.53 | 1.70 |
| $\beta = 0.98$ | 0.08 | 17.79 | 16.92 | 0.13 | 0.19 | 0.17 | 0.23 | 1.17 |

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents. Cost premium refers to the cost of options above the actuarially fair price such that the welfare gains from hedging are zero.

many other crude oil-exporting countries) as a country that exports crude oil and imports refined petroleum derivatives, embedded in the way we estimate and calibrate the parameters of the oil price process. While crude oil prices and the import price of oil derivatives co-move, they follow different short-run dynamics, due to differences in markets, evolution of refining costs, and the underlying petroleum derivative (e.g., diesel, gasoline, etc.), hence resulting in parameter values that look different to those that would be estimated from time-series on international crude oil prices (e.g., Brent or WTI).

**Other Parameters.** We also conduct a robustness analysis of other parameters in our model in Table 4. Generally speaking, the hedging benefits are robust to different parameter values. In particular, the gains increase with the volume of oil production, $\alpha$ because a larger fraction of income is protected. Moreover, risk spreads decline as the risk of default is lower. Gains are also larger when consumers are more risk averse, i.e. higher $\gamma$ since they dislike income fluctuations more. Welfare gains also increase with $G$ since higher growth in non-oil income increases the desire to borrow, whose cost is reduced by hedging. Welfare gains decline when the international risk-free rate, $r^*$, increases, which in turn makes borrowing more expensive, reducing the desire to borrow. With lower borrowing, the hedging benefits through the borrowing cost channel weaken. Welfare gains also decline when the income loss from default is lower. This is represented in the table by increasing $y^*$. The result is analogous to what happens in the right regions of Figure 3 depicting the outcomes from our two-period model. Note in Table 4 that for a sufficiently low cost of default, i.e. sufficiently high $y^*$, the welfare gains vanish since hedging in those cases increases default incentives (Proposition 3). In particular, the welfare gains become a negative number in the case of $y^* = 1.05E[y]$ since hedging increases the default probability from 1.14 % in the economy without hedging to 10.53 % in the economy with hedging. Welfare gains decrease with the probability of redemption $\lambda$. Since losing access to international financial markets is one component of the cost of default, increasing $\lambda$ is equivalent to reducing the cost of default; therefore, the result is consistent with what happens when $y^*$ is higher. Intuitively, when the default is less costly, the benefits of hedging decline in the presence of defaultable debt, which serves also as a hedging and consumption smoothing instrument. Welfare gains decline with the discount factor, $\beta$, since the more patient consumers become, the less incentive they have to borrow, and the weaker the borrowing costs channel of welfare gains.[23]

The above robustness checks with model parameters help us understand the tradeoff for hedging oil price shocks, i.e. the cost and the benefit of hedging. In our baseline economy with put options, the cost is the upfront foregone consumption $\xi(\bar{p}_t)$ to purchase put options with a strike price $\bar{p}_t$. The benefit consists of two parts: one is a smoother income and the other is a lower borrowing cost due to a lower default incentive. The above parameters change the tradeoff of hedging and thus the size of welfare gains.

Note that our model did not feature an optimal level of hedging. In our baseline calibration, we assume that the hedging share is constant at 29%, the average value from 2006 to 2016. Even though the hedging share is weakly correlated with oil prices, the correlation is only -0.49 and statistically insignificant. In this regard, we use a constant 29% hedging volume as our baseline calibration. We did not analyze the optimal hedging level because 1) it is computationally challenging; 2) we want to calibrate the model to a policy that is known to be implementable in the data. Therefore, Our model compares gains from hedging (at an optimal level implied by the data) vs. no hedging by focusing on the extensive margin of hedging.

One may be tempted to infer the optimal level of hedging by varying $\alpha$, the share of oil production that is hedged. However, doing so may not be the right approach. Welfare gains monotonically

[23] While we also show the implications of $\beta$=0.98 on welfare gains, this level of the discount factor annually would bring the degree of impatience to unrealistic levels for a country like Mexico, given its level of social and economic development (the level of the U.S. risk-free interest rate is 0.64% in the baseline calibration).

increase with $\alpha$ in our baseline calibration. This is a feature of risk-sharing property between risk-neutral international investors and risk-averse domestic agents. Intuitively, domestic agents have the incentive to hedge all oil risk related to production, i.e. $\alpha^* = 1$. This can be seen from the following first-order condition for $\alpha$. Had we allowed the agents to choose $\alpha \in [0,1]$, it would be a corner solution of $\alpha^* = 1$ or 0. Our baseline calibration implies that hedging cost is lower than hedging benefit, and thus $\alpha^* = 1$.

$$- \underbrace{\xi(\bar{p}_t)}_{\text{Hedging cost}} + \underbrace{E_t\left[\beta\left(G\frac{c_{t+1}}{c_t}\right)^{-\gamma}\max\{\bar{p}_t - p_{t+1}, 0\}|\text{No Default}\right]}_{\text{Hedging benefit}} \tag{15}$$

To generate an interior solution for $\alpha = 0.29$ as in the data, one can introduce an adjustment cost of choosing $\alpha$, a common assumption for portfolio problems (such as in Jermann and Yue 2018). This is a reasonable assumption because the more volumes hedged, they may end up influencing the cost of options since it means that counterparts will take larger exposures on their balance sheets (given also they don't use standard, more liquid instruments such as options on the WTI or Brent). Indeed, we have shown that as the cost of options increases, welfare gains decline. This suggests that there may be an optimal level of hedging but to find it, one would have to consider deeper microfoundations for the counterpart side (the microstructure of the options market) as the above argument would imply that markets are not deep enough to not be influenced by the size of Mexico's transactions. There is some evidence that this may be the case as Mexico decided to stop disclosing details of the hedging program fearing it could influence the price of the options in subsequent periods.

Our analysis suggests a potentially sizable welfare gain from hedging oil-price risk using options but if so, why don't many more countries use hedging instruments? We suspect that the political economy considerations may prevent many countries from using these instruments. These political economy considerations may exacerbate the tradeoffs involved in hedging through options. Acquiring put options means incurring an upfront cost with certainty while the benefits of this decision are obtained only if oil prices decline. While in principle this is no different from homeowners forgoing consumption to pay the annual premium to insure their homes against a fire, political economy considerations at the country level make the hedging decision more complex. First, it involves many more people (e.g., Congress), and second, their incentives may not be well aligned. Some may represent constituencies with urgent needs and would prefer using the money to build schools or fund social programs rather than purchasing insurance. Mexico seems to have found a good balance between these political economy constraints and the benefits of market instruments to hedge oil price risk.

# 6 Extensions

## 6.1 Selling Oil Forward

In our baseline model, the upfront cost of options generates a tradeoff. On the one hand, hedging helps smooth income, but on the other, it implies devoting resources in the current period to hedge. We contrast the welfare gains with an alternative hedging vehicle: selling oil forward at a predetermined price. There is no upfront cost of insurance, but the country gives up any revenue windfall if oil prices rise unexpectedly. We maintain the one-year horizon of the hedge. We model this variant of hedging by assuming that the country sells a fraction $\alpha$ of oil production at the conditional mean of the oil price in each period. The new budget constraint and dynamics of the wealth are given by

$$w_t = c_t + q_t G b_{t+1}$$

$$w_{t+1} = 1 + Q\{(1-\alpha)p_{t+1} + \alpha E_t[p_{t+1}]\} + b_{t+1}$$

To understand the benefits/costs of forward, we first modify our two-period model to include forwards and derive the following proposition:

**Proposition 4. Forwards and default incentives**

*Define $y^{def}$ as the income under default in the no-hedging economy such that the economy does not default when $y^{def} < \hat{y}^{def}$, i.e when the cost of default is sufficiently high. Introducing forwards at a price equal to the conditional mean increases $\hat{y}^{def}$.*

*Proof.* The proof is given in Appendix B.4. □

One implication from Proposition 4 is that the introduction of forwards can reduce default incentives. A similar plot to Figure 3 is presented in Figure 9 in the appendix.[24] Similar to options, if default never happens, hedging through forwards increases welfare only through the income smoothing channel (Region A in Figure 9), when the no-hedging economy defaults in the low-income state of the world, introducing forwards reduces the likelihood of default to zero since income is locked in at a level above the income level under default. In this case, forwards increase welfare through income smoothing and lower borrowing costs (Region B in Figure 9). For completeness, we also describe the implications of the model when default costs are sufficiently low, meaning that $y^H > \hat{\hat{y}}^{def} > \hat{y}^{def,forward} > \hat{y}^{def} > y^L$. In this case, forwards worsen default incentives. However, it is not an interesting case to examine since it would imply locking in through forwards a level of income below what the economy would get if it defaulted.

Turning now to the quantitative analysis, we find welfare gains from hedging through forwards equivalent to a permanent increase in consumption of 0.89 percent, roughly twice as large as those from our baseline model. However, recall that in our baseline calibration, the strike price for the put options is set at $\bar{p}_t = \mu E_t[p_{t+1}|p_t]$, with $\mu = 0.77$, while in this section, the economy hedges

[24]The figure shows the case where $\hat{y}^{def,forward} < \hat{y}^{def}$. However, it is theoretically possible that $\hat{y}^{def,forward} > \hat{\hat{y}}^{def}$.

**Table 5** Welfare Gains from Selling Oil Forward

| | Welfare Gains (%) | Debt (%) | | Default Spreads (%) | | Default Prob. (%) | |
|---|---|---|---|---|---|---|---|
| | Overall | Hedging | No Hedging | Hedging | No Hedging | Hedging | No Hedging |
| Forwards ($\mu = 1$) | 0.89 | 14.19 | 10.50 | 0.96 | 1.59 | 0.92 | 1.41 |
| Put Options ($\mu = 1$) | 0.75 | 13.32 | 10.50 | 1.14 | 1.59 | 1.06 | 1.41 |

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents.

through selling oil forward at a price equal to $E_t[p_{t+1}|p_t]$. Therefore, to conduct a more appropriate comparison between forwards and options, we compute the welfare gains from hedging through put options after setting $\mu = 1$. As shown in Table 5, the resulting welfare gains from options are equivalent to a permanent increase in consumption of 0.75, higher than in the baseline calibration, but still below those from forwards. With forwards, the probability of default and risk spreads are lower than in the model with options, while the economy can afford to borrow more (Table 5).

## 6.2 Risk-Averse Investors

This last extension is intended to understand the effects of hedging in a world where global changes in risk appetite affect commodity and other asset prices simultaneously. For simplicity, we model the situation as having risk-averse international investors with a time-variant pricing kernel $m_t$, i.e. the intertemporal marginal rate of substitution. Follow Arellano (2008), $m_t$ is an i.i.d. random variable. The pricing of sovereign bonds and options is given by

$$q_t(b_{t+1}, p_t) = E_t[m_{t+1}(1 - D(y_{t+1} + b_{t+1}, p_{t+1}))] \tag{16}$$

$$\xi_t(p_t) = E_t[m_{t+1} \max(\bar{p} - p_{t+1}, 0)] \tag{17}$$

with $m_{t+1} = e^{-r^* - \nu\varepsilon_{t+1}}$ to ensure $m_{t+1}$ is non-negative. Given that $\varepsilon_t$ is the same shock to oil prices, foreign investors discount more when the oil price is lower.

Table 6 reports the quantitative results for various values of $\nu$, the degree of risk aversion.[25] A higher risk aversion implies a higher premium charged by foreign investors to compensate for default risk, making debt more expensive and discouraging borrowing. In this situation, hedging might be more beneficial if it lowers the borrowing cost. However, the cost of hedging is also higher as foreign investors demand extra compensation when selling put options. It is thus unclear how risk aversion will change the case for hedging. Our quantitative analysis suggests that welfare gains are larger when foreign investors become more risk averse, i.e. $\nu$ increases, despite the higher upfront cost of insurance.

# 7 Conclusion

The sharp unexpected decline in oil prices during 2014-2016 renewed the interest in designing policies to manage such risks in countries highly exposed to swings in commodity prices. Discussions about the various alternatives to countries often start with Mexico, given its longstanding practice of hedging through put options, but analyses of the welfare gains of such policy have been limited. This paper attempts to fill this gap and derives lessons about the benefits and costs for

[25] Calibrating the risk aversion parameter using global shocks is an interesting exercise, which we leave for future research.

**Table 6** Risk Averse Investors: Hedging and No-hedging Economies

| | Welfare Gains (%) | Debt (%) | | Default Spreads (%) | | Default Prob. (%) | | Cost Premium |
|---|---|---|---|---|---|---|---|---|
| | Overall | Hedging | No Hedging | Hedging | No Hedging | Hedging | No Hedging | Hedging |
| $\nu = 0$ | 0.44 | 11.97 | 10.50 | 1.40 | 1.59 | 1.27 | 1.41 | 2.52 |
| $\nu = 0.25$ | 0.44 | 11.77 | 10.59 | 1.24 | 1.89 | 0.86 | 1.30 | 2.10 |
| $\nu = 0.5$ | 0.59 | 13.03 | 11.35 | 1.45 | 2.42 | 0.50 | 1.10 | 2.06 |
| $\nu = 0.75$ | 0.98 | 17.82 | 14.15 | 2.20 | 2.44 | 0.32 | 0.46 | 2.08 |

Note: We run 100 Monte Carlo simulations of 2,000 periods each for the economies with and without hedging. The initial 500 periods are dropped for each simulation. The reported debt ratio and spreads are calculated as the average values, across periods and simulations, excluding default episodes for each economy. The probability of default is calculated as the average fraction, across periods and simulations, of default periods. Welfare gains are calculated by constructing simulations for both economies subject to the same stochastic shocks and initial conditions, and then computing the present discounted value of the utility of consumption to ultimately express the difference in terms of consumption equivalents.

commodity exporters of using market insurance to hedge commodity price risk.

We have focused our analysis on the role of hedging instruments as a complement to defaultable debt, which in and of itself can be seen as a hedging strategy. Our quantitative assessment concludes that the welfare gains from hedging, in the presence of defaultable debt, can be equivalent to a permanent increase in consumption of about 0.44 percent. We also find that about 90 percent of these gains stem from a reduction in borrowing costs and the difference from income smoothing. The beneficial role of hedging is robust to numerous sensitivity analyses.

In terms of lessons for the design of a program like Mexico's, the welfare gains are lower when option prices exceed their actuarially fair value, a circumstance that may become more likely when using relatively illiquid, over-the-counter options. It may then be worth accepting some base risk to ensure hedging is welfare-enhancing. Nevertheless, the model suggests that the premium above the actuarially fair price would have to be very large for the welfare gains to decline to zero.

The model also suggests that selling oil forward generates larger welfare gains than hedging through put options. However, political economy considerations cannot be ignored since selling oil forward implies giving up any potential revenue windfall if oil prices rise. Mexico, through the use of options, seems to have found a good balance between these political economy constraints and the benefits of market instruments to hedge oil price risk.

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# A Normalized Economy

In this appendix, we show the derivation of the normalized economy starting from the original setup. As described in the main text, the original economy has the following structure:

**Preference.**

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right]$$

**Total Income.**

$$Y_t = F_t + p_t Q_t$$

**Budget Constraint under No Default.**

$$C_t + q_t B_{t+1} + \alpha Q_{t+1}\xi(\bar{p}_t) = Y_t + B_t$$

where agents hedge $Q_{t+1}$ production of oil at period $t$.

**Budget Constraint under Default.**

$$C_t = Y_t - H(Y_t)$$

where $H(Y_t) = h(y_t)F_t$.

Given that $F_t$ grows at a constant rate $G$ in every period, $C_t$ and $B_{t+1}$ grow at the same rate as $F_t$ and $F_{t+1}$ respectively. In order to solve a stationary problem, we can normalize the consumers' preferences, total income, and budget constraints as follows.

**Normalized Preference.**

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(F_tc_t)^{1-\gamma}}{1-\gamma}\right] = F_0^{1-\gamma}E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left(\frac{F_t}{F_{t-1}}\frac{F_t-1}{F_{t-2}}\cdots\frac{F_1}{F_0}c_t\right)^{1-\gamma}}{1-\gamma}\right] = F_0^{1-\gamma}E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(G^tc_t)^{1-\gamma}}{1-\gamma}\right]$$

**Normalized Total Income.**

$$y_t = \frac{Y_t}{F_t} = \frac{F_t + p_t Q_t}{F_t} = 1 + p_t \frac{Q_t}{F_t} = 1 + p_t Q$$

**Normalized Budget Constraint under No Default.**

$$y_t + b_t = \frac{Y_t + B_t}{F_t} = \frac{C_t + q_t B_{t+1} + \alpha Q_{t+1} \xi(\bar{p}_t)}{F_t} = c_t + q_t \frac{F_{t+1}}{F_t} \frac{B_{t+1}}{F_{t+1}} + \alpha \frac{Q_{t+1}}{F_{t+1}} \frac{F_{t+1}}{F_t} \xi(\bar{p}_t) = c_t + q_t G b_{t+1} + \alpha Q G \xi(\bar{p}_t)$$

**Normalized Budget Constraint under Default.**

$$c_t = \frac{C_t}{F_t} = \frac{Y_t - H(Y_t)}{F_t} = y_t - h(y_t)$$

Given the normalized preferences, total income, and budget constraints, we can solve the normalized economy problem knowing that the original problem can always be recovered by multiplying all variables by $F_t$.

# B Proofs

## B.1 Proof of Proposition 1

*Proof.* If the economy defaults in the H state, it must default in the L state since $y^H > y^L$. Conditional on no default, optimal borrowing $d^*$ satisfies the following condition:

$$\frac{1}{y + d^*} = \beta \left( \frac{p}{y^H - d^*} + \frac{1 - p}{y^L - d^*} \right)$$

The economy finds it optimal not to default iff $y^L - d^* > y^{def}$.

Similarly, when the economy defaults only in L state, optimal borrowing $d^{**}$ satisfies

$$\frac{p}{y+pd^{**}} = \beta \frac{p}{y^H - d^{**}}$$

which is consistent iff $y^H - d^{**} > y^{def}$. Define $\hat{y}^{def} = y^L - d^*$ and $\hat{\hat{y}}^{def} = y^H - d^{**}$ and we establish the first part of the proposition.

In an economy with hedging, optimal borrowing $d^{*,hedge}$ with no default satisfies

$$\frac{1}{y + d^{*,hedge} - \xi} = \beta \left( \frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}} \right)$$

It is easy to find $d^{*,hedge} > d^*$ since the marginal benefit of borrowing increases and the marginal cost declines. Using the same logic, we find that $d^{**,hedge} > d^{**}$. This implies that $\bar{\bar{y}}^{def,hedge} < \hat{\hat{y}}^{def}$. However, $\bar{y}^{def,hedge} > \hat{y}^{def}$ since $c_1^{L,hedge} > c_1^L$ due to the presence of hedging. □

## B.2 Proof of Proposition 2

*Proof.* If the economy does not default in equilibrium, the optimal allocation is given by

$$\frac{1}{y + d^*} = \beta \left( \frac{p}{y^H - d^*} + \frac{1-p}{y^L - d^*} \right).$$

With the introduction of hedging, the optimality condition becomes

$$\frac{1}{y + d^{*,hedge} - \xi} = \beta \left( \frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}} \right).$$

It is easy to see that $d^{*,hedge} > d^*$ since the income in the first period is reduced by $\xi$ and income in the L state has increased by $\bar{y} - y^L$.

We also need to establish the results that social welfare has been increased. Intuitively, hedging does not change the PDV of the income stream but reduces the variance of income. This is beneficial since it increases the welfare in the second period. Denote the social welfare without and with hedging by $U_0(d^*)$ and $U_0^{hedge}(d^{*,hedge})$ respectively. We want to show $U_0^{hedge}(d^{*,hedge}) > U_0(d^*)$ by proving $U_0^{hedge}(d^* + \xi(\bar{y})) > U_0(d^*)$. To see it, we have

$$\begin{aligned} U_0^{hedge}(d^* + \xi(\bar{y})) - U_0(d^*) &= \beta\left[p\log\left(y^H - \xi(\bar{y}) - d^*\right) + (1-p)\log\left(\bar{y} - \xi(\bar{y}) - d^*\right)\right] \\ &\quad -\beta\left[p\log\left(y^H - d^*\right) + (1-p)\log\left(y^L - d^*\right)\right] \\ &= \beta\left[p\log\left(y^H - (1-p)(\bar{y} - y^L) - d^*\right) + (1-p)\log\left(y^L + p(\bar{y} - y^L) - d^*\right)\right] \\ &\quad -\beta\left[p\log\left(y^H - d^*\right) + (1-p)\log\left(y^L - d^*\right)\right] > 0 \end{aligned}$$

where the last inequality holds since the function

$$f(x) = \beta\left[p\log\left(y^H - (1-p)x - d^*\right) + (1-p)\log\left(y^L + px - d^*\right)\right]$$

increases in $x \in [0, y^H - y^L]$. □

## B.3 Proof of Proposition 3

*Proof.* When the economy defaults only in the low-income state of nature, hedging is beneficial if it reduces default incentives. If the economy does not default in equilibrium, social welfare increases with hedging (See Proposition 2). However, debt might increase or decrease. One can see that from the first order conditions with debt $d^{**}$ and $d^{*,hedge}$ satisfying

$$\frac{p}{y+pd^{**}} = \beta \frac{p}{y^H - d^{**}}$$
$$\frac{1}{y+d^{*,hedge}-\xi} = \beta\left(\frac{p}{y^H - d^{*,hedge}} + \frac{1-p}{\bar{y} - d^{*,hedge}}\right)$$

It is hard to sign $d^{**}$ and $d^{*,hedge}$ since both the marginal benefit and the marginal cost of borrowing increase with hedging.

If hedging does not change default incentives, social welfare is lower since the economy borrows more (See the proof of Proposition 2 in Appendix B.2) and consumption streams are unambiguously lower. Clearly, social welfare is further reduced if hedging increases default incentives. There is no borrowing in this case. □

### B.4 Proof of Proposition 4

*Proof.* The two-period model with forwards changes into the following form

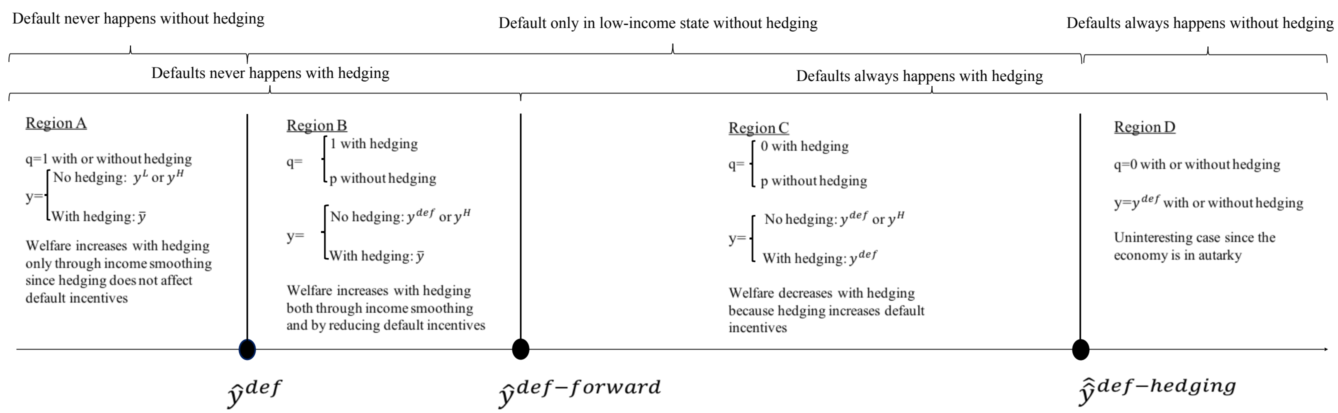
$$\begin{aligned} U_0^{forwards} &= \max_d \log c_0 + \beta \log c_1 \\ \text{s.t.} \quad & c_0 = y + d, \\ & c_1 = \max\{\bar{y} - d, y^{def}\} \end{aligned}$$

where $\bar{y} = py^H + (1-p)y^L$.

The optimality condition when the economy does not default implies that

$$\frac{1}{y+d^{*,forwards}} = \beta \frac{1}{\bar{y} - d^{*,forwards}}$$

**Figure 9** Two-period Model with Forwards



It is easy to show that $d^* < d^{*,forwards}$ since the marginal cost of borrowing decreases with forwards.[26] Therefore, we have

$$\beta\frac{1}{\bar{y}-d^{*,forwards}} = \frac{1}{y+d^{*,forwards}} < \frac{1}{y+d^*} = \beta\left(\frac{p}{y^H-d^*}+\frac{1-p}{y^L-d^*}\right) < \beta\frac{1}{y^L-d^*},$$

which implies that $\bar{y}-d^{*,forwards} > y^L-d^*$. It follows that $\hat{y}^{def,forwards} > \hat{y}^{def}$. As to welfare, if both economies never default in both states, the economy with forwards has larger utility due to the concavity of the log function. If the no-hedging economy defaults and forwards avoid default, welfare is larger with forward. If the no-hedging economy defaults in the low state and the forward makes the economy to default in both states, welfare is lower. □

# C Option Pricing

The payoff of the put options is given by $\max\{\bar{p}-p_{t+1},0\}$ for strike price $\bar{p}$ and current price $p_t$ at time $t$. For a risk-neutral investor, the put option is priced according to the following formula:

$$\xi(p_t) = \frac{E_t\left[\max\{\bar{p}-p_{t+1},0\}\right]}{1+r^*}$$

Since we assume that $\log p_t$ follows an AR(1) process, i.e.

$$\log p_{t+1} \sim N\left(\underbrace{(1-\rho)\left[\log(p)-\frac{1}{2}\frac{\sigma^2}{1-\rho^2}\right]+\rho\log p_t}_{\mu_t},\sigma^2\right)$$

[26]One can see that from the inequality

$$f(x) = \frac{p}{y^H-d}+\frac{1-p}{y^L-d}-\frac{1}{\bar{y}-d} = \frac{p(1-p)\left(y^H-y^L\right)^2}{(y^H-d)(y^L-d)(\bar{y}-d)} > 0$$

Hence, we have

$$
\begin{aligned}
\xi(p_t) &= \frac{E_t\left[\max\{\bar{p}-p_{t+1},0\}\right]}{1+r^*} \\
&= \frac{\int_{\log p_{t+1}\leq\log\bar{p}}(\bar{p}-p_{t+1})}{1+r^*} \\
&= \frac{1}{1+r^*}\int_{-\infty}^{\log\bar{p}}(\bar{p}-p_{t+1})\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(\log p_{t+1}-\mu_t)^2}{2\sigma^2}}d\log p_{t+1} \\
&= \frac{\bar{p}}{1+r^*}\Phi\left(\frac{\log\bar{p}-\mu_t}{\sigma}\right)-\frac{1}{1+r^*}\int_{-\infty}^{\log\bar{p}}p_{t+1}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(\log p_{t+1}-\mu_t)^2}{2\sigma^2}}d\log p_{t+1} \\
&= \frac{\bar{p}}{1+r^*}\Phi\left(\frac{\log\bar{p}-\mu_t}{\sigma}\right)-\frac{1}{1+r^*}e^{\mu_t+\frac{\sigma^2}{2}}\Phi\left(\frac{\log\bar{p}-\mu_t-\sigma^2}{\sigma}\right)
\end{aligned}
$$