Foreign Reserves and Capital Controls: A Financial Development Perspective *

Chang Ma[†] Hidehiko Matsumoto[‡]

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Abstract

We provide a financial development perspective for the joint use of foreign reserves and capital controls. Empirical evidence indicates that countries with intermediate levels of financial development have the highest reserve-to-GDP ratios, but capital controls are negatively correlated with financial development. To explain this pattern, we develop a small-open-economy model featuring endogenous growth and liquidity shocks. Both reserves and capital controls are necessary to mitigate fire-sale externalities resulting from asset liquidation during liquidity crises. Our model captures financial development by the magnitude of liquidity shocks, thereby replicating the observed relationship between reserves, capital controls, and financial development.

Keywords: Capital Controls; Foreign Reserves; Sudden Stops; Liquidity Crisis. **Financial Development**

JEL: F32, F41, F44.

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†Fanhai International School of Finance, Fudan University (changma@fudan.edu.cn).

‡National Graduate Institute for Policy Studies (hmatsu.hm@gmail.com.)

1 Introduction

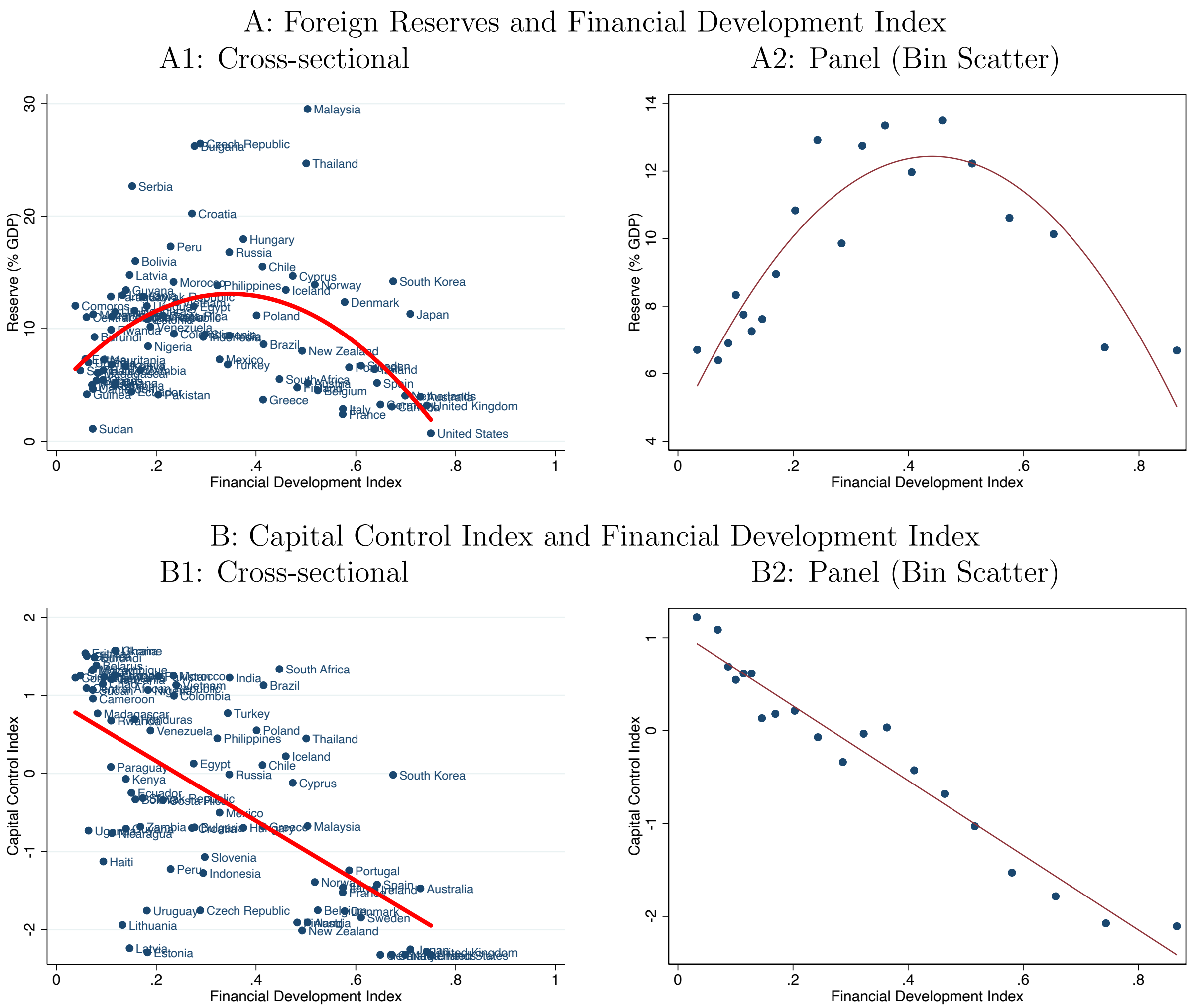
Emerging markets and developing economies are subject to volatile capital flows driven by the global financial market conditions and policies of major advanced economies (Rey 2015). For those economies, capital controls and foreign reserves are two important policy tools to smooth impacts from the global financial market (Basu, Boz, Gopinath, Roch, and Unsal 2020). Accordingly, there is a growing literature on the optimal design of these policy tools (Bianchi and Lorenzoni 2022). However, there is a wide cross-country variation in how actively each country uses these policy tools, and important questions remain unanswered: What is the optimal combination of capital controls and reserve policy? What is the key determinant of the optimal combination, and what explains the cross-country variation?

In this paper, we provide a financial development perspective for the joint use of capital controls and foreign reserves. Our analysis is motivated by the empirical relationship between financial development, capital controls, and foreign reserves shown in Figure 1. On the one hand, the relationship between financial development and the reserve-to-GDP ratio is non-monotonic. Countries with an intermediate level of financial development tend to have a higher reserve-to-GDP ratio than other countries. On the other hand, countries with high financial development use capital controls less actively.

We develop a dynamic small-open-economy model to understand these empirical facts. Our main contributions are twofold. First, we provide a novel model of sudden stops that necessitates a joint use of capital controls and foreign reserve policy. Unlike preceding papers such as Bianchi (2011) that model sudden stops as an occasionally binding borrowing constraint, we model sudden stops as liquidity shocks that require a part of foreign debt to be repaid before new borrowing. Domestic agents are forced to sell some of their assets at a fire-sale price to make this repayment. Fire-sale externalities arising from the asset liquidation justify a joint use of capital controls and foreign reserve interventions. Second, our model rationalizes the observed relationship between capital controls, foreign reserves, and financial development. Building on the literature such as Mendoza, Quadrini, and Rios-Rull (2009) that identifies financial development as a key determinant of capital flows, our paper connects financial development to the optimal policy designs regarding capital flows. To the best of our knowledge, our model is the first to explain the observed cross-country patterns of financial development, capital controls, and foreign reserves.

In our model, households produce and consume tradable goods, borrow from abroad, hold liquid foreign assets (private foreign reserves), and invest in productive assets. Pro-

Figure 1 Reserves, Capital Control Index, and Financial Development Index



Note. Panel A (B) presents the relationship between reserves (capital control index, i.e. the negative of Chinn-Ito Index in Chinn and Ito 2006) and the financial development index for a panel of 87 economies in 1980-2019. A1 and B1 plot the cross-sectional relationship where each data point is the time-average for a country. A2 and B2 are bin scatter plots for the panel dataset.

ductive assets accumulate over time, and the model features endogenous growth, following the literature that connects business cycles and growth (Comin and Gertler 2006, Queralto 2020, Benigno, Fornaro, and Wolf 2022). There are two particular reasons why we introduce endogenous growth. First, it allows us to capture persistent impacts of crises observed in the data (Cerra and Saxena 2008, Reinhart and Rogoff 2009). Second, changes in permanent income are important for business cycles and crises in emerging economies (Aguiar and Gopinath 2007, Guntin, Ottonello, and Perez 2023).

The key element of the model is a liquidity shock. At the beginning of each period before new borrowing and production, households may be required to repay a certain fraction of outstanding foreign debt with an exogenous probability. When this happens, households can liquidate part of their productive assets for repayment. However, liquidation is costly because assets can be sold only at a fire-sale price during crises (Aguiar and Gopinath 2005). To reduce this costly liquidation, households have an incentive to hold foreign reserves as a liquidity buffer. In this regard, our model highlights the liquidity role of foreign reserves (Arce, Bengui, and Bianchi 2019, Céspedes and Chang 2020, Jeanne and Sandri 2023). Indeed, a survey conducted by IMF (2013) shows that about 75% of the country authorities hold reserves for precautionary liquidity purposes.

The amount of asset liquidation during crises is determined to cover liquidity shortage, defined as required debt repayments minus reserve holdings. Households can reduce liquidation by decreasing debt or increasing reserves ex ante, but the latter option is more efficient. If households increase reserve holdings by one unit, it directly reduces the next-period liquidity shortage by the same amount. In contrast, decreasing debt by one unit reduces the next-period liquidity shortage only by the fraction of debt subject to liquidity shocks, which can be less than one. Therefore, reserves have a relative advantage over debt in reducing liquidity risk. We term this as the 'liquidity advantage' of reserves. However, reserve holdings entail an opportunity cost because the interest rate on reserves is lower than the interest rate on debt (Jeanne and Rancière 2011). Households thus choose debt and reserves to strike a balance between the liquidity advantage and the opportunity cost.

The need for policy interventions arises from a fire-sale externality associated with asset liquidation. Foreign buyers purchase liquidated assets and use them to produce tradable goods. The price is equal to the marginal product of assets, which declines with the aggregate amount of liquidated assets. However, domestic households take this price as given, leading to a fire-sale externality. This externality distorts households' decisions on debt and reserves as follows. When a liquidity shock hits, households need to liquidate assets to cover the liquidity shortage. If households marginally reduce debt or increase reserves ex ante, the need for asset liquidation is reduced, which increases the liquidation price and reduces liquidation even further. As individual households do not internalize the impact of their decisions on the liquidation price, they borrow excessively and hold an insufficient level of reserves. The optimal policy to correct these distortions involves a tax on foreign debt and either a subsidy on reserve holdings or official foreign reserve accumulation by the public sector.[1]

[1] In Section 3.4, we show that a subsidy on reserve holdings and official foreign reserve accumulation,

We show that the potential size of liquidity shocks, which is a fraction of foreign debt to be repaid in the event of a liquidity shock, is the key determinant of reserve holdings and capital controls. If the size of liquidity shocks is small, the resulting liquidity shortage is also small, and there is no need to hold a large amount of reserves. Conversely, if the size of liquidity shocks is large, the liquidity advantage of reserves over debt diminishes as reducing debt would also effectively decrease the liquidity shortage. In this case, households simply reduce debt rather than holding reserves. Therefore, the amount of reserve holdings is the largest when the size of liquidity shocks is intermediate. In contrast, the optimal debt tax rate monotonically increases in the size of liquidity shocks. We interpret the potential size of liquidity shocks as an indicator of the degree of financial development, where large liquidity shocks imply low financial development. This interpretation comes from the observation that in a well-developed financial market, high enforceability and strong commitment enable households to borrow even during sudden stops. As a result, a well-developed financial market effectively reduces the magnitude of liquidity shocks. This notion of financial development aligns with theoretical works such as Chang and Velasco (2001), Mendoza et al. (2009), Maggiori (2017), as well as empirical results in Section 2.

In the quantitative analysis, we calibrate the model to average emerging economies and solve it numerically using a global method. The severity of asset fire sales is calibrated using the empirical estimates by Aguiar and Gopinath (2005). In a stochastic simulation with interest rate and liquidity shocks, a liquidity shock triggers a sharp current account reversal, asset liquidation, and persistently low output, consumption, and investment relative to pre-crisis trends. The current account reversal arises from a permanent loss in income, due to asset liquidation that persistently lowers productivity. The mechanism of sudden stops in our model is in line with Aguiar and Gopinath (2007) and supported by the empirical evidence in Guntin et al. (2023). The persistent negative impacts on the real economy are also due to endogenous productivity loss and are consistent with the empirical regularities of sudden stops. Our model also captures procyclical gross capital flows: both private inflows and public outflows increase when the interest rate is low, and vice versa (Broner, Didier, Erce, and Schmukler 2013, Avdjiev, Hardy, Kalemli-Özcan, and Serven 2022). Compared with the decentralized economy, the social planner who internalizes the fire-sale externalities holds a larger amount of reserves to reduce liquidation. Consequently, the share of liquidated assets during crises is only 0.7% of total assets in contrast to 2.5% in the decentralized economy. Output after crises is persistently higher by more than 1% under the planner's allocation.

when combined with a tax of foreign debt, can achieve the same constrained efficient allocation.

Finally, we solve the model with a wide range of liquidity shock sizes and compute the average debt tax rate and the amount of reserves. The amount of reserves is maximized at 33% of GDP when the size of a liquidity shock is intermediate, whereas the tax rate is monotonically increasing with the size of liquidity shocks. Interpreting the size of a liquidity shock as the level of financial development, these patterns are consistent with our empirical findings. The expected welfare gain by these policies is also non-monotonic in the size of liquidity shocks, reaching 0.4% of permanent consumption at its peak. The size of the welfare gain is substantially higher than that suggested by preceding models. In our model, the optimal policy mitigates persistent negative impacts of crises on the real economy. This result emphasizes the importance of considering the persistent impacts of crises, as often observed in the data when designing optimal policies.

Literature Review This paper contributes to a broad literature on capital controls and foreign reserve policy for a precautionary motive. One strand of literature focuses on capital controls to correct excessive foreign borrowing. A typical assumption in this literature is that a pecuniary externality through a drop in the collateral asset price induces overborrowing by private agents and calls for a tax on private debt. Papers in this literature include Bianchi (2011), Benigno et al. (2013, 2016), Bianchi and Mendoza (2018), and Jeanne and Korinek (2020), among others. Ma (2020) introduces endogenous growth and studies how capital controls should be designed when they affect growth.

Another strand of literature focuses on foreign reserves. Papers in this literature introduce different assumptions to motivate reserve accumulation, such as shocks to borrowing limit (Jeanne and Rancière 2011, Céspedes and Chang 2020, Matsumoto 2022), liquidity shocks (Hur and Kondo 2016), capital flow shocks (Cavallino 2019), sovereign default and endogenous borrowing cost (Hernández 2017, Bianchi, Hatchondo, and Martinez 2018, Bianchi and Sosa-Padilla 2020), self-fulfilling currency crisis (Bocola and Lorenzoni 2020), growth externality (Benigno et al. 2022), and collateral constraint on foreign borrowing (Shousha 2017). Jeanne and Sandri (2023) develop a model with liquidity shocks and pecuniary externalities to show that foreign reserve policy is necessary for countries with intermediate levels of financial development, similar to our result. In contrast to these papers that study capital controls and reserve policies separately, we study the optimal combination of capital controls and reserve policy in a unified framework, and rationalize the observed cross-country pattern.

Several recent papers study the relationship between capital controls and reserve policies.

Arce et al. (2019) shows that public reserve accumulation can be used as a macroprudential policy tool similar to capital controls against sudden stops. Davis et al. (2021a), Davis et al. (2021b), and Fanelli and Straub (2021) assume financial frictions on private foreign debt similar to Gabaix and Maggiori (2015) and show that foreign reserve policy can be used as a substitute for capital controls to manage private capital flows. In these papers, reserve policy is at best a perfect substitute for capital controls, and could be a less efficient policy tool if reserve policy is associated with a higher cost. Our model deviates from these papers by rationalizing a joint use of capital controls and reserve policy. Lutz and Zessner-Spitzenberg (2023) rationalize a joint use of capital controls and reserve policy by imposing a cash-in-advance constraint for trade financing. We take a different approach by incorporating liquidity shocks and asset fire sales. In this regard, our model is also related to Lorenzoni (2008) and Dávila and Korinek (2018) who focus on fire-sale externalities to justify policy interventions. More importantly, to the best of our knowledge, our model is the first to explain the observed cross-country patterns of financial development, capital controls, and foreign reserves.

Our paper is also related to the literature on gross capital flows. Broner et al. (2013) document a positive correlation between gross inflows and outflows. Davis and van Wincoop (2018) show that this positive correlation is driven by financial globalization and develop a model to explain their observations. Caballero and Simsek (2020) assume liquidity shocks similar to ours and show that capital retrenchment helps to stabilize the economy when foreign capital flows out, explaining the positive correlation. Our model differs from these papers in that capital inflows are driven by the private sector while capital outflows are driven by the public sector through reserve accumulation. This is consistent with the empirical pattern in emerging economies as shown by Avdjiev et al. (2022), and models by Arce et al. (2019) and Jeanne and Sandri (2023).

The rest of the paper is organized as follows. Section 2 shows empirical facts about foreign reserves and capital controls. Section 3 lays out the model. Section 4 calibrates the model and conducts quantitative analyses, and Section 5 shows how the size of a liquidity shock affects the optimal policy. Section 6 concludes.

2 Motivating Empirical Facts

In this section, we present motivating empirical facts on financial development, foreign reserves, and capital controls. We document three patterns in particular:

Fact 1. *The relationship between the level of financial development and the foreign reserve-to-GDP ratio is non-monotonic. Countries with an intermediate level of financial development tend to have higher reserve-to-GDP ratios than other countries.*

Fact 2. *A negative correlation exists between financial development and capital control measures. Countries with higher financial development impose fewer restrictions on capital flows.*

Fact 3. *A positive correlation exists between financial development and external liability (debt). Countries with high financial development tend to have a high external liability-to-GDP (debt-to-GDP) ratio.*

To establish these facts, we combine data sets from several sources for 87 economies from 1980 to 2019: the financial development index constructed by the IMF, the External Wealth of Nations data set from Lane and Milesi-Ferretti (2007), and the capital control index constructed by Chinn and Ito (2006). We also add into our regressions important country-level controls using data from the World Bank's World Development Indicator database, the Worldwide Governance Indicators, and exchange rate regimes constructed by Shambaugh (2004). The details on the country lists and the variable constructions are in Appendix A.

Our primary focus is on the measure of each country's financial development. Ideally, it should capture the ability and efficiency of a country's financial market in helping domestic agents deal with external shocks. The IMF's financial development measure is constructed to capture how developed financial institutions and financial markets are in terms of depth (size and liquidity), access (the ability of individuals and companies to access financial services), and efficiency (the ability of institutions to provide financial services at low cost and with sustainable revenue and the level of activity of capital markets). Unlike the traditional empirical proxy for financial depth, such as the private credit-to-GDP ratio or stock market capitalization-to-GDP ratio, the IMF's financial development index takes into account the complex multidimensional nature of financial development (Svirydzenka 2016).

Columns (1) and (2) in Table 1 present the relationship between foreign reserves and financial development measures. Consistent with Figure 1, there is a non-monotonic relationship between these two variables: Countries with an intermediate level of financial development tend to have higher reserve-to-GDP ratios than other countries. Column (1) controls for country-fixed effects that absorb all time-invariant cross-country differences.

Table 1 Reserve, Capital Control Index and External Borrowing: Relationship with Financial Development Index in 1980-2019

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.78***	0.91***	-0.56***	-0.24**	0.89***	0.50**	0.64***	0.63***
	(0.15)	(0.24)	(0.08)	(0.11)	(0.11)	(0.21)	(0.10)	(0.22)
Financial Development^2	-0.22***	-0.23***						
	(0.07)	(0.09)						
GDP per capita (log)		-0.16		-0.28		-0.87***		-0.81***
		(0.35)		(0.26)		(0.32)		(0.29)
Trade (% GDP)		0.18		-0.24**		0.24***		0.08
		(0.13)		(0.09)		(0.08)		(0.11)
Institutional Quality		0.20		-0.55**		-0.39*		-0.26
		(0.26)		(0.21)		(0.21)		(0.25)
Peg		-0.06		0.00		-0.02		0.02
		(0.15)		(0.08)		(0.07)		(0.08)
CA (% GDP)		0.07		-0.02		-0.03		0.01
		(0.04)		(0.02)		(0.04)		(0.04)
Constant	0.16**	0.29**	0.04***	-0.04	-0.08***	0.16*	-0.08***	-0.01
	(0.06)	(0.14)	(0.01)	(0.07)	(0.01)	(0.10)	(0.01)	(0.08)
Year FE	N	Y	N	Y	N	Y	N	Y
Country FE	Y	Y	Y	Y	Y	Y	Y	Y
Observations	3212	1853	3141	1852	3137	1819	3138	1813
Adjusted R^2	0.480	0.681	0.685	0.869	0.533	0.747	0.504	0.697

Note. The dependent variable is reserve-to-GDP in columns (1) and (2), capital control index (the negative value of the Chinn-Ito Index constructed by Chinn and Ito 2006) in columns (3) and (4), external liability-to-GDP in columns (5) and (6), and external debt-to-GDP in columns (7) and (8) respectively. The independent variables include a financial development index constructed by the IMF, GDP per capita (log), trade-to-GDP ratio, the institutional quality measure, the exchange rate regime constructed by Shambaugh (2004), and the current account-to-GDP ratio. We standardize all variables (except for the exchange rate regime variable). Standard errors are clustered by countries and reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are in Appendix A.

Column (2) adds year-fixed effects that absorb all common time-variant factors along with important cross-country characteristics, including GDP per capita (a proxy for economic development), trade (a proxy for economic openness), institutional quality measures, exchange rate regimes, and current account-to-GDP ratio. As is conventional, we cluster the standard errors at the country level. The non-monotonic relationship is robust, statistically significant, and economically strong. For ease of interpretation, we standardize all variables in the regression (except for the exchange rate regime dummy variable). Using the point estimate in column (1) as an illustration, an improvement in the financial development index by one standard deviation increases the reserve-to-GDP ratio by 0.56 unit standard deviation, including a linear positive effect of 0.78 unit and a non-linear negative effect of 0.22 unit.

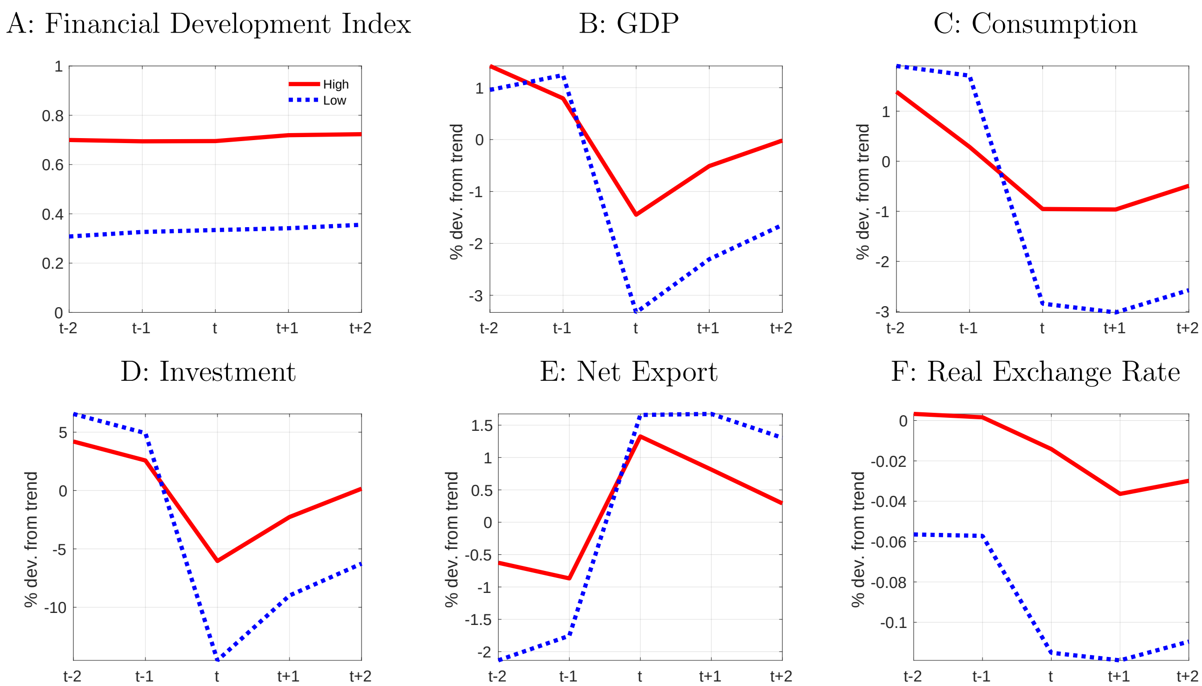
Columns (3) and (4) present the relationship between the capital control index and financial development measures. We use the negative value of the Chinn-Ito index constructed by Chinn and Ito (2006) as a measure for capital controls due to its long time series back to 1980. We observe a clear pattern that countries with higher financial development tend to impose fewer restrictions on capital flows, as in Figure 1. The negative relationship is statistically significant and economically meaningful. Using column (3) with county-fixed effects as an illustration, a one-unit standard deviation improvement in financial development measures reduces 0.56 unit standard deviation in capital control measures. Both the statistical and economic power are lower once we include year-fixed effects along with important country-level variables in column (4). This is understandable as the capital control measure is sticky and may correlate with important country-level characteristics (Acosta-Henao et al. 2020, Ma and Wei 2020). However, the negative relationship is robust to those controls.

Why does financial development exhibit a non-monotonic relationship with reserves? Our interpretation is that the level of financial development determines the effective size of liquidity shocks. Liquidity shocks require a part of external debt to be repaid before new borrowing. A well-developed financial market enables domestic agents to borrow even during such liquidity shocks, thereby reducing the effective size of liquidity shocks. In this case, the need for reserves is low. Conversely, low financial development implies limited access to financing during liquidity shocks. The effective size of liquidity shocks is so large that domestic agents do not borrow much ex ante. In this case again, the need for reserves is low. In contrast, an intermediate level of financial development allows domestic agents to borrow some amount during liquidity shocks. This induces agents to borrow relatively a large amount ex ante, but it is associated with some liquidity risk. This is the case where reserves are most needed. Our model formalizes this idea.

Figure 2 provides supporting evidence that links the level of financial development to the size of liquidity shocks, measured by the severity of liquidity crises. We use sudden stop episodes identified by Korinek and Mendoza (2014) and group them based on the financial development index two years before the crisis. Low financial development countries have a severe decline in GDP, consumption, investment, and real exchange rate in crises compared to high financial development countries. Although we do not provide a causal statement, the endogeneity concern is mitigated by the persistent dynamics of the financial development index in the 5-year window.

Our interpretation of financial development is also consistent with the data pattern on

Figure 2 Financial Development and the Severity of Sudden Stop Dynamics



Note. The data of sudden stop episodes comes from Korinek and Mendoza (2014). We group those episodes into two groups based on the financial development index two years before the crises.

external borrowing. Columns (5)-(8) in Table 1 present the relationship between financial development and external liability (debt). Countries with high financial development tend to have a high external liability-to-GDP ratio and debt liability-to-GDP ratio. This relationship is robust to country-fixed effects in columns (5) and (7), as well as year-fixed effects along with other important country-level characteristics in columns (6) and (8). Using column (5) as an illustration, a one-unit standard deviation improvement in financial development measures increases 0.89 unit standard deviation in the external liability-to-GDP ratio. This suggests a tight connection between a country's financial development level and its external borrowing, consistent with our interpretation that the advanced domestic financial market reduces the liquidity risk and thus encourages more external borrowing.

Motivated by the above empirical facts, we will provide a theory to rationalize the optimal use of reserves and capital controls, connecting them to financial development as in the data. To discipline our model mechanism, we document the business cycle features of private capital flows and reserve flows. We use data from 47 emerging economies from 1987 to 2019. Data for private and reserve flows is taken from the updated dataset in Alfaro, Kalemli-

Table 2 Capital flows, Reserve flows, and Borrowing Cost

	Private Flows (% GDP)		Private Flows (% GDP)		Reserve Flows (% GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Reserve Flows (% GDP)	0.25***	0.28***				
	(0.07)	(0.06)				
EMBI Spread (in log)			-0.32***	-0.12*	-0.22***	-0.26***
			(0.08)	(0.07)	(0.06)	(0.08)
GDP per capita		0.29**		0.39		-0.28
		(0.14)		(0.27)		(0.17)
Trade (% GDP)		0.02		0.04		0.14
		(0.07)		(0.10)		(0.17)
Institutional Quality		0.13		0.35		0.14
		(0.12)		(0.22)		(0.18)
Peg		-0.12		-0.10		-0.05
		(0.09)		(0.11)		(0.19)
CA (% GDP)		-0.37***		-0.28***		0.34***
		(0.06)		(0.08)		(0.09)
Constant	-0.00***	0.04	0.05***	-0.04	0.02***	0.10
	(0.00)	(0.04)	(0.00)	(0.09)	(0.00)	(0.09)
Year FE	N	Y	N	Y	N	Y
Country FE	Y	Y	Y	Y	Y	Y
Observations	1269	871	663	627	664	628
Adjusted R^2	0.101	0.407	0.159	0.354	0.077	0.234

Note. The dependent variable is private flows (% GDP) in columns (1)-(4) and reserve flows (% GDP) in columns (5) and (6) respectively. The independent variables include reserve flows (% GDP), EMBI spread (in log), GDP per capita (log), trade-to-GDP ratio, the institutional quality measure, the exchange rate regime constructed by Shambaugh (2004), and the current account-to-GDP ratio. We standardize all variables (except for the exchange rate regime variable) for ease of comparison. Standard errors are clustered by countries and reported in parentheses. *, ** and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. All variable constructions are reported in Appendix A.

Ozcan, and Volosovych (2014). The Emerging Markets Bond Index (EMBI) spreads measure external borrowing costs and are taken from the World Bank's Global Economic Monitor. Other independent variables are the same as those in Table 1.

There are two patterns observed between private flows, reserve flows, and borrowing costs. First, private capital flows are positively correlated with reserve accumulation, consistent with Jeanne and Sandri (2023). Columns (1) and (2) in Table 2 show that the relationship is robust to country-fixed effects, year-fixed effects, and important country-level characteristics.

Economically, a unit standard deviation increase in reserve flows is associated with a 0.25 unit standard deviation increase in private flows as in column (1). Moreover, the point estimate barely changes with additional controls in column (2).

Second, both private capital and reserve flows are negatively correlated with the borrowing cost measure, as shown in columns (3)-(6) in Table 2. The EMBI spreads capture the *cost* difference for a country between external borrowing and holding reserves. When the spread is high, both the capital flows and the reserve flows decline. Again, the relationship holds even after controlling for other important country-level characteristics. Economically, one unit standard deviation increase in EMBI spread is associated with a 0.12 unit standard deviation decline in private flows as in column (4), and a 0.26 unit standard deviation decline in reserve flows in column (6). This suggests that the interest rate shock is important in driving both private and reserve flows.

3 Model

We now develop a model that is motivated by the salient empirical patterns documented in the previous section. Our small-open-economy model features endogenous growth and liquidity shocks. Financial development is represented by the size of the liquidity shocks.

3.1 Setup

Our small open economy is inhabited by a unit measure of identical infinitely-lived households. They produce and consume tradable goods, borrow from abroad, hold liquidity in safe foreign assets (reserves), and invest in productive assets. Their utility is given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

where β is the discount factor and the utility function u is strictly increasing and strictly concave. The budget constraint is

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^{\ell} \tag{2}$$

where c_t denotes consumption of tradable goods, b_t is foreign bond holdings, s_t is reserve holdings, and z_t is investment in productive assets. R_t is the gross interest rate on foreign bonds, and R^s is the fixed gross interest rate on reserves. Households are not allowed to borrow using reserves, implying that s_t cannot be negative. In the quantitative analysis below, we set parameter values such that b_t is always negative. Therefore, we call b_t foreign debt or simply debt henceforth. a_t is productive asset holdings and also output because we assume a linear production function $y_t = a_t L$ with fixed labor supply $L = 1$. $q_t a_t^\ell$ is the amount of resource obtained by liquidating a part of asset holdings when a liquidity shock hits the economy, which will be explained in detail below.

Productive assets a_t in the model are broad assets that include capital and technology used for the production of tradable goods. It can also be interpreted as the productivity level of the economy. The amount of productive assets a_t grows endogenously as households invest z_t units of tradable goods. The law of motion for a_t is given as follows:

$$a_t = a_{t-1} + \eta(z_{t-1})^\gamma \left(a_{t-1} + \kappa a_{t-1}^*\right)^{1-\gamma} - a_t^\ell \tag{3}$$

a_{t-1}^* is the level of foreign productive assets, which is assumed to grow at a fixed rate $1 + \bar{g}$. The term in the second parenthesis implies that both domestic and foreign assets, a_{t-1} and a_{t-1}^*, promote the accumulation of productive assets. κ captures the degree of technological spillover from foreign technology as in Gavazzoni and Santacreu (2020) and Gornemann, Guerrón-Quintana, and Saffie (2020). Agents can increase the amount of domestic productive assets by their investment z_{t-1}. The parameters $\gamma \in (0, 1)$ and $\eta > 0$ govern the accumulation process and will be calibrated to the data. By introducing spillover from exogenous a_t^*, a_t endogenously fluctuates around the exogenous path of a_t^*, but will not deviate far from it. This implies that the long-run average growth rate of a_t is exogenously given by $1 + \bar{g}$.[2] We assume that households internalize that the current assets a_t facilitate future growth through (3), so that there is no externality associated with growth. a_t^ℓ denotes the amount of liquidated assets. As explained below, households may need to liquidate a part of their assets to repay foreign debt when a liquidity shock hits the economy.

There are two stochastic shocks. One is a shock to the interest rate on foreign debt, a key driver of capital flows as in Section 2. The interest rate R_t is given by:

$$R_t = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\frac{b_t}{a_t} - \bar{b}\right) - 1\right] \tag{4}$$

[2]If there is no spillover, a_t could deviate substantially far from a_t^*. Large deviations of a_t from a_t^* lead to extremely high or low liquidation price q_t, because q_t is affected by a_t^* as shown below in (8). In quantitative analyses, we set κ by targeting the level and elasticity of q_t.

where R^b is the baseline interest rate, ε_t^R is a stochastic shock, and the second term is a debt-elastic component with $\psi^b > 0$ and $\bar{b} \in \mathcal{R}$ as in Schmitt-Grohé and Uribe (2003). The debt-elastic component is not essential but improves the quantitative performance of the model.[3] We assume that households internalize the effect of b_t and a_t on the interest rate R_t, so that there is no externality through the interest rate.

The other shock is a liquidity shock, the key component of our model. At the beginning of each period before households obtain new borrowing b_t and production a_t, households may be required by foreign lenders to repay a fraction $\theta_t \in [0, 1]$ of the existing debt b_{t-1}, where θ_t is a stochastic variable. This means that there is a rollover risk for external borrowing determined by θ_t. Any stochastic process of $\theta_t \in [0, 1]$ is consistent with our model, but for simplicity and clarity of the mechanism, we assume that θ_t is a binary stochastic variable which takes either 0 or $\theta \in [0, 1]$ with an exogenous probability. In this case, the parameter θ determines the size of liquidity shocks. We interpret θ as an indicator for the degree of financial development, with a lower value of θ indicating higher financial development. This is because a well-developed financial market provides alternative financing channels and enables households to borrow even during sudden stops, thereby reducing the effective size of liquidity shocks. Further discussion will be provided in Section 5 when we study how the value of θ affects the optimal policies.

When the liquidity shock hits, households can use reserves s_{t-1} from the last period to make the early repayment, $-\theta b_{t-1}$. If the reserve s_{t-1} is not enough, households need to liquidate a part of their productive assets to finance the liquidity shortage, defined by $-\theta b_{t-1} - s_{t-1} > 0$.[4] Let a_t^ℓ denote the amount of liquidated assets, and q_t denote its price. The proceeds from asset liquidation $q_t a_t^\ell$ need to be enough to cover the liquidity shortage.

$$q_t a_t^\ell \geq -\theta b_{t-1} - s_{t-1}. \tag{5}$$

We call this inequality a liquidity constraint. An inequality sign implies that households can potentially liquidate more assets than required for the early repayment. In the quantitative analyses, however, households never liquidate assets more than necessary because the liquidation price q_t is low and liquidation is costly. Therefore, the liquidity constraint binds whenever there is a liquidity shortage. We also assume that households cannot buy

[3] Without a debt-elastic component of R_t, foreign debt b_t and reserves s_t would become substantially larger and more volatile. We set the values for ψ^b and $\bar{b}$ by targeting the average debt-to-GDP ratio and the standard deviation of the current account.

[4] Notice that we do not allow households to default on their foreign debt.

productive assets from foreigners.

$$q_t a_t^\ell \geq 0. \tag{6}$$

Because domestic households are homogeneous and equally short of liquidity, foreign agents are the only possible buyers of liquidated assets. We assume that foreign agents are competitive and less efficient than domestic agents in producing goods from domestic assets. In particular, they combine liquidated assets a_t^ℓ with their own assets a_t^* to produce tradable goods. Their profit maximization problem is given as follows:

$$\pi_t^* = \max_{a_t^\ell} \left\{ (a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - q_t a_t^\ell - F a_t^* \right\} \tag{7}$$

with $0 < \zeta < 1$ being a parameter for the production function. $F a_t^*$ is an entry cost to enter the market of liquidated assets, which is explained below. The first-order condition gives a demand equation for liquidated assets:

$$q_t = (1-\zeta) \left(\frac{a_t^*}{a_t^\ell} \right)^\zeta \tag{8}$$

This equation implies that the liquidation price q_t goes down as liquidation a_t^ℓ increases, indicating a downward-sloping demand by foreign agents. This is meant to capture asset fire sales during crises. Importantly, atomistic households take the liquidation price q_t as given and do not internalize the effect of their liquidation on prices through (8). This is a fire-sale externality and the only source of externality in the model. As shown below, this externality distorts the debt and reserve decisions by households and calls for policy interventions.

The demand equation in (8) also implies that q_t becomes very high when a_t^ℓ is very small. This means that domestic households have an incentive to sell a small amount of assets even if there is no liquidity shortage. To avoid such asset sales in normal times, we set the entry cost parameter F such that foreign buyers are willing to buy liquidated assets only when a liquidity shock hits the economy and a_t^ℓ is large enough to cover the fixed cost.

3.2 Decentralized Equilibrium

In the decentralized equilibrium, atomistic households choose $\{c_t, b_t, s_t, z_t, a_t^\ell\}$ to maximize expected utility (1) subject to the budget constraint (2), the law of motion for productive assets (3), debt-elastic interest rate (4), the liquidity constraint (5), the non-negativity constraints on liquidation (6) and reserves, taking the liquidation price q_t as given but it is determined by (8). The recursive maximization problem by households is set up as follows:

$$
\begin{aligned}
V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) = \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} & u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) && (9) \\
& - \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a^\ell_t \right] && (10) \\
& - \xi_t \left[a_t - a_{t-1} - \eta z^\gamma_{t-1} \left(a_{t-1} + \kappa a^*_{t-1} \right)^{1-\gamma} + a^\ell_t \right] && (11) \\
& + \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] && (12) \\
& + \varphi_t q_t a^\ell_t && (13) \\
& + \nu_t \frac{s_t}{R^s} && (14)
\end{aligned}
$$

Θ_t is a set of stochastic shocks $\Theta_t = \left\{ \theta_t, \varepsilon^R_t \right\}$. Foreign assets a^*_t follows an exogenous path $a^*_t = (1 + \bar{g}) a^*_{t-1}$. The last term (14) is the non-negativity constraint on reserve holdings.

The first-order conditions and the definition of the decentralized equilibrium are given in Appendix B.1. Arranging the first-order conditions leads to the following equations:

$$
u'(c_t) = \beta \mathbb{E}_t \left[\xi_{t+1} \eta \gamma \left(\frac{z_t}{a_t + \kappa a^*_t} \right)^{\gamma - 1} \right] \tag{15}
$$

$$
\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t) \tag{16}
$$

$$
\begin{aligned}
\xi_t = & u'(c_t) \left[1 + \left(\frac{b_t / a_t}{R_t} \right)^2 \psi^b \exp \left(- \frac{b_t}{a_t} - \bar{b} \right) \right] \\
& + \beta \mathbb{E}_t \left[\xi_{t+1} \left\{ 1 + \eta (1 - \gamma) \left(\frac{z_t}{a_t + \kappa a^*_t} \right)^\gamma \right\} \right]
\end{aligned} \tag{17}
$$

$$
u'(c_t) = \beta \underbrace{\frac{R_t}{1 + \psi^b \exp \left(- \frac{b_t}{a_t} - \bar{b} \right) \frac{b_t / a_t}{R_t}}}_{\equiv \tilde{R}_t} \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \theta_{t+1} \right] \tag{18}
$$

$$
u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right] + \nu_t \tag{19}
$$

(15) is the Euler equation regarding investment z_t. (16) is the first-order condition regarding liquidation a^ℓ_t. (17) is the first-order condition regarding asset a_t, and ξ_t captures a shadow value of one unit of asset. It consists of a marginal utility of an additional unit of consumption and a contribution to next-period asset accumulation. (18) and (19) are the Euler equations

regarding debt and reserves respectively.

The first-order condition regarding liquidation (16) needs some explanation. ψ_t is a Lagrange multiplier for the liquidity constraint (12), and φ_t is a multiplier for the non-negativity constraint on liquidation (13). It is technically possible that neither constraint binds and $\psi_t = \varphi_t = 0$, which means that households liquidate assets more than necessary to cover the liquidity shortage (including the case of no liquidity shortage). But in the quantitative analyses below, we set the parameter values such that q_t is substantially lower than ξ_t, and the right-hand side of (16) is always positive. It follows that either (12) or (13) binds in each period, depending on whether there is a liquidity shortage or not. Specifically,

(1) When a liquidity shock hits the economy and there is a liquidity shortage, households need to liquidate assets, implying $a_t^\ell > 0$. In this case, q_t is determined by the demand equation (8). Because $a_t^\ell > 0$ implies a slack non-negativity constraint and $\varphi_t = 0$, ψ_t is equal to the right-hand side of (16). In this case, ψ_t captures the shadow value of an additional unit of liquidity. With one unit of additional liquidity, households can reduce asset liquidation by $1/q_t$ units, whose value is the first term in the right-hand side where ξ_t is the shadow value of one unit of productive assets. At the same time, a $1/q_t$-unit reduction in asset liquidation reduces households' available budget by 1 unit, whose value is the second term. We call ψ_t a 'private' value of liquidity to distinguish it from a 'social' value of liquidity discussed below in Section 3.4.

(2) When there is no liquidity shortage, either because $\theta_t = 0$ or households have enough reserves, the liquidity constraint (12) is slack and $\psi_t = 0$. $a_t^\ell = 0$ because foreign buyers would not buy assets due to the entry cost and a_t^ℓ cannot be negative. In this case, q_t and φ_t are irrelevant for the other part of the model. Just for model consistency, we assume a positive value for q_t that makes the right-hand side of (12) positive when $a_t^\ell = 0$. This asset price q_t would be the price if domestic households could buy assets from foreign agents. Then φ_t is given by the right-hand side of (16).

In the Euler equation regarding debt (18), the interest rate is adjusted to internalize the effect of debt on the interest rate through (4). We denote this adjusted interest rate by $\tilde{R}_t$ henceforth. This Euler equation can be understood as follows. By giving up one unit of consumption at period t, households can reduce debt by $\tilde{R}_t$ units and increase the resource at $t+1$ by the same amount. This will bring the expected utility given by the first term in the right-hand side. In addition, when $\theta_{t+1} = \theta$ and there is a liquidity shortage $-\theta b_t - s_t > 0$ at $t+1$, a reduction in debt by $\tilde{R}_t$ units will reduce the liquidity shortage by $\theta\tilde{R}_t$ units.

This will bring the expected utility given by the last term in (18), where a reduction in the liquidity shortage is evaluated by a private value of liquidity ψ_{t+1}.

The Euler equation regarding reserves (19) can be understood in the same way. By giving up one unit of consumption at period t, households can increase reserves and the resource at $t+1$ by R^s units, whose value is captured by the first term on the right-hand side. R^s units of reserves at $t+1$ will also reduce the liquidity shortage by R^s units if there is a liquidity shortage, captured by the second term. Note that ψ_{t+1} is not multiplied by θ_{t+1}, in contrast to the Euler equation regarding debt (18), because R^s units of reserves will reduce the liquidity shortage one-to-one by R^s units. This difference between (18) and (19) plays a critical role in the model, which is explained in the next subsection. ν_t is a Lagrange multiplier for the non-negativity constraint on reserves and is positive when $s_t = 0$.

3.3 Reserve Holdings

In this subsection, we examine the mechanism of how households choose reserves in detail. Combining the two Euler equations (18) and (19), we obtain the key equation of the model:

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \theta_{t+1}\tilde{R}_t)\psi_{t+1}] + \nu_t \tag{20}$$

We set parameter values such that $\tilde{R}_t > R^s$ always holds, as is typically the case in emerging economies.[5] The left-hand side captures an opportunity cost for reserves. Households can use one unit of tradable good either to buy R^s units of reserves or reduce $\tilde{R}_t$ units of debt. If households choose the former, they receive R^s units at $t+1$ but lose the opportunity to reduce the interest payment on debt $\tilde{R}_t$. This gap is the opportunity cost of holding reserves.

The first term on the right-hand side captures a relative advantage of holding reserves over reducing debt in liquidity management. As explained above, by holding R^s units of reserves, households can reduce a liquidity risk $-\theta b_t - s_t$ (a potential liquidity shortage at $t+1$) by R^s units, and its expected value is given by $\beta\mathbb{E}_t[R^s\psi_{t+1}]$. In contrast, by reducing $\tilde{R}_t$ units of debt, households can reduce a liquidity risk by $\theta\tilde{R}_t$ units, and its expected value is $\beta\mathbb{E}_t[\theta_{t+1}\tilde{R}_t\psi_{t+1}]$. If θ satisfies $R^s > \theta\tilde{R}_t$, then increasing reserves will reduce the liquidity risk more effectively than reducing debt. This gap is the relative advantage of holding reserves over reducing debt in liquidity management, and we call it a liquidity advantage

[5] If $\tilde{R}_t = R^s$, then households choose debt and reserves such that there is no liquidity shortage in any states at $t+1$, implying $\psi_{t+1} = 0$. Then all the terms in (20) become zero. In this case, the model is reduced to a standard small open-economy model in which only the net foreign asset position $b_t + s_t$ matters.

of reserves for simplicity. Recall that ψ_{t+1} is a private value of liquidity, which is positive only when there is a liquidity shortage and the liquidity constraint binds. It follows that the liquidity advantage has a positive value only when there is a positive probability that a liquidity shock triggers a liquidity shortage $-\theta b_t - s_t > 0$ at $t+1$. Overall, the key equation (20) says that households choose debt and reserves to equalize the opportunity cost and the liquidity advantage of reserves. When the cost is high and/or the benefit is low, and it is not possible to equalize these two, households choose $s_t = 0$, and the Lagrange multiplier for the non-negativity constraint on reserves $\nu_t > 0$ fills the gap between the cost and benefit.

The value of θ is a critical determinant of the liquidity advantage of reserves and thus the amount of reserve holdings. We can derive the following three propositions regarding reserve holdings from (20).

Proposition 1. *If $\theta = 0$, households do not hold reserves, $s_t = 0$ $\forall t$.*

Proof. If $\theta = 0$, there is no liquidity shortage, and $\psi_{t+1} = 0$ for every period. The first term on the right-hand side of (20) is zero, which implies $\nu_t > 0$ and thus $s_t = 0$. □

The intuition is straightforward: if there is no liquidity risk at all, there is no reason to hold reserves because it comes with an opportunity cost.

Proposition 2. *If $\theta \geq R^s/\tilde{R}_t$, households do not hold reserves, $s_t = 0$ $\forall t$.*

Proof. If this condition holds, the first term on the right-hand side of (20) is non-positive. Because the left-hand side is positive, $\nu_t > 0$ and thus $s_t = 0$. □

Intuitively, if θ is close to one, reserves lose their liquidity advantage over debt because reducing debt is as effective as increasing reserves in reducing liquidity risk. Households simply reduce debt to manage liquidity risks as reserves come with an opportunity cost.

Proposition 3. *Households never hold enough reserves to eliminate a liquidity risk. In other words, households always choose b_t and s_t such that $-\theta b_t - s_t > 0$ holds.*

Proof. Suppose to the contrary that households hold enough reserves to eliminate a liquidity risk, $-\theta b_t \leq s_t$. Then there is no liquidity shortage and $\psi_{t+1} = 0$ at $t+1$. In addition, $\nu_t = 0$ because $s_t \geq -\theta b_t > 0$. This implies that the right-hand side of (20) is zero. But the left-hand side is positive, and this cannot be an equilibrium. □

Intuitively, if the amount of reserves is enough to eliminate a liquidity risk, there is no additional benefit of holding reserves by reducing a liquidity risk at the margin. This is too much reserve holdings given the opportunity cost of holding reserves. As long as there is an opportunity cost of holding reserves, households accept a positive liquidity risk to achieve the balance between the marginal cost and benefit of holding reserves.

An important implication of these propositions is that households hold a positive amount of reserves only when θ is an intermediate value between 0 and $R^s/\tilde{R}_t$. More generally, the amount of reserves is small when θ is close to zero or close to $R^s/\tilde{R}_t$. On the one hand, if θ is close to zero, the size of the early repayment $-\theta b_t$ is small. Proposition 3 says that households never hold enough reserves to cover the entire early repayment, i.e., $-\theta b_t > s_t$. Then reserve holdings become small. On the other hand, if θ is close to $R^s/\tilde{R}_t$, the liquidity advantage of reserves is small. In this case, households reduce debt rather than increase reserves to manage a liquidity risk. In addition, high θ implies that a liquidity risk quickly increases with debt, and households do not borrow much ex ante. For these reasons, reserve holdings become small.

In contrast, when θ takes an intermediate value, the size of an early repayment to be covered by reserves is relatively large, and the liquidity advantage of reserves is also high. In this case, reserve holdings can become large. This mechanism leads to a non-monotonic relationship between reserve holdings and the value of θ. As we interpret the value of θ as the degree of financial development, this non-monotonic relationship is consistent with the empirical finding in Section 2. We show this relationship quantitatively in Section 5.

3.4 Social Planner's Allocation

We next examine the social planner's allocation. The only difference between the decentralized economy and the planner's allocation is that the planner internalizes the price of liquidated assets q_t decreasing in the amount of liquidation a_t^ℓ as in (8). The setup of the maximization problem and first-order conditions are in Appendix B.2. The key first-order condition is the one regarding liquidation a_t^ℓ, corresponding to (16) in the decentralized equilibrium:

$$\psi_t^{SP} + \varphi_t^{SP} = \frac{\xi_t}{q_t - \zeta q_t} - u'(c_t) \tag{21}$$

The difference from (16) is $-\zeta q_t$ in the denominator, obtained by $\frac{\partial q_t}{\partial a_t^\ell} a_t^\ell = -\zeta q_t < 0$. It implies that the social value of liquidity ψ_t^{SP} is higher than the private value ψ_t given everything else equal. Intuitively, an additional unit of liquidity will reduce liquidation and increase the liquidation price q_t, thereby reducing liquidation even further. The planner internalizes this effect and assigns a higher value to liquidity compared to decentralized households.

Although this externality appears in the first-order condition regarding liquidation a_t^ℓ, the decision on a_t^ℓ itself is not distorted by this externality. This is because the amount of liquidation a_t^ℓ is determined by the binding liquidity constraint (5), given debt b_{t-1} and reserves s_{t-1} chosen in the previous period. When there is a liquidity shortage, a larger ψ_t^{SP} than ψ_t implies that the planner has a higher incentive to reduce liquidation than decentralized households. It follows that whenever the liquidity constraint (5) binds in the decentralized equilibrium, the planner has no incentive to liquidate more, and chooses the same liquidation a_t^ℓ as decentralized households.

What is distorted by the externality is households' decisions on debt and reserves. Recall that the private value of liquidity ψ_{t+1} appears in the right-hand sides of the Euler equations (18) and (19), and affects how much households borrow and hold reserves. The corresponding Euler equations by the planner are given as follows:

$$u'(c_t) = \beta \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \theta_{t+1} \right] \tag{22}$$

$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP} \right] + \nu_t \tag{23}$$

where ψ_{t+1}^{SP} is given by (21). ψ_{t+1}^{SP} being larger than ψ_{t+1} implies that individual households underestimate the value of liquidity when the liquidity constraint binds in the next period. Due to the externality, households borrow excessively and hold an insufficient amount of reserves compared with the planner's allocation. Combining the two Euler equations (22) and (23), we can obtain an equation similar to the key equation (20). All the discussions and propositions on reserve holdings in Section 3.3 apply to the social planner's allocation. In particular, the socially optimal amount of reserves is non-monotonic to θ: it becomes high when θ is an intermediate value, and it is low when θ is close to zero or $R^s/\tilde{R}_t$.

The social planner's allocation can be decentralized by the following policy instruments. Overborrowing can be corrected by a tax on foreign debt, which is capital controls. Insufficient reserve holdings can be corrected by a subsidy on reserve holdings. Any fiscal surplus or deficit resulting from these policies is balanced through lump-sum transfers to or from households. Let τ_t^b and τ_t^s denote a tax on debt and a subsidy on reserve holdings respectively.

Introducing these policies, the Euler equations by decentralized households become

$$u'(c_t) = \beta(1+\tau_t^b)\tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1}\right] \tag{24}$$

$$u'(c_t) = \beta(1+\tau_t^s)R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\right] + \nu_t \tag{25}$$

with

$$1+\tau_t^b = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP}\theta_{t+1}\right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1}\right]} \tag{26}$$

$$1+\tau_t^s = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP}\right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\right]} \tag{27}$$

where ψ_{t+1} and ψ_{t+1}^{SP} are given by (16) and (21) respectively. These policies make decentralized households internalize the externalities and achieve the social planner's allocation. Because $\psi_{t+1}^{SP} > \psi_{t+1}$, both τ_t^b and τ_t^s are positive.

Alternatively, the planner can achieve the same optimal allocation by a tax on debt and official foreign reserve accumulation. In this policy scheme, the planner accumulates official reserves through a lump sum tax on households in normal times. When a liquidity shock hits the economy and there is a liquidity shortage, the planner rebates official reserves to households through a lump sum transfer. As the planner accumulates official reserves in normal times, households reduce private reserves one to one by Ricardian equivalence. However, once the planner accumulates official reserves more than the amount of reserves individual households would hold without policy interventions, a non-negativity constraint binds and households cannot reduce private reserves anymore. Because the socially optimal amount of reserves is more than the amount of reserves chosen by individual households, the planner accumulates the socially optimal amount of reserves and private reserves become zero. Combined with the capital control τ_t^b characterized by (26), this policy scheme achieves the planner's allocation. We provide a formal proof in Appendix B.3. In quantitative analyses, we focus on this policy scheme because it corresponds to our empirical observations.

In Appendix D, we analytically show that the optimal debt tax rate τ_t^b increases with θ in a simplified two-period model. This can be understood from equation (26). Given $\psi_{t+1}^{SP} > \psi_{t+1}$, a higher θ directly increases the tax rate in (26), given everything else fixed.[6]

[6]In the two-period model in the appendix, there is another channel where θ increases τ_t^b. A higher θ increases the liquidation a_t^ℓ and increases ψ_{t+1}^{SP} more than ψ_{t+1}. But this channel does not necessarily work in the full model because a_t^ℓ is also affected by the endogenous state variables b_{t-1} and s_{t-1}, whereas, in the two-period model, the initial state is fixed for any θ.

Interpreting θ as the degree of financial development, our model rationalizes the empirical patterns of financial development, capital controls, and foreign reserves in Section 2. We show this relationship quantitatively in Section 5.

4 Quantitative Analyses

This section conducts quantitative analyses of the model. We solve the model numerically using a global method to deal with an occasionally binding non-negativity constraint on reserves. To reduce the number of state variables, we define a net wealth of households $w_t = b_{t-1} + s_{t-1} + q_t a_t^\ell$ as a state variable instead of keeping track of b_{t-1} and s_{t-1} individually. This is possible because b_{t-1} and s_{t-1} determine the amount of asset liquidation a_t^ℓ and its price q_t through the binding liquidity constraint (5) and the asset price equation (8), but they are not individually relevant for the households' decisions once we know w_t after liquidation. The detail of the numerical solution is presented in Appendix E.

4.1 Calibration

One period in the model is meant to be one year. We assume log utility $u(c_t) = \ln(c_t)$ for households and set the standard parameters to the conventional values in the literature as in Table 3. The discount factor β is 0.91 following Bianchi (2011). The baseline gross interest rate on foreign debt R^b is 1.06, standard in the literature. We assume no interest on reserves and set $R^s = 1$. The curvature of investment γ is 0.8 following Comin and Gertler (2006). The exogenous growth rate of foreign asset $\bar{g}$ is 2.61%, the average growth rate of the 47 sample emerging economies over the sample period in the empirical analysis in Section 2.

There are two stochastic shocks in the model, an interest rate shock ε_t^R and a liquidity shock θ_t. For the interest rate shock, we follow Mendoza (2010) and assume that ε_t^R takes either of two values $\{\varepsilon^R, -\varepsilon^R\}$ with $\varepsilon^R = 0.0196$. The liquidity shock takes either θ or 0. We assume a perfect correlation between two shocks: when a liquidity shock hits the economy, the interest rate shock takes a high value ε^R. This means that there are three possible realizations of shocks, $(\varepsilon_t^R, \theta_t) = \{(\varepsilon^R, 0), (-\varepsilon^R, 0), (\varepsilon^R, \theta)\}$. This shock follows a three-state Markov process, and the probability transition matrix is based on Mendoza (2010) and Bianchi and Mendoza (2018). Specifically, in normal times with no liquidity shock, the same interest shock continues with probability 0.54, the interest rate shock changes with probability 0.36, and a liquidity shock occurs with probability 0.1. When a liquidity shock

Table 3 EXTERNALLY DETERMINED PARAMETERS

Parameter		Value	Source
β	Discount factor	0.91	Bianchi (2011)
R^b	Gross interest rate on debt	1.06	Standard
R^s	Gross interest rate on reserves	1	Standard
γ	Investment curvature	0.8	Comin and Gertler (2006)
$\bar{g}$	Foreign growth rate	0.0261	Data
ε^R	Interest rate shock	0.0196	Mendoza (2010)

Table 4 CALIBRATED PARAMETERS

Parameter		Value	Data target		Model
η	Investment efficiency	0.102	Mean CA-to-GDP	-0.017	-0.016
κ	Productivity spillover	0.3	Fire-sale price/normal price	0.37	0.35
ζ	Share of foreign assets	0.42	Elasticity of fire-sale price	1.74	1.70
ψ^b	Debt-elasticity of spread	0.01	S.D. of CA-to-GDP	0.063	0.063
$\bar{b}$	Baseline debt-to-GDP	0.8	Mean debt-to-GDP	0.53	0.53
θ	Size of liquidity shock	0.45	Mean reserve-to-GDP	0.17	0.17

occurs, the economy goes back to a normal state and $\varepsilon_t^R = \varepsilon^R$ with probability 0.9, and a liquidity shock occurs again with probability 0.1.

Given the externally determined parameter values, the remaining six parameter values are jointly determined to match the simulation moments of the decentralized economy to the corresponding data in Section 2 or empirical estimates in other papers. These parameters are (1) the investment efficiency η, (2) the productivity spillover coefficient κ, (3) the share of foreign assets in foreign production ζ, (4) the debt-elasticity of the spread ψ^b, (5) the baseline debt-to-GDP ratio for the debt-elastic component of the spread $\bar{b}$, and (6) the size of a liquidity shock θ. η and $\bar{b}$ determine the households' incentive to borrow from abroad and are set to match the simulation mean of the current account and the debt-to-GDP ratio to the data. ψ^b is set to target the standard deviation of the current account. κ and ζ are closely related to the liquidation price and its elasticity to liquidity holdings. Aguiar and Gopinath (2005) show that during the Asian currency crisis, firms were acquired by foreign investors at a fire-sale price, which is on average equal to 37% of the market price before the crisis. They also estimate the elasticity of the fire-sale price to firm liquidity holdings to be 1.74. We set κ and ζ so that the fire-sale price and its elasticity during sudden stops in the model match these numbers.[7] Finally, θ is determined to match the mean reserve-to-GDP ratio in the model simulation to the data in Section 2. The parameter values and the

[7] For the market price of productive assets, we use the simulation mean of the domestic value of assets ξ_t.

calibration results are summarized in Table 4.

4.2 Business Cycle Moments

We simulate the model with stochastic shocks for 100,000 periods and compute the business cycle moments, dropping the first 10,000 periods. Table 5 displays the mean and standard deviations of key variables under the decentralized economy (DE) and the social planner's allocation (SP). Consumption, investment, debt, reserves, and current account are expressed in the ratios to GDP. Standard deviations are divided by the mean of each variable. Comparing debt and reserves between DE and SP, we observe that debt is slightly higher in DE, but the reserve is substantially larger in SP. Consequently, the mean net foreign asset position-to-GDP ratio is -0.361 in DE and -0.319 in SP. The current account is more volatile and larger in deficit in DE than in SP. A small gap in debt does not mean that a debt tax is unimportant. If a subsidy on reserves is provided but a tax on debt is not imposed, decentralized households would hold larger debt, offsetting the stabilization effect of a subsidy on reserves. We study the case of one policy instrument in Appendix C and show that households offset the policy effects when only one of a debt tax or a reserve subsidy is introduced.

The simulation means of tax and subsidy rates are 4.35% and 9.48% respectively. There is a rationale for setting the subsidy rate on reserves higher than the tax rate on debt. Combining the two Euler equations with a tax and a subsidy (24) and (25), we obtain the following equation, which is a variant of the key equation (20):

$$\beta((1+\tau_t^b)\tilde{R}_t - (1+\tau_t^s)R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[((1+\tau_t^s)R^s - \theta_{t+1}(1+\tau_t^b)\tilde{R}_t)\psi_{t+1}] + \nu_t \quad (28)$$

As we discussed in Section 3.3, the left-hand side is the opportunity cost of holding reserves, and the right-hand side is the liquidity advantage of reserve holdings. In the case of $\tau_t^s > \tau_t^b$, the opportunity cost becomes lower and the liquidity advantage becomes larger. Both these effects induce households to hold a larger amount of reserves and take a safer position. In contrast, setting a debt tax rate higher than a reserve subsidy rate would discourage private reserve holdings by increasing the opportunity cost and reducing the liquidity advantage. In the last line in Table 5, the crisis probability is substantially lower in SP.[8]

Figure 3 plots changes in debt and reserves as a ratio to GDP, $(-b_t + b_{t-1})/a_t$ and

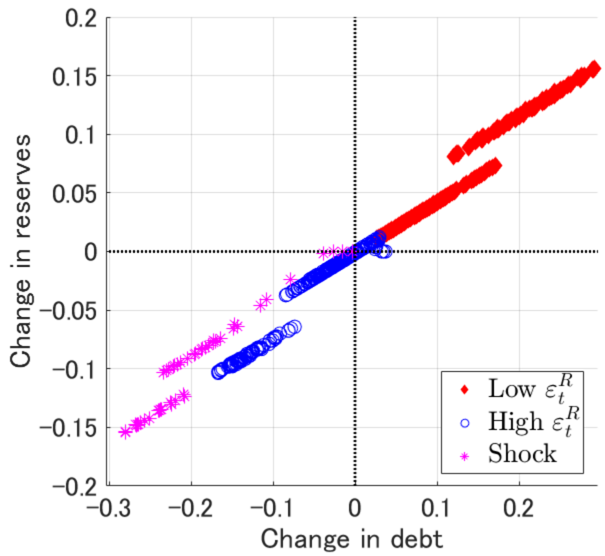
[8]Following the literature, a crisis in the model is identified when the current account is more than two standard deviations above its mean.

Table 5 Stochastic simulation moments

	Decentralized economy		Social planner	
	Mean	S.D.	Mean	S.D.
Consumption	0.807	0.035	0.811	0.036
Investment	0.180	0.164	0.171	0.172
Debt	−0.533	0.373	−0.530	0.341
Reserve	0.172	0.573	0.211	0.402
Current account	−0.016	0.063	−0.008	0.055
Mean tax on debt	...		4.35%	
Mean subsidy on reserve	...		9.48%	
Crisis probability	3.20%		0.18%	

Note: Consumption, investment, debt, reserves, and the current account are in GDP ratios. Standard deviations are divided by the mean of each variable.

Figure 3 Changes in debt and reserves over stochastic simulation



Note: This figure plots changes in debt and reserves at each period in a stochastic simulation of the decentralized economy. The horizontal line indicates changes in debt $(-b_t + b_{t-1})/a_t$, and the vertical axis indicates changes in reserves $(s_t - s_{t-1})/a_t$.

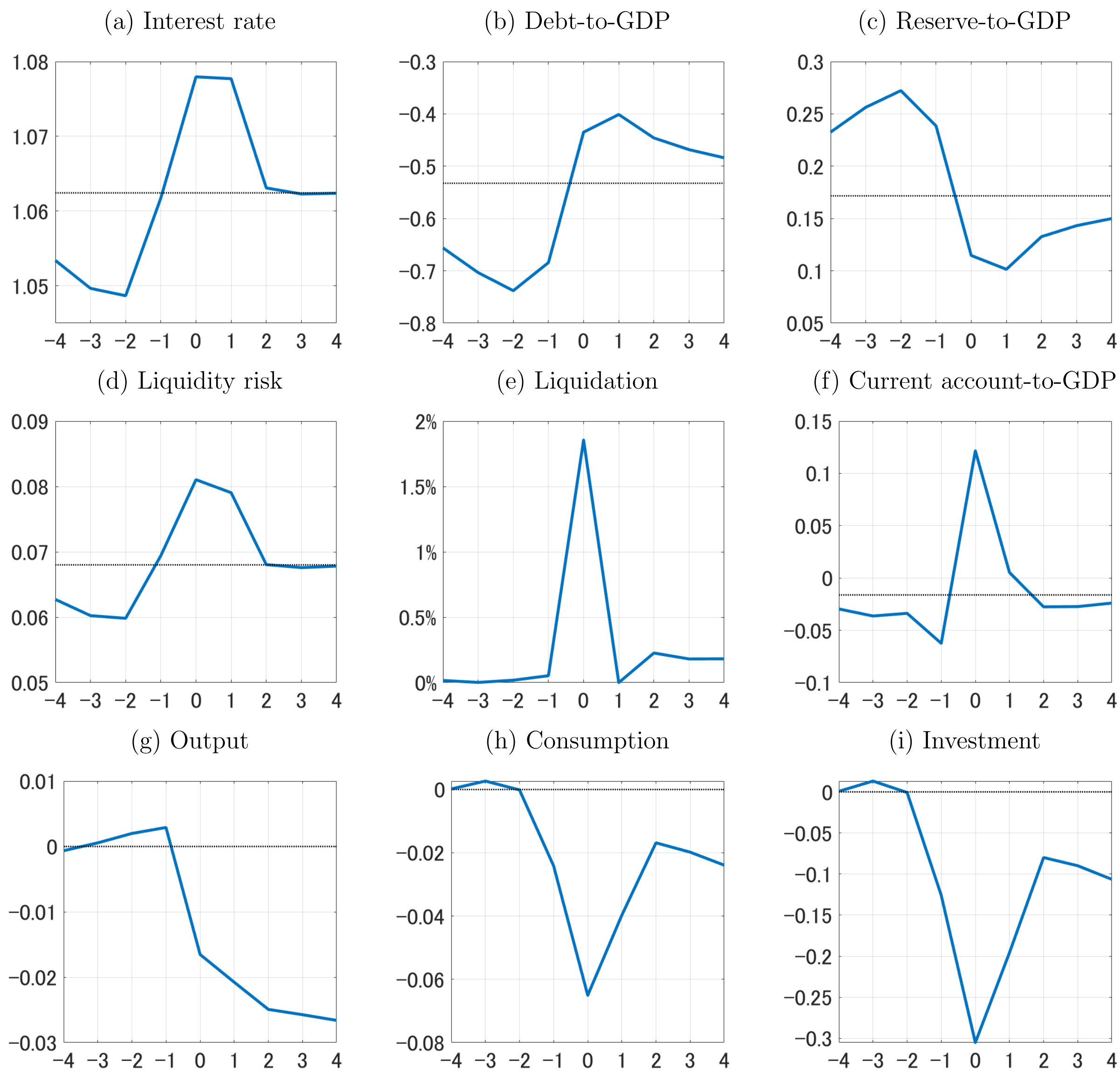
$(s_t - s_{t-1})/a_t$ respectively, at each period in a stochastic simulation of the decentralized economy. Dots in the top-right (bottom-left) quadrant indicate periods when both debt and reserves increase (decrease). There are two observations, both of which are consistent with our empirical findings in Section 2. First, changes in debt and reserves are positively correlated with each other, indicated by the fact that most dots are located in the top-right and bottom-left quadrants. The correlation between changes in debt and reserves is 0.99 in our model. Second, both debt and reserves increase when the interest rate is low, indicated by solid diamonds, and both decrease when the interest rate is high, indicated by circles and asterisks. The correlation between the interest rate and changes in debt is −0.84, and the correlation between the interest rate and changes in reserves is −0.81. The negative correlation between the interest rate and changes in debt is straightforward as it implies that households borrow more when the interest rate is low. There are two reasons why households increase reserves when the interest rate is low. First, as households borrow more, a liquidity risk $-\theta b_t - s_t$ becomes larger, and the value of liquidity ψ_{t+1} becomes higher. This induces households to hold more reserves. Second, a low interest rate on debt implies that the opportunity cost of holding reserves is low and the liquidity advantage of reserves is high. This also induces households to hold more reserves. The same logic applies when the interest rate is high. The dynamics under SP show the same patterns.

4.3 Crisis Dynamics

We next show the crisis dynamics of the model. Following the literature, we identify a crisis in the model when the current account-to-GDP ratio is more than two standard deviations above its long-run mean. We pick up all crisis events from a 100,000-period stochastic simulation and compute the average dynamics of the model variables around crises. Figure 4 plots the average crisis dynamics under the decentralized economy.

Panel (a) plots the interest rate dynamics. The interest rate is low from period −4 to −2, and increases one period before a crisis at period −1. Responding to these interest dynamics, Panels (b) and (c) show that debt and reserves are large up to period −2, and slightly shrink at period −1. Panel (d) plots a liquidity risk $-\theta b_t - s_t$ in terms of the ratio to GDP, which is the size of a liquidity shortage if a liquidity shock occurs in the next period. The risk increases at period −1 as the interest rate increases and the reserve shrinks. Given this heightened risk, a liquidity shock at period 0 triggers a large asset liquidation and a severe crisis. Panel (e) shows that about 1.9% of assets are liquidated upon a liquidity shock, and the current account sharply reverses in Panel (f). Panels (g), (h), and (i) show the dynamics of output, consumption, and investment. Because these variables grow over time, we compute the 10-period pre-crisis log-linear trend of each variable, and plot log deviations from the trend. Panel (g) shows that output drops by about 2% through a liquidation, and the deviation

Figure 4 Crisis dynamics



Note: These panels plot the average crisis dynamics under the decentralized economy. Debt, reserves, liquidity risk, and the current account are in GDP ratios. A dotted horizontal line in Panels (a), (b), (c), (d), and (f) indicates the simulation mean of each variable. Output, consumption, and investment are log deviations from the 10-period pre-crisis log-linear trend of each variable.

from the pre-crisis trend even widens in the following periods. This widening output loss is due to the fact that asset liquidation slows down future asset accumulation by lowing the investment efficiency in the law of motion (3).[9] Panels (h) and (i) show that consumption

[9]The level of asset and output will recover the pre-crisis trend in the long run through a spillover from

and investment fall below the pre-crisis trend by about 6% and 30% respectively. Although there is a partial recovery from the bottom, both consumption and investment stay lower than the pre-crisis trend in the following periods.

In these crisis dynamics, endogenous growth plays key roles in two respects. First, a sharp reversal in the current account and a drop in consumption are caused by a permanent income loss through asset liquidation. This mechanism is consistent with Aguiar and Gopinath (2007), and different from sudden stops caused by an occasionally binding borrowing constraint. Guntin et al. (2023) show that even top-income households with liquid assets reduce consumption substantially during sudden stops, supporting the view that a consumption drop is caused by a permanent income loss rather than by a borrowing constraint. Second, persistently low productivity after sudden stops leads to output, consumption, and investment persistently lower than the pre-crisis trend. Persistent negative impacts of sudden stops and financial crises on output are consistent with the empirical fact documented by Cerra and Saxena (2008) and Reinhart and Rogoff (2009) among others.

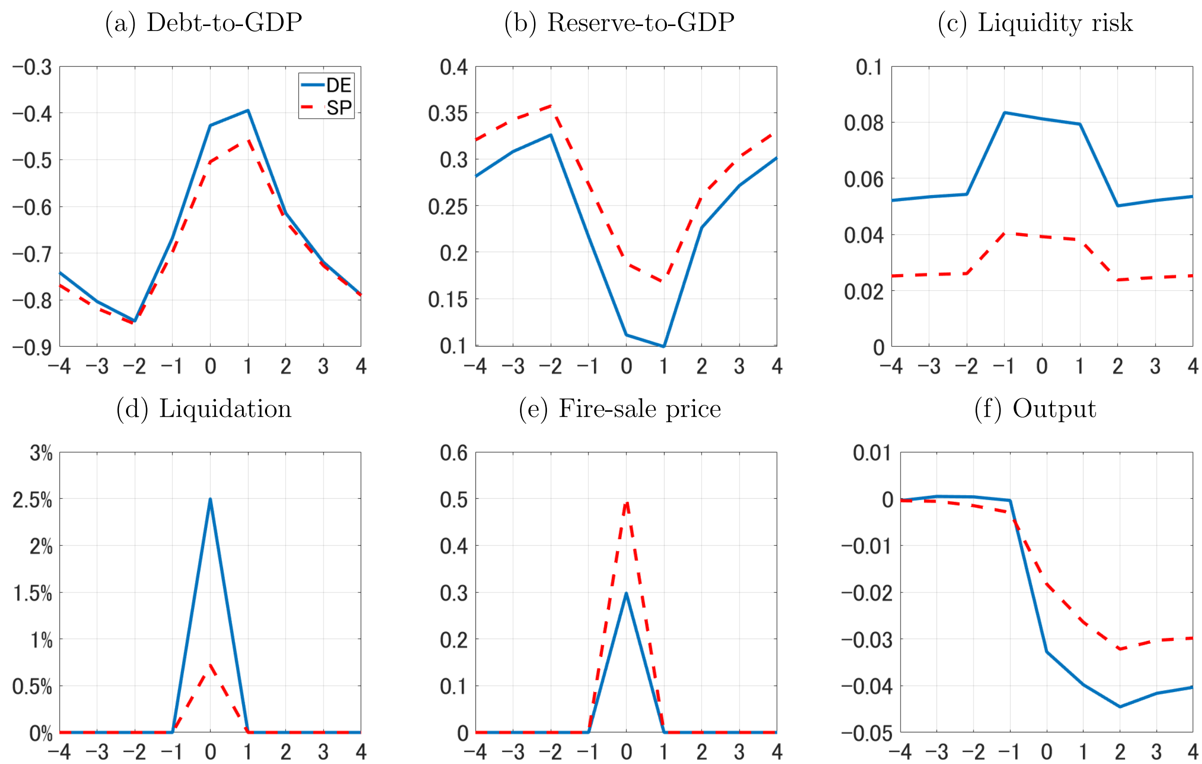
Next, we compare the crisis dynamics under the decentralized economy (DE) and the social planner's allocation (SP). We set the initial state of the economy at the values in period -4 of the crisis dynamics in Figure 4. Then we feed the mean path of stochastic shocks from period -4 to period 4 to the two economies.[10]

Figure 5 plots the result of this exercise. The solid lines are the dynamics under DE, and the dashed lines are those under SP. Panel (a) shows that the planner borrows slightly more than decentralized households. As explained above, this does not mean that a debt tax is unimportant. If the planner does not impose a debt tax, decentralized households would borrow substantially more, given the larger amount of reserves. Panel (b) shows that the planner holds more reserves than decentralized households. Due to this gap in reserves, the liquidity risk $-\theta b_t - s_t$ under SP is roughly half of that in DE in Panel (c), implying that the planner chooses a substantially safer position. Consequently, there is a substantial gap in the amount of asset liquidation in Panel (d). Decentralized households sell 2.5% of assets during a crisis, whereas the planner sells only 0.7%. There is also a sizable gap in the asset fire-sale price. Panel (e) plots the price at which assets are sold, divided by the simulation mean of the domestic value of an asset ξ_t. The numbers in Panel (e) thus indicate the severity of fire sales. It shows that decentralized households sell assets at a price equivalent to 30% of the

foreign technology a_t^*.

[10] More specifically, the interest rate is low from period -4 to -2, high in period -1, and a liquidity shock occurs at period 0. In the following periods, the interest rate is high in period 1, and low from period 2 to 4.

Figure 5 Crisis dynamics under DE and SP



Note: These panels plot the crisis dynamics under the decentralized economy (DE in blue) and the planner's allocation (SP in red dashed). Fire-sale price in (e) is the price at which assets are sold, divided by the simulation mean of the domestic value of one unit of asset ξ_t. Output in (f) is a log deviation from the pre-crisis trend under the decentralized economy.

domestic value in normal times, whereas the planner sells at 50%.

Finally, the substantial gap in liquidation in Panel (d) leads to a large and persistent gap in output after the crisis, as shown in Panel (f). Panel (f) plots the dynamics of output in DE and SP, both in terms of a log deviation from the pre-crisis log-linear trend in the decentralized economy. It shows that output in SP is persistently higher than that in DE by more than 1%, even 4 periods after the crisis. This large and persistent gap in output suggests a sizable welfare gain by the policy intervention, which we study in Section 5.2.

5 Financial Development and Optimal Policy

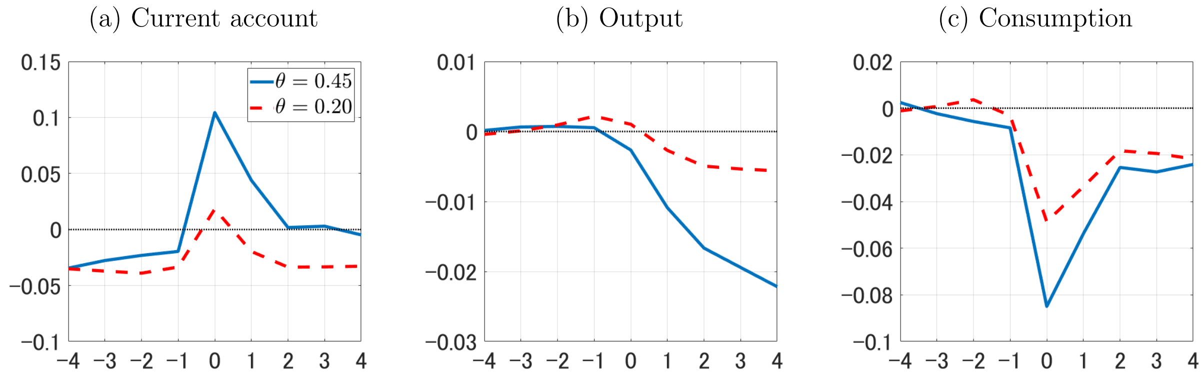
In the quantitative analyses in the previous section, the size of a liquidity shock θ is calibrated and fixed at 0.45. In this section, we change the value of θ and examine how financial development measured by θ affects the optimal capital controls and reserve holdings. We also study the welfare implications of these policies.

As mentioned in Section 2 and 3, we interpret θ as the degree of financial development, where low θ corresponds to high financial development. In general, a well-developed financial market provides diverse financial channels and better risk management tools, thereby reducing a liquidity risk. More specifically, θ in our model is closely linked to two interpretations of financial development in theoretical work. The first is the degree of commitment or enforceability of financial contracts. Chang and Velasco (2001) develop a model of bank runs à la Diamond and Dybvig (1983) for emerging economies and show that the limited commitment to foreign debt repayment may induce foreign lenders to refuse additional loans for rollover, thereby causing a liquidity crisis. Our model is in line with this mechanism if we interpret high θ as a low degree of commitment to foreign debt repayment. A low degree of commitment implies a limited amount of additional foreign loans for rollover, and thus a large fraction of foreign debt needs to be repaid by domestic resources. Mendoza et al. (2009) and Maggiori (2017) model financial development as the degree of financial contract enforceability. Similar to low commitment, low enforceability implies a limited amount of additional foreign loans for rollover. Our model is consistent with these papers as well once we interpret high θ as low enforceability.

Another measure of financial development is the interbank market efficiency in dealing with liquidity risk. Bianchi et al. (2022) develop a model of international banks subject to deposit withdrawal shocks. When a bank is short of liquidity due to a withdrawal shock, it can borrow only a fraction of its deficit in the international interbank market. The fraction is determined by the relative size of the aggregate deficit to the aggregate surplus through a matching function. θ in our model can be interpreted as the matching efficiency in the international interbank market in Bianchi et al. (2022). Low θ corresponds to the high efficiency of the interbank market, enabling banks to cover a larger portion of deficits by borrowing from other banks.

Based on these discussions, we interpret θ as country-level financial development, where low θ indicates high financial development. To ensure consistency with the data, Figure 6 compares the sudden stop dynamics under different values of θ (0.45 and 0.20), which corresponds to Figure 2 in the empirical section. Consistent with Figure 2, low financial development captured by high θ is associated with a more severe crisis characterized by large current account reversals and declines in output and consumption.

Figure 6 CRISIS DYNAMICS FOR HIGH AND LOW θ



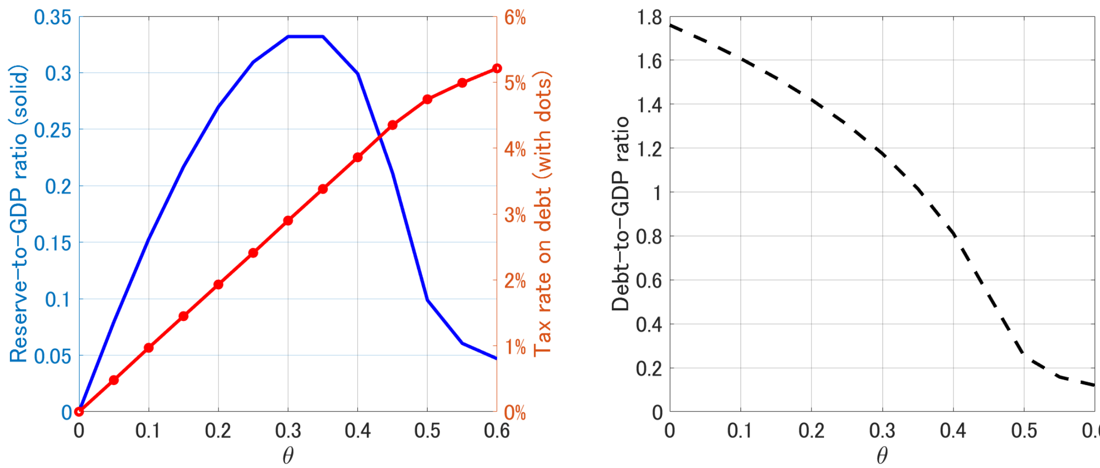
Note: These panels plot the crisis dynamics under the planner's allocation when $\theta = 0.45$ and 0.20. Output and consumption are log deviations from the log-linear pre-crisis trend.

5.1 Capital Controls and Reserve Holdings

We numerically solve the model under the planner's solution with different θ, and simulate the model for 100,000 periods with stochastic shocks. We then compute the simulation mean of reserves and the debt tax rate. The left panel of Figure 7 plots the result, with the value of θ on the horizontal axis ranging from 0 to 0.6 with an interval of 0.05. Reserves are in terms of the ratio to GDP and scaled on the left axis. The debt tax rate is scaled on the right axis. The key observation is that the tax rate monotonically increases in θ, whereas reserves move non-monotonically with θ. Reserve holdings are small when θ is either low or high, and reach the maximum of 33% of GDP at $\theta = 0.30$ and 0.35. 33% of GDP is consistent with the largest reserve-holding countries such as Malaysia in panel A1 of Figure 1. As we interpret θ as the degree of country-level financial development, our model successfully replicates the cross-country differences in the use of capital controls and reserve holdings shown in Introduction and Section 2. The right panel of Figure 7 plots the debt-to-GDP ratio over the same range of θ. It shows that debt monotonically decreases in θ, which is also consistent with the empirical finding in Section 2.

Figure 8 shows the mechanism of the non-monotonic relation between reserves and θ that we discussed in Section 3.3. The solid curve, scaled on the left axis, is $-\theta b$ for each value of θ, where b is the simulation mean of the debt-to-GDP ratio plotted in the right panel of Figure 7. $-\theta b$ indicates the size of a liquidity risk without reserves. When it is high, the planner has an incentive to hold a large amount of reserves to lower the liquidity risk. The line with dots, scaled on the right axis, is $R^s - R\theta$ for each θ. This value indicates the size of

Figure 7 RESERVES, CAPITAL CONTROL, AND DEBT ACROSS DIFFERENT θ



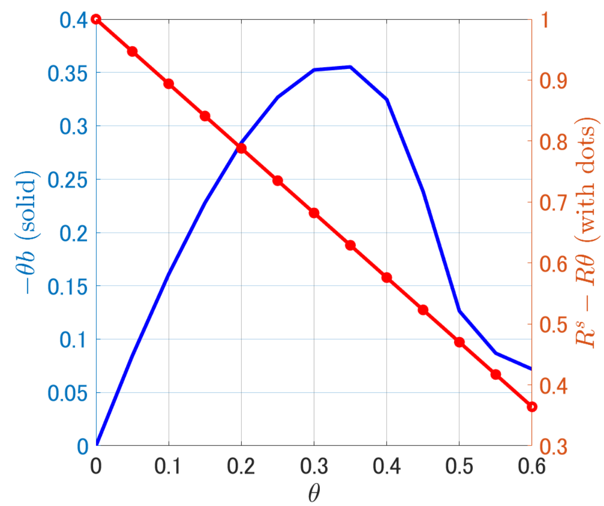
Note: The horizontal axis is θ in both panels. The left panel plots the stochastic simulation mean of the reserve-to-GDP ratio (scaled on left axis) and the debt tax rate (scaled on right axis) across different θ. The right panel plots the stochastic simulation mean of the debt-to-GDP ratio across different θ.

a liquidity advantage of reserves over debt. The panel shows that when θ is low, the liquidity advantage is high but the liquidity risk is low. Therefore, the planner holds a small amount of reserves. As θ becomes higher, the liquidity risk $-\theta b$ increases and peaks at $\theta = 0.35$. The liquidity advantage decreases linearly in θ, but it still takes an intermediate value of 0.63 at $\theta = 0.35$. These two factors induce the planner to hold a large amount of reserves when θ is an intermediate value around 0.35. As θ becomes even higher, the liquidity advantage keeps decreasing, implying a low incentive to hold reserves. In addition, high θ implies a high risk of holding debt, and debt quickly shrinks as θ becomes higher than 0.4, pushing down the liquidity risk $-\theta b$. These two factors explain why reserves become small when θ is high.

5.2 Welfare Analysis

Finally, we study the welfare implications of the optimal policy across different values of θ. For each value of θ ranging from 0 to 0.6 with an interval of 0.05, we compute a welfare gain by the optimal policy in two steps. First, we create grid points over the state space and compute an expected welfare gain by the planner's allocation relative to the decentralized economy on each grid point as an initial state for stochastic simulations. The welfare gain is measured by a permanent consumption compensation that makes households indifferent between the decentralized equilibrium and the planner's allocation. In the second step, we

Figure 8 Determinants of reserve holdings



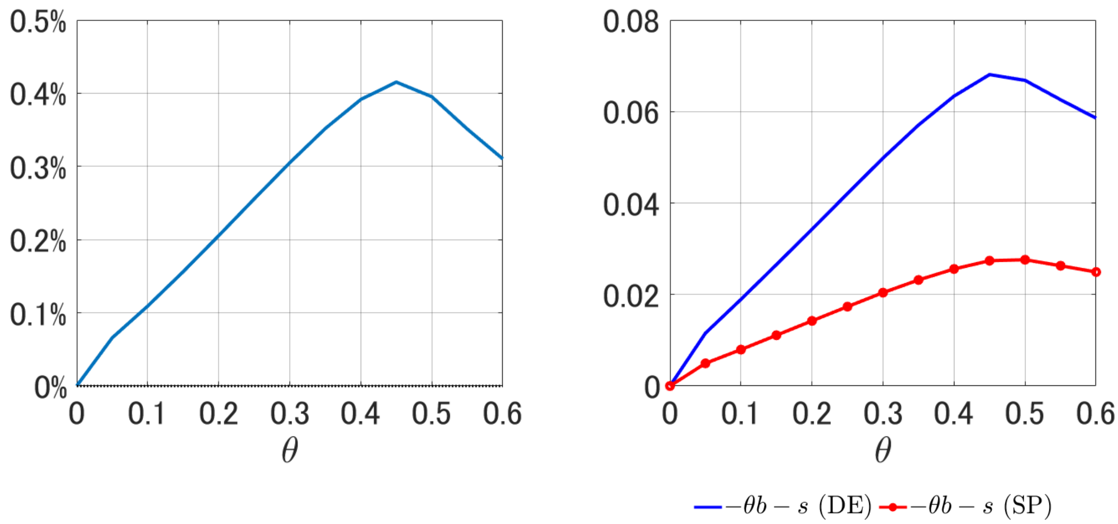
Note: This panel plots $-\theta b$ (scaled on the left axis) and $R^s - R\theta$ (scaled on the right axis) across different θ on the horizontal axis. b is the simulation mean of the debt-to-GDP ratio.

simulate the decentralized economy with stochastic shocks for 100,000 periods and compute the ergodic distribution of the state variables. We then compute the weighted average of the expected welfare gains over the state space, where the weight is given by the ergodic distribution of the state variables.

The left panel of Figure 9 plots the expected welfare gains for each value of θ. It shows that welfare gain is the greatest at 0.42% when $\theta = 0.45$. The size of this welfare gain is substantially greater than welfare gains in many preceding papers in the literature on sudden stops. The key difference from preceding papers is that sudden stops in our model cause persistent negative impacts on output and consumption through an endogenous productivity loss, and policy interventions mitigate these persistent negative impacts.

The right panel of Figure 9 explains why the welfare gain is the greatest when $\theta = 0.45$. The solid line plots the liquidity risk $-\theta b - s$ across different θ under the decentralized economy, where b and s are the simulation mean of debt and reserves in terms of the ratio to GDP. The line with dots plots the liquidity risk under the planner's allocation. The liquidity risk under the decentralized economy peaks at 0.068 when $\theta = 0.45$, whereas the risk under the planner's allocation peaks at 0.028 when $\theta = 0.50$. The risk is substantially higher under the decentralized economy for any θ due to the fire-sale externality. The gap between the two lines indicates excessive risk-taking by the decentralized economy, as the gap illustrates

Figure 9 Welfare analysis and risk taking



Note: The horizontal axis is θ in both panels. The left panel plots a permanent consumption gain by the planner's allocation in percentage points on the vertical axis. In the right panel, the solid line plots the liquidity risk $-\theta b - s$ in the decentralized economy, where b and s are the simulation mean of debt and reserves in terms of the ratio to GDP. The line with dots plots the liquidity risk in the planner's allocation.

how much larger liquidity risk individual households take relative to the socially optimal level of risk. The gap reaches its maximum value of 0.041 when $\theta = 0.45$, indicating that the size of excessive risk-taking corrected by the policies is largest when $\theta = 0.45$. This explains why the welfare gain is the greatest when $\theta = 0.45$.

6 Conclusion

In this paper, we provide a financial development perspective on optimal capital flow management policies. We first provide empirical facts on the relationship between financial development, capital controls, and reserve holdings. On the one hand, the relationship between financial development and reserve holdings is non-monotonic. Countries with an intermediate level of financial development accumulate more reserves than other countries. On the other hand, countries with high financial development use capital controls less actively.

We then develop a small-open-economy model to rationalize these findings. The model features endogenous growth and a liquidity shock that requires households to repay a part of outstanding foreign debt before new borrowing. This assumption motivates households to hold reserves for debt repayment because a liquidity shortage forces households into costly liquidation of productive assets. Because asset liquidation is associated with a fire-sale externality, individual households overborrow and hold too little reserves. The optimal policy calls for public foreign reserve accumulation and a tax on debt.

In the quantitative analysis, we show that the crisis dynamics of our model are in line with the empirical regularities of sudden stops in emerging economies. Current account reversals occur due to a permanent loss in income, and endogenous productivity losses lead to persistent negative impacts on output, consumption, and investment. The social planner internalizes the fire-sale externality and accumulates more reserves than decentralized households to prepare for a liquidity shock. Consequently, asset liquidation is substantially smaller, and the persistent negative effects of crises are mitigated.

Our model can successfully replicate the empirical relationships between financial development, reserve holdings, and capital controls. Welfare gains of optimal policies can be as large as 0.4% of permanent consumption. This result emphasizes the importance of considering persistent impacts of crises on productivity and growth in policy designs.

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Internet Appendix

‘Foreign Reserves and Capital Controls: A Financial Development Perspective’

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by C. Ma, and H. Matsumoto

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A Data Appendix

A.1 Country List

Our sample used in Figure 1 and Table 1 is the same as in Bianchi and Lorenzoni (2022). We exclude countries with extreme reserve-to-GDP ratios and external liability-to-GDP ratios such as Lebanon, Liberia, Luxembourg, Malta, Saudi Arabia, Singapore, and Switzerland. Our final sample thus includes 87 economies from 1980 to 2019, including 30 advanced economies such as Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Latvia, Lithuania, Netherlands, New Zealand, Norway, Portugal, San Marino, Slovak Republic, Slovenia, South Korea, Spain, Sweden, United Kingdom, United States; 31 emerging economies such as Belarus, Brazil, Bulgaria, Chile, Colombia, Costa Rica, Croatia, Ecuador, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Paraguay, Peru, Philippines, Poland, Romania, Russia, Serbia, South Africa, Thailand, Turkey, Ukraine, Uruguay, Venezuela, Vietnam; and 26 low-income economies such as Bolivia, Burundi, Cameroon, Central African Republic, Chad, Comoros, Eritrea, Ethiopia, Ghana, Guinea, Guyana, Haiti, Honduras, Kenya, Madagascar, Malawi, Mauritania, Mozambique, Nicaragua, Rwanda, Sierra Leone, Somalia, Sudan, Tanzania, Uganda, Zambia.

The sample used in Table 2 consists of 47 countries: Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Georgia, Ghana, Hungary, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russia, Senegal, South Africa, South Korea, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam.

A.2 Variable Construction

CA (% GDP) Current account balance to GDP ratio from World Development Indicators.

Capital controls index The negative value of Chinn-Ito index from Chinn and Ito (2006).

EMBI spread (in log) The log value of one plus EMBI spreads from the World Bank's Global Economic Monitor Database.

External debt (% GDP) Total external debt liability divided by GDP, based on Lane and Milesi-Ferretti (2007).

External liability (% GDP) Total external liability divided by GDP, based on Lane and Milesi-Ferretti (2007).

Financial development index Measures how developed financial institutions and financial markets are in each country. Source: IMF.

GDP per capita The log of gross domestic product divided by midyear population from World Development Indicators.

Institutional quality The average of the six Worldwide Governance Indicators from the World Bank Institute including control of corruption, government effectiveness, political stability and absence of violence/terrorism, rule of law, regulatory quality, and voice and accountability.

Reserves (% GDP) Foreign reserves (excluding gold) divided by GDP, based on Lane and Milesi-Ferretti (2007).

Reserve Flows (% GDP) Changes in reserve stocks excluding gold (% GDP) from Alfaro et al. (2014).

Trade (% GDP) The sum of exports and imports of goods and services measured as a share of gross domestic product from World Development Indicators.

Peg A dummy variable for a country to have a fixed exchange rate from Shambaugh (2004).

Private Flows (% GDP) Total private capital inflows (% GDP) from Alfaro et al. (2014).

A.3 Summary Statistics

Table C1 SUMMARY STATISTICS

Panel A: Variables used in Figure 1 and Table 1					
	Obs	Mean	S.D.	Min	Max
Reserves (% GDP)	3279	9.58	8.58	0	68.35
Capital control index	3188	-0.23	1.59	-2.32	1.92
External liability (% GDP)	3204	1.11	1.05	0.16	10.23
External debt (% GDP)	3205	0.74	0.66	0.11	5.6
Financial development index	3400	0.31	0.24	0	0.97
GDP per capita (in log)	3219	8.11	1.71	4.55	11.54
Trade/GDP	3126	0.68	0.39	0.06	3.21
Institutional quality	2088	0.11	1	-2.41	1.95
Peg	3118	0.34	0.47	0	1
Current account/GDP	3222	-0.03	0.07	-0.84	0.31
Panel B: Variables used in Table 2					
	Obs	Mean	S.D.	Min	Max
Private flow (% GDP)	1275	6.24	12.91	-161.68	126.09
Reserve flow (% GDP)	1274	1.51	3.70	-17.11	26.27
EMBI (in log)	844	1.51	0.64	0.00	4.60
GDP per capita (in log)	1499	8.03	0.99	4.55	10.42
Trade/GDP	1471	0.74	0.36	0.00	2.20
Institutional quality	1128	-0.20	0.58	-1.90	1.22
Peg	1432	0.39	0.49	0.00	1.00
Current account (% GDP)	1414	-1.69	6.30	-26.21	38.79

B Model Appendix

B.1 Decentralized Equilibrium

The recursive maximization problem by households is set up as follows:

$$
\begin{aligned}
& V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \\
&= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\
&- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a^\ell_t \right] \\
&- \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^\gamma (a_{t-1} + \kappa a^*_{t-1})^{1-\gamma} + a^\ell_t \right] \\
&+ \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\
&+ \varphi_t q_t a^\ell_t \\
&+ \nu_t \frac{s_t}{R^s}
\end{aligned} \tag{1}
$$

The first-order conditions are as follows:

$$ c_t : \lambda_t = u'(c_t) \tag{2} $$

$$ b_t : \lambda_t = \beta \frac{R_t}{1 + \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right) \frac{b_t/a_t}{R_t}} \mathbb{E}_t V_b(t+1) \tag{3} $$

$$ s_t : \lambda_t - \nu_t = \beta R^s \mathbb{E}_t V_s(t+1) \tag{4} $$

$$ z_t : \lambda_t = \beta \mathbb{E}_t V_z(t+1) \tag{5} $$

$$ a^\ell_t : \psi_t q_t + \varphi_t q_t = \xi_t - q_t \lambda_t \tag{6} $$

$$ a_t : \xi_t = \lambda_t \left[1 + \left(\frac{b_t/a_t}{R_t} \right)^2 \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b} \right) \right] + \beta \mathbb{E}_t V_a(t+1) \tag{7} $$

$$ \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] = 0,\ \psi_t \geq 0 \tag{8} $$

$$ \varphi_t q_t a^\ell_t = 0,\ \varphi_t \geq 0 \tag{9} $$

$$ \nu_t \frac{s_t}{R^s} = 0,\ \nu_t \geq 0 \tag{10} $$

The envelope conditions are given as follows:

$$V_b(t) = \lambda_t + \psi_t \theta_t \tag{11}$$

$$V_s(t) = \lambda_t + \psi_t \tag{12}$$

$$V_z(t) = \xi_t \eta \gamma \left(\frac{z_{t-1}}{a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma - 1} \tag{13}$$

$$V_a(t) = \xi_t \left[1 + \eta(1-\gamma) \left(\frac{z_{t-1}}{a_{t-1} + \kappa a^*_{t-1}} \right)^{\gamma} \right] \tag{14}$$

Forwarding the envelope conditions one period and plugging them into the first-order conditions, we obtain the equilibrium conditions in the text, (15), (16), (17), (18), and (19). Foreign asset a^*_t follows the exogenous law of motion $a^*_t = (1+\bar{g})a^*_{t-1}$.

The decentralized equilibrium is defined by allocations $\{c_t,\, b_t,\, s_t,\, z_t,\, a^\ell_t,\, a_t,\, a^*_t\}_{t=0}^{\infty}$, the Lagrange multipliers $\{\psi_t,\, \varphi_t,\, \xi_t,\, \nu_t\}_{t=0}^{\infty}$, and the liquidation price $\{q_t\}_{t=0}^{\infty}$ that satisfy (2), (3), (8), (15), (16), (17), (18), (19), (8), (9), (10), and the law of motion for a^*_t, given the initial conditions and sequences of exogenous shocks $\{\varepsilon^R_t, \theta_t\}_{t=0}^{\infty}$.

B.2 Social Planner's Problem

The only difference from the decentralized equilibrium is that the planner internalizes that the liquidation price q_t is a function of liquidation a^ℓ_t as in (8). Accordingly, the setup of the planner's problem is identical to the maximization problem in the decentralized economy, except that q_t is a function of a^ℓ_t. Formally,

$$\begin{aligned}
&V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \qquad (15)\\
&= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t)\\
&- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t(a^\ell_t; a^*_{t-1}) a^\ell_t \right]\\
&- \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^{\gamma} (a_{t-1} + \kappa a^*_{t-1})^{1-\gamma} + a^\ell_t \right]\\
&+ \psi^{SP}_t \left[q_t(a^\ell_t; a^*_{t-1}) a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right]\\
&+ \varphi^{SP}_t q_t(a^\ell_t; a^*_{t-1}) a^\ell_t\\
&+ \nu_t \frac{s_t}{R^s}
\end{aligned}$$

The first-order condition regarding liquidation a_t^ℓ is:

$$\psi_t^{SP} + \varphi_t^{SP} = \frac{\xi_t}{q_t + (\partial q_t/\partial a_t^\ell) a_t^\ell} - u'(c_t) \tag{16}$$

$$= \frac{\xi_t}{(1-\zeta) q_t} - u'(c_t) \tag{17}$$

This is (21) in the main text. As explained in the main text, the social value of liquidity ψ_t^{SP} is higher than the private value of liquidity ψ_t because the planner internalizes the effect of liquidation a_t^ℓ on the liquidation price q_t, which is captured by $\partial q_t/\partial a_t^\ell < 0$.

B.3 Public Reserve Holdings

In Section 3.4, we show that the planner's allocation can be decentralized by a tax on debt and a subsidy on reserves. In this subsection, we show that public reserve holdings instead of a subsidy on private reserves can achieve the same allocation. The logic is similar to the one discussed in Lutz and Zessner-Spitzenberg (2023).

Consider a Ramsey planner's problem whose policy instruments are a tax on debt and direct reserve holdings. In particular, this planner can collect a tax from households in a lump sum, buy foreign reserves with the interest R^s, and rebate the proceeds to households in a lump sum. The planner can also provide reserves to households when a liquidity shock hits the economy. Let $\hat{s}_t$ denote the planner's reserve holdings. Given this setup, the households' budget constraint can be written as follows:

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + \frac{\hat{s}_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + \hat{s}_{t-1} + q_t a_t^\ell + T_t \tag{18}$$

where T_t is a lump-sum transfer by the planner to finance the tax on debt. Liquidation is subject to the liquidity constraint:

$$q_t a_t^\ell \geq -\theta_t b_{t-1} - s_{t-1} - \hat{s}_{t-1} \tag{19}$$

Private and public reserves are subject to the non-negative constraint respectively:

$$s_t \geq 0 \tag{20}$$

$$\hat{s}_t \geq 0 \tag{21}$$

Given these policies, the first-order conditions in the decentralized equilibrium remain the same as those in the main text (except the Euler equation regarding debt due to the tax), because public reserve holdings do not distort the households' decisions. The Ramsey planner chooses all the endogenous variables including public reserve holdings $\hat{s}_t$ to maximize the households' expected utility, given the households' first-order conditions as the implementability constraints, and internalizing the effect of liquidation a_t^ℓ on the liquidation price q_t through (8).

In the budget constraint (18) and the liquidation constraint (19), private and public reserve holdings appear only in the form of $s_t + \hat{s}_t$. This means that s_t and $\hat{s}_t$ are perfect substitutes for households, and they only care about the sum of private and public reserves in their decisions. Let s_t^0 denote reserve holdings chosen by households when $\hat{s}_t = 0\ \forall t$, i.e. no public reserves. As the planner increases public reserves $\hat{s}_t$ from zero, households reduce private reserves to achieve $s_t + \hat{s}_t = s_t^0$, because s_t^0 is the individually optimal amount of total reserves. But this is possible only if $s_t \geq 0$ and equivalently $\hat{s}_t \leq s_t^0$. Once public reserves $\hat{s}_t$ exceed the individually optimal reserves s_t^0, households choose $s_t = 0$, and the planner can choose any arbitrary amount of total reserves, which is only public reserves.

Now, suppose the planner introduces the optimal tax on debt τ_t^b given by (26). Given this policy, private reserves chosen by households are smaller than the planner's optimal amount of reserves because of the gap between ψ_{t+1} and ψ_{t+1}^{SP} discussed in Section 3.4. This means that the planner can choose the optimal amount of reserves, and private reserves are zero. These policies achieve the socially optimal allocation discussed in 3.4.

C Case of One Policy Instrument

We show in the main text that the planner needs to use two policy instruments to achieve the socially optimal allocation, one to correct overborrowing and the other to correct too little reserves. In this section, we study the case where only one policy instrument is available. We consider three types of policy instruments one by one: a tax on foreign debt, a subsidy on private reserve holdings, and official reserve accumulation. In all cases, we set $\theta = 0.45$ and compare simulation moments and welfare impacts with those under the decentralized equilibrium and the planner's allocation in the main text.

The first policy is a tax on foreign debt. In this case, the planner cannot directly affect households' decisions on reserve holdings. The optimal tax design thus needs to internalize its impact on households' decisions on reserves. This introduces time inconsistency in the policy design because an expectation for future policies affects households' decisions today. We could consider a time-consistent policy in a Markov perfect equilibrium as in Bianchi and Mendoza (2018), but instead, we introduce the same tax policy (26) as in the main text. While this is not the optimal tax policy in the case of one policy instrument, this exercise helps to underscore the importance of using two policy instruments jointly by simply dropping one of them. Table C2 presents the simulation mean of debt, reserves, liquidity risk $-\theta b - s$, and the tax rate under this policy regime along with those under the decentralized equilibrium and the planner's allocation in the main text. When the debt tax is introduced, debt is substantially smaller than debt in DE and SP, and reserves become zero. As we discussed in Section 4, a debt tax without a reserve subsidy increases the opportunity cost of holding reserves and reduces the liquidity advantage of reserves, both of which discourage private reserve holdings. Although households offset the stabilization effect of a debt tax by reducing reserves, it is only partial because households cannot reduce reserves to be negative. Thus, the resulting liquidity risk is lower than in DE but higher than in SP. The welfare impact of this policy is only a 0.01% gain in permanent consumption.

The second policy is a reserve subsidy. We again consider the same subsidy policy (27) as in the main text. Table C2 shows that this policy induces much larger debt and reserves. A reserve subsidy without a debt tax lowers the opportunity cost of reserves and increases the liquidity advantage of reserves, thereby inducing large reserve holdings. At the same time, high reserves lower the liquidity risk and the private value of liquidity ψ_{t+1}, inducing households to take larger foreign debt. Although this policy induces households to take a substantially safer position with large reserves and low liquidity risk, its welfare impact is a more than 2% loss in permanent consumption. There are two main reasons for this result. First, large debt increases the interest payment to foreign lenders because the interest rate is elastic to the amount of debt, and this is a large loss in domestic resources. Second, 70% of GDP is invested in low-yielding reserves ($R^s = 0$ in our model), which is an inefficient investment and causes a large opportunity cost of reserves. These costs substantially dominate a benefit of the stabilization effect of this policy, resulting in a large welfare loss.

The third policy is official reserve accumulation. As in the main text, we assume that the planner accumulates reserves through a lump sum tax on households, and rebates through a lump sum transfer when there is a liquidity shortage. We assume the same decision rules on reserves as the planner in the main text. Namely, given the state of the economy, the planner here chooses the same amount of reserves as the planner in the main text does. The

Table C2 ONE POLICY-INSTRUMENT CASE

	DE	SP	Debt tax	Reserve subsidy	Official reserve
Debt	-0.533	-0.530	-0.072	-1.589	-0.828
Reserve	0.172	0.211	0	0.706	0.318
Liquidity risk	0.068	0.027	0.032	0.001	0.055
Mean τ_t^b	...	4.35%	4.56%	...	...
Mean τ_t^s	...	9.48%	...	9.41%	...
Welfare gain	...	0.42%	0.01%	-2.17 %	-0.05 %

Note: Debt and reserve are measured in terms of the ratios to GDP. The numbers in this table are the simulation mean under each policy regime. Welfare gain is a permanent consumption gain/loss compared to the expected utility under the decentralized equilibrium.

only difference from the planner in the main text is that the planner here does not impose a tax on debt. Table C2 shows that households under this policy borrow more than in DE and SP, and the planner holds reserves corresponding to 32% of GDP. A resulting liquidity risk is 0.055, which is lower than in DE but higher than in SP. Again, households partially offset the planner's reserve policy by taking larger foreign debt. A welfare impact of this policy is a 0.05% loss in permanent consumption. On the one hand, the stabilization effect of this policy is limited because households largely offset the effect by increasing foreign debt. On the other hand, large debt and reserves imply a high cost of foreign debt repayment and the opportunity cost of holding reserves. These two factors result in an overall welfare loss.

D Two-Period Model

In this section, we introduce a two-period model and show analytical results to facilitate understanding of the full model in the main text. In particular, we prove that the optimal tax rate on debt τ_b is increasing in θ in this two-period model, and provide some intuitions.

D.1 Model Setup

We consider a two-period model with $t = 0$ and 1. At $t = 0$, households choose consumption c_0, foreign bond b_0 (negative b_0 is borrowing), and reserves s_0. Households own a_0 units of productive asset, which is exogenously given and used for production at $t = 1$. There is no production or endowment at $t = 0$, and households need to borrow to consume and buy reserves, implying $b_0 < 0$.

At $t = 1$, households produce goods using productive assets. However, at the beginning of period 1 before production, a liquidity shock may hit the economy with probability π. A liquidity shock forces households to repay a θ fraction of the period-0 debt b_0. Households make this repayment by reserves s_0 and liquidating some of their assets if reserves are not enough for the repayment. Liquidation of assets is costly because liquidated assets can be sold only at a fire-sale price. We assume that the liquidation price q_1 decreases in the amount of liquidated asset a_1^ℓ, satisfying $q_1'(a_1^\ell) < 0$ for any a_1^ℓ. We also assume that liquidation proceeds $q_1(a_1^\ell)a_1^\ell$ are increasing in a_1^ℓ. This assumption implies that the elasticity of q_1 is not too high, so that households can make the repayment by liquidation when reserves are not enough. These two assumptions are satisfied by the functional form assumed in the main text. Individual households take q_1 as given when they make decisions.

Later at $t = 1$, households produce goods and consume. Production is linear in the amount of asset and given as $y = A(a_0 - a_1^\ell)$ with productivity $A > 0$.

The period-by-period budget constraints are given as follows:

$$t = 0 : c_0 + \frac{b_0}{R^b} + \frac{s_0}{R^s} = 0 \tag{22}$$

$$t = 1 : c_1 = A(a_0 - a_1^\ell) + b_0 + s_0 + q_1 a_1^\ell \tag{23}$$

We assume $R^b > R^s$. Liquidation a_1^ℓ needs to satisfy:

$$q_1 a_1^\ell \geq -\theta_1 b_0 - s_0 \tag{24}$$

$$q_1 a_1^\ell \geq 0 \tag{25}$$

where θ_1 is stochastic and takes θ with probability π and 0 with probability $1 - \pi$.

We assume the households' utility function as follows:

$$U(c_0, c_1^N, c_1^L) = \log(c_0) + \beta \left[(1 - \pi)c_1^N + \pi c_1^L\right]$$

β is a discount factor, c_1^N is consumption when no liquidity shock at $t = 1$, and c_1^L is consumption when a liquidity shock hits at $t = 1$. We assume a linear utility at period 1 for analytical tractability.

D.2 Period 1 with No Liquidity Shock ($\theta_1 = 0$)

We solve the decentralized equilibrium of the model backward from period 1. The state variables at period 1 are debt b_0, reserves s_0, and a realization of a liquidity shock θ_1. We first consider the case of no liquidity shock, $\theta_1 = 0$. The value function is given by

$$\begin{aligned} V^N(b_0, s_0) = \max_{c_1^N, a_1^\ell} & \, c_1^N \\ & - \lambda_1^N \left[c_1^N - A(a_0 - a_1^\ell) - b_0 - s_0 - q_1 a_1^\ell \right] \\ & + \psi_1^N \left[q_1 a_1^\ell + s_0 \right] \\ & + \varphi_1^N \left[q_1 a_1^\ell \right] \end{aligned}$$

Combining the first-order conditions regarding c_1^N and a_1^ℓ leads to the following equation:

$$\psi_1^N + \varphi_1^N = \frac{A}{q_1} - 1$$

Note that the liquidity constraint (24) never binds because $\theta_1 = 0$ and $s_0 \geq 0$, implying $\psi_1^N = 0$. We assume $A > q_1(a_1^\ell)$ for any a_1^ℓ, implying $\varphi_1^N > 0$ and $a_1^\ell = 0$. This implies that households do not liquidate unless it is necessary.

Envelope conditions are given by

$$V_b^N(b_0, s_0) = 1 \tag{26}$$

$$V_s^N(b_0, s_0) = 1 \tag{27}$$

where the subscripts b and s indicate the variable with which a partial derivative is taken. The second condition utilizes $\psi_1^N = 0$.

D.3 Period 1 with Liquidity Shock ($\theta_1 = \theta$)

Next, we consider the case of a liquidity shock hitting the economy, $\theta_1 = \theta$. The value function is given by

$$\begin{aligned} V^L(b_0, s_0) = \max_{c_1^L, a_1^\ell} c_1^L \\ &- \lambda_1^L \left[c_1^L - A(a_0 - a_1^\ell) - b_0 - s_0 - q_1 a_1^\ell\right] \\ &+ \psi_1^L \left[q_1 a_1^\ell + \theta b_0 + s_0\right] \\ &+ \varphi_1^L \left[q_1 a_1^\ell\right] \end{aligned}$$

Again, combining the first-order conditions leads to the same equation:

$$\psi_1^L + \varphi_1^L = \frac{A}{q_1} - 1 \tag{28}$$

If $-\theta b_0 > s_0$ and reserves are not enough to cover the early repayment (which is always the case as shown below), there is positive liquidation $a_1^\ell > 0$ and $\varphi_1^L = 0$. Assuming $A > q_1$, ψ_1^L is given by (28), and households liquidate investment just enough to cover the liquidity shortage, implying $q_1 a_1^\ell = -\theta b_0 - s_0$.

If $-\theta b_0 \leq s_0$ and reserves are enough to cover the early repayment (which never happens in the equilibrium as shown below), then $a_1^\ell = 0$ and $\psi_1^L = 0$.

Envelope conditions are given as follows:

$$V_b^L(b_0, s_0) = 1 + \psi_1^L \theta \tag{29}$$

$$V_s^L(b_0, s_0) = 1 + \psi_1^L \tag{30}$$

D.4 Period 0

We go back to period $t = 0$. The value function is given by

$$\begin{aligned} V_0 = \max_{c_0, b_0, s_0} \log(c_0) + \beta\left[(1-\pi)V^N(b_0, s_0) + \pi V^L(b_0, s_0)\right] \\ - \lambda_0 \left[c_0 + \frac{b_0}{R^b} + \frac{s_0}{R^s}\right] \\ + \nu_0 \frac{s_0}{R^s} \end{aligned}$$

The first-order conditions are given as follows:

$$c_0 : \frac{1}{c_0} = \lambda_0 \tag{31}$$

$$b_0 : \lambda_0 = \beta R^b \left[(1-\pi) V_b^N(b_0, s_0) + \pi V_b^L(b_0, s_0) \right] \tag{32}$$

$$s_0 : \lambda_0 - \nu_0 = \beta R^s \left[(1-\pi) V_s^N(b_0, s_0) + \pi V_s^L(b_0, s_0) \right] \tag{33}$$

D.5 Decentralized Equilibrium

Plugging the envelope conditions (26), (27), (29), and (30) into the Euler equations (32) and (33), we obtain the explicit expressions for the Euler equations regarding debt and reserves:

$$\frac{1}{c_0} = \beta R^b \left[(1-\pi) + \pi \left\{ 1 + \psi_1^L \theta \right\} \right] \tag{34}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s \left[(1-\pi) + \pi \left\{ 1 + \psi_1^L \right\} \right] \tag{35}$$

and ψ_1^L is given by

$$\psi_1^L = \begin{cases} \frac{A}{q_1} - 1 & \text{if } -\theta b_0 - s_0 > 0 \text{ and } a_1^\ell > 0. \\ 0 & \text{if } -\theta b_0 - s_0 \leq 0 \text{ and } a_1^\ell = 0. \end{cases} \tag{36}$$

If $a_1^\ell > 0$, it is implicitly given by

$$q_1(a_1^\ell) a_1^\ell = -\theta b_0 - s_0 \tag{37}$$

which is the binding liquidity constraint. Combining the two Euler equations (34) and (35),

$$\beta(R^b - R^s) = \pi \beta \psi_1^L (R^s - \theta R^b) + \nu_0 \tag{38}$$

which is a simplified version of the key equation (20) in the main text. It is straightforward to prove the three propositions in Section 3.3. In particular, Proposition 3 says that households never hold enough reserves to cover the entire early repayment, given the opportunity cost of holding reserves $R^b > R^s$. This means that ψ_1^L takes a positive value in (36). As explained in the main text, ψ_1^L is the *private* value of one unit of liquidity when there is a liquidity shortage. Henceforth, we denote ψ_1^L as ψ_1^{DE} to distinguish it from the *social* value of liquidity

ψ_1^{SP}. Namely,

$$\psi_1^{DE} = \frac{A}{q_1} - 1 \tag{39}$$

The decentralized equilibrium of this model is the six variables $\{c_0, b_0, s_0, a_1^\ell, \psi_1^{DE}, \nu_0\}$ that satisfy the six equations (22), (34), (35), (37), (39), and $\nu_0 s_0/R^s = 0$ with $\nu_0 \geq 0$. c_1^N and c_1^L are given by the resource constraint (23).

D.6 Relation between a_1^ℓ and θ

We now show that the equilibrium size of liquidation a_1^ℓ increases as θ becomes higher. Plugging ψ_1^{DE} in (39) into the key equation (38) leads to the following equation:

$$\beta(R^b - R^s) = \pi\beta\left(\frac{A}{q_1} - 1\right)(R^s - \theta R^b) + \nu_0 \tag{40}$$

If the parameter values are such that $s_0 > 0$ and $\nu_0 = 0$ in the equilibrium, (40) with $\nu_0 = 0$ alone pins down a_1^ℓ. In this case, we can compute how a_1^ℓ changes as θ increases by the implicit function theorem. Formally,

$$\frac{\partial a_1^\ell}{\partial \theta} = -\frac{\pi\beta R^b\left(\frac{A}{q_1} - 1\right)}{-\pi\beta(R^s - \theta R^b)\frac{-Aq_1'}{q_1^2}} > 0$$

It shows that when $s_0 > 0$, the size of liquidation a_1^ℓ increases as θ becomes higher.

If, instead, the parameter values are such that $s_0 = 0$ and $\nu_0 > 0$ in the equilibrium, the liquidation equation (37) reduces to

$$q_1(a_1^\ell)a_1^\ell = -\theta b_0 \tag{41}$$

Because of the assumption that the liquidation proceeds $q_1 a_1^\ell$ increase in a_1^ℓ, a_1^ℓ and b_0 have a one-to-one relation. Plugging this equation into the Euler equation regarding debt (34) with $c_0 = -b_0/R^b$, we obtain

$$\frac{\theta R^b}{q_1(a_1^\ell)a_1^\ell} = \beta R^b\left[(1-\pi) + \pi\left\{1 + \theta\left(\frac{A}{q_1} - 1\right)\right\}\right] \tag{42}$$

This equation pins down a_1^ℓ. Applying the implicit function theorem,

$$\frac{\partial a_1^\ell}{\partial \theta} = -\frac{\frac{R^b}{q_1 a_1^\ell} - \beta R^b \pi \left(\frac{A}{q_1} - 1\right)}{-\theta R^b \frac{1}{(q_1 a_1^\ell)^2} \frac{\partial (q_1 a_1^\ell)}{\partial a_1^\ell} - \beta R^b \pi \theta \left(-\frac{A q_1'}{q_1^2}\right)} > 0$$

The denominator is negative given $\partial(q_1 a_1^\ell)/\partial a_1^\ell > 0$ and $q_1' < 0$. The numerator is positive because multiplying θ gives $\theta R^b / q_1 a_1^\ell - \beta R^b \pi \theta (A/q_1 - 1)$ and this is equal to $\beta R^b > 0$ from (42). Therefore, we have $\partial a_1^\ell / \partial \theta > 0$ when $s_0 = 0$. It follows that whether $s_0 > 0$ or $s_0 = 0$ in the equilibrium, the size of liquidation a_1^ℓ increases as θ becomes higher. This relation will be used later when we analyze the relation between the optimal tax on debt τ_b and θ.

D.7 Social Planner' Allocation

The only difference between the decentralized equilibrium and the social planner's allocation is that the planner internalizes that the liquidation price q_1 is decreasing in the amount of liquidation a_1^ℓ, i.e. $q_1' < 0$. This affects the problem of period 1 with a liquidity shock. The setup is the same as above, but the first-order condition w.r.t. a_1^ℓ leads to the following equation:

$$\psi_1^{SP} + \varphi_1^{SP} = \frac{A}{q_1 + q_1' a_1^\ell} - 1$$

Accordingly, the Euler equations by the planner are given as follows:

$$\frac{1}{c_0} = \beta R^b \left[(1-\pi)1 + \pi \left\{1 + \psi_1^{SP} \theta\right\}\right] \tag{43}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s \left[(1-\pi)1 + \pi \left\{1 + \psi_1^{SP}\right\}\right] \tag{44}$$

and ψ_1^{SP} in these Euler equations is given by

$$\psi_1^{SP} = \frac{A}{q_1 + q_1' a_1^\ell} - 1 \tag{45}$$

$q_1' < 0$ implies $\psi_1^{SP} > \psi_1^{DE}$ given the same a_1^ℓ.

Comparing the Euler equations under the decentralized equilibrium (34) and (35) and those under the planner's allocation (43) and (44), the optimal tax on debt and subsidy on reserves are characterized as follows:

$$\frac{1}{c_0} = \beta R^b(1+\tau_b)\left[1+\pi\psi_1^{DE}\theta\right] \tag{46}$$

$$\frac{1}{c_0} - \nu_0 = \beta R^s(1+\tau_s)\left[1+\pi\psi_1^{DE}\right] \tag{47}$$

with the tax and subsidy given by

$$1+\tau_b = \frac{1+\pi\psi_1^{SP}\theta}{1+\pi\psi_1^{DE}\theta} \tag{48}$$

$$1+\tau_s = \frac{1+\pi\psi_1^{SP}}{1+\pi\psi_1^{DE}} \tag{49}$$

and ψ_1^{DE} given by (39) and ψ_1^{SP} given by (45). Because $\psi_1^{SP} > \psi_1^{DE}$, both τ_b and τ_s are positive, implying a tax on debt and a subsidy on reserves.

D.8 Relation between τ_b and θ

We now show how τ_b changes as θ increases. Because τ_b in (48) is a function of only a_1^ℓ given θ, we can directly take the derivative of τ_b with respect to θ, taking into account the positive effect of θ on a_1^ℓ shown above:

$$\frac{\partial\tau_b}{\partial\theta} = \frac{(\pi\psi_1^{SP}+\pi\theta\frac{\partial\psi_1^{SP}}{\partial\theta})(1+\pi\theta\psi_1^{DE}) - (\pi\psi_1^{DE}+\pi\theta\frac{\partial\psi_1^{DE}}{\partial\theta})(1+\pi\theta\psi_1^{SP})}{(1+\pi\psi_1^{DE}\theta)^2} \tag{50}$$

The sign of this expression is given by the sign of the numerator. First, it is useful to examine the terms without partial derivatives, because it shows how θ affects τ_b given the size of liquidation a_1^ℓ fixed. Collecting the terms without derivatives,

$$\pi\psi_1^{SP}(1+\pi\theta\psi_1^{DE}) - \pi\psi_1^{DE}(1+\pi\theta\psi_1^{SP}) = \pi(\psi_1^{SP}-\psi_1^{DE}) > 0$$

This result is straightforward from (48). With ψ_1^{SP} and ψ_1^{DE} being fixed and given $\psi_1^{SP} > \psi_1^{DE}$, an increase in θ increases the numerator proportionally more than the denominator, increasing the optimal tax rate τ_b.

Another effect of θ on τ_b works through its impacts on a_1^ℓ, which are captured by the partial derivatives. Collecting the terms with derivatives in the numerator of (50),

$$\pi\theta\left[\frac{\partial\psi_1^{SP}}{\partial\theta}(1+\pi\theta\psi_1^{DE})-\frac{\partial\psi_1^{DE}}{\partial\theta}(1+\pi\theta\psi_1^{SP})\right]$$

Using (39) and (45), we can rewrite the bracketed terms explicitly as follows:

$$\begin{aligned}&\left(-\frac{A}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}\frac{\partial a_1^\ell}{\partial\theta}\right)\left[1+\pi\theta\left\{\frac{A}{q_1}-1\right\}\right]\\-&\left(-\frac{A}{(q_1)^2}q_1'\frac{\partial a_1^\ell}{\partial\theta}\right)\left[1+\pi\theta\left\{\frac{A}{q_1+q_1'a_1^\ell}-1\right\}\right]\end{aligned} \tag{51}$$

To obtain further analytical results, we introduce an explicit functional form for the liquidation price $q_1(a_1^\ell)$. We assume essentially the same function as in the main text, $q_1(a_1^\ell) = C(a_1^\ell)^{-\zeta}$ with a constant $C > 0$ and $0 < \zeta < 1$. Then we obtain the following equations:

$$\begin{aligned}q_1' &= -\zeta C(a_1^\ell)^{-\zeta-1} < 0\\\frac{\partial(q_1a_1^\ell)}{\partial a_1^\ell} &= q_1+q_1'a_1^\ell = (1-\zeta)C(a_1^\ell)^{-\zeta} = (1-\zeta)q_1 > 0\\\frac{\partial^2(q_1a_1^\ell)}{\partial(a_1^\ell)^2} &= \frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell} = -\zeta(1-\zeta)C(a_1^\ell)^{-\zeta-1} = (1-\zeta)q_1' < 0\end{aligned}$$

Using these expressions, we compute the gap between the two terms in parentheses in (51), which is actually $\partial\psi_1^{SP}/\partial\theta - \partial\psi_1^{DE}/\partial\theta$:

$$\begin{aligned}A\frac{\partial a_1^\ell}{\partial\theta}\left(-\frac{1}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}+\frac{1}{(q_1)^2}q_1'\right) &= A\frac{\partial a_1^\ell}{\partial\theta}\left(-\frac{q_1'}{(1-\zeta)(q_1)^2}+\frac{q_1'}{(q_1)^2}\right)\\&= A\frac{\partial a_1^\ell}{\partial\theta}\frac{-\zeta q_1'}{(1-\zeta)(q_1)^2} > 0\end{aligned}$$

The sign comes from $\partial a_1^\ell/\partial\theta > 0$ shown above and $q_1' < 0$. This result shows that both ψ_1^{SP} and ψ_1^{DE} increase as θ becomes higher, but ψ_1^{SP} increases more than ψ_1^{DE} does. Intuitively, as θ becomes higher, both the private and social value of liquidity increase, but the social value increases more as the planner internalizes that higher θ induces larger liquidation a_1^ℓ and decreases the liquidation price more. This is the second effect of θ on τ_b.

Lastly, we examine the remaining terms in (51), which are the terms in parentheses times the fractions in the curly brackets. The sign of these terms is given by

$$\left(-\frac{1}{(q_1+q_1'a_1^\ell)^2}\frac{\partial(q_1+q_1'a_1^\ell)}{\partial a_1^\ell}\right)\frac{A}{q_1}-\left(-\frac{1}{(q_1)^2}q_1'\right)\frac{A}{q_1+q_1'a_1^\ell}$$
$$=-\frac{q_1'}{(1-\zeta)(q_1)^2}\frac{A}{q_1}+\frac{q_1'}{(q_1)^2}\frac{A}{(1-\zeta)q_1}=0$$

These terms cancel out each other. Therefore, the expression in (51) is positive and the entire term in (50) is positive. This completes the proof that $\partial\tau_b/\partial\theta > 0$.

To summarize, an increase in θ leads to a higher tax rate τ_b through two channels. The first is the direct channel, which is that the right-hand side of the Euler equation increases more in the planner's allocation than in the decentralized equilibrium given $\psi_1^{SP} > \psi_1^{DE}$. The second channel is indirect through the effect on a_1^ℓ. As θ increases, the size of liquidation a_1^ℓ also increases, and both the private and social value of liquidity ψ_1^{DE} and ψ_1^{SP} increase. But ψ_1^{SP} increases more than ψ_1^{DE}, because the planner internalizes that larger liquidation causes a larger drop in the fire-sale price, and additional liquidity becomes more valuable.

E Numerical Solution

E.1 Stationarized Competitive Equilibrium

We first stationarize the equilibrium conditions. Below is the complete list of the competitive equilibrium conditions:

$$a_t^* = (1+\bar{g})a_{t-1}^* \tag{52}$$

$$\psi_t \left[q_t a_t^\ell + \theta_t b_{t-1} + s_{t-1}\right] = 0,\ \psi_t \geq 0 \tag{53}$$

$$q_t = (1-\zeta)\left(\frac{a_t^*}{a_t^\ell}\right)^\zeta \tag{54}$$

$$\varphi_t q_t a_t^\ell = 0,\ \varphi_t \geq 0 \tag{55}$$

$$a_t = a_{t-1} + \eta(z_{t-1})^\gamma (a_{t-1} + \kappa a_{t-1}^*)^{1-\gamma} - a_t^\ell \tag{56}$$

$$\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t) \tag{57}$$

$$c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t = a_t + b_{t-1} + s_{t-1} + q_t a_t^\ell \tag{58}$$

$$\frac{1}{c_t} = \beta \tilde{R}_t \mathbb{E}_t\left[\frac{1}{c_{t+1}} + \psi_{t+1}\theta_{t+1}\right] \tag{59}$$

$$\frac{1}{c_t} - \nu_t = \beta R^s \mathbb{E}_t\left[\frac{1}{c_{t+1}} + \psi_{t+1}\right] \tag{60}$$

$$\frac{1}{c_t} = \beta \mathbb{E}_t\left[\xi_{t+1}\eta\gamma\left(\frac{z_t}{a_t + \kappa a_t^*}\right)^{\gamma-1}\right] \tag{61}$$

$$\nu_t s_t = 0,\ \nu_t \geq 0 \tag{62}$$

$$\begin{aligned} \xi_t &= u'(c_t)\left[1 + \left(\frac{b_t/a_t}{R_t}\right)^2 \psi^b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right)\right] \\ &\quad + \beta \mathbb{E}_t\left[\xi_{t+1}\left\{1 + \eta(1-\gamma)\left(\frac{z_t}{a_t + \kappa a_t^*}\right)^\gamma\right\}\right] \end{aligned} \tag{63}$$

$$R_t = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\frac{b_t}{a_t} - \bar{b}\right) - 1\right] \tag{64}$$

We divide thirteen endogenous variables into two groups. The first group contains six variables that are automatically determined given a liquidity shock θ_t without households' endogenous choices; the second group contains seven variables that are determined by households' endogenous choices.

First, a_t^* is determined by (52). Given θ_t and the state variables b_{t-1} and s_{t-1}, the liquidity constraint (53) indicates whether there is a liquidity shortage and $a_t^\ell > 0$ or $a_t^\ell = 0$.[A1]

- In the case of $a_t^\ell > 0$, the binding liquidity constraint (53) and (54) pin down q_t and a_t^ℓ. $\varphi_t = 0$ by (55), and a_t is determined by (56). ψ_t is computed by (57) after obtaining c_t and ξ_t later.

- In the case of $a_t^\ell = 0$, $\psi_t = 0$ by (53), q_t and (54) are irrelevant, and $\varphi_t > 0$ by (55). a_t is determined by (56). φ_t is computed by (57) after obtaining c_t and ξ_t later.

Therefore, values for a_t^ℓ, q_t, a_t^*, a_t, and signs for ψ_t and φ_t are automatically determined at the beginning of a period when θ_t is realized, independent of endogenous choices at this period. These six variables are in the first group. The remaining seven variables, c_t, b_t, s_t, z_t, ξ_t, ν_t, and R_t are jointly determined to satisfy seven equations (58)-(64). In these seven equations, the state variables are only b_{t-1} and s_{t-1} in (58), and $q_t a_t^\ell$ are already determined. This means that when agents make decisions to determine seven variables in the second group, the relevant endogenous state variable is $b_{t-1} + s_{t-1} + q_t a_t^\ell$ in (58), and there is no need to know b_{t-1}, s_{t-1}, and z_{t-1} individually.

We utilize this fact to simplify our numerical solution. In particular, we denote $w_t = b_{t-1} + s_{t-1} + q_t a_t^\ell$, and solve the model as if agents first choose endogenous variables determining c_t, b_t, s_t, z_t, ξ_t, ν_t, and R_t given the state variables (w_t, a_t^*, θ_t, ε_t^R); then a liquidity shock θ_{t+1} is realized and a_{t+1}^ℓ, q_{t+1}, a_{t+1}^*, a_{t+1}, ψ_{t+1}, φ_{t+1}, and w_{t+1} are automatically determined.

We stationarize the equilibrium conditions by dividing the equilibrium conditions by the productivity level. $\hat{c}_t$, $\hat{b}_t$, $\hat{s}_t$, $\hat{z}_t$, $\hat{w}_t$, $\hat{a}_t^\ell$, and $\hat{a}_t^*$ denote each variable divided by a_t. $\hat{\psi}_t$, $\hat{\varphi}_t$, $\hat{\nu}_t$, and $\hat{\xi}_t$ denote each variable multiplied by a_t, because these variables shrink overtime at the same rate as the marginal utility $1/c_t$. R_t and q_t are stationary in the first place. g_t denotes the growth rate of the productivity level $a_t/a_{t-1} - 1$. Below is the list of the stationarized

[A1] Technically, agents may liquidate assets more than necessary or even when there is no liquidity shock, and $a_t^\ell > 0$ and $\psi_t = 0$ may hold at the same time. In our calibrated model, however, $\psi_t > 0$ if and only if $a_t^\ell > 0$, implying that agents never liquidate more than necessary to cover the liquidity shortage.

equilibrium conditions:

$$R_t = R^b \exp(\varepsilon_t^R) + \psi^b \left[\exp\left(-\hat{b}_t - \bar{b}\right) - 1\right] \tag{65}$$

$$\hat{c}_t + \frac{\hat{b}_t}{R_t} + \frac{\hat{s}_t}{R^s} + \hat{z}_t = 1 + \hat{w}_t \tag{66}$$

$$\frac{1}{\hat{c}_t} = \beta \tilde{R}_t \mathbb{E}_t \left[\frac{1}{\hat{c}_{t+1}(1+g_{t+1})} + \frac{\hat{\psi}_{t+1}}{1+g_{t+1}} \theta_{t+1}\right] \tag{67}$$

$$\frac{1}{\hat{c}_t} - \hat{\nu}_t = \beta R^s \mathbb{E}_t \left[\frac{1}{\hat{c}_{t+1}(1+g_{t+1})} + \frac{\hat{\psi}_{t+1}}{1+g_{t+1}}\right] \tag{68}$$

$$\frac{1}{\hat{c}_t} = \beta \mathbb{E}_t \left[\frac{\hat{\xi}_{t+1}}{1+g_{t+1}} \eta\gamma \left(\frac{\hat{z}_t}{1+\kappa \hat{a}_t^*}\right)^{\gamma-1}\right] \tag{69}$$

$$\hat{\nu}_t \hat{s}_t = 0 \tag{70}$$

$$\hat{\xi}_t = \frac{1}{\hat{c}_t}\left[1 + \left(\frac{\hat{b}_t}{R_t}\right)^2 \psi^b \exp\left(-\hat{b}_t - \bar{b}\right)\right]$$

$$+ \beta \mathbb{E}_t \left[\frac{\hat{\xi}_{t+1}}{1+g_{t+1}} \left\{1 + \eta(1-\gamma)\left(\frac{\hat{z}_t}{1+\kappa \hat{a}_t^*}\right)^{\gamma}\right\}\right] \tag{71}$$

$$\hat{\psi}_{t+1}\left[q_{t+1}\hat{a}_{t+1}^{\ell}(1+g_{t+1}) + \theta_{t+1}\hat{b}_t + \hat{s}_t\right] = 0 \tag{72}$$

$$q_{t+1} = (1-\zeta)\left(\frac{\hat{a}_{t+1}^*}{\hat{a}_{t+1}^{\ell}}\right)^{\zeta} \tag{73}$$

$$\hat{\varphi}_{t+1} q_{t+1} \hat{a}_{t+1}^{\ell} = 0 \tag{74}$$

$$1 + g_{t+1} = 1 + \eta(\hat{z}_t)^{\gamma}(1+\kappa \hat{a}_t^*)^{1-\gamma} - \hat{a}_{t+1}^{\ell}(1+g_{t+1}) \tag{75}$$

$$\hat{a}_{t+1}^* = \frac{1+\bar{g}}{1+g_{t+1}} \hat{a}_t^* \tag{76}$$

$$\hat{w}_{t+1} = \frac{\hat{b}_t}{1+g_{t+1}} + \frac{\hat{s}_t}{1+g_{t+1}} + q_{t+1}\hat{a}_{t+1}^{\ell} \tag{77}$$

$$\hat{\psi}_{t+1} + \hat{\varphi}_{t+1} = \frac{\hat{\xi}_{t+1}}{q_{t+1}} - \frac{1}{\hat{c}_{t+1}} \tag{78}$$

The state variables at the beginning of a period are $(\hat{w}_t,\ \hat{a}_t^*,\ \theta_t,\ \varepsilon_t^R)$.

E.2 Numerical Solution Algorithm

We solve this stationarized model numerically by a policy-function iteration.

1. We create equally-spaced 101 grid points for $\hat{w}_t$ and 11 grid points for $\hat{a}^*_t$. For each grid point, we create initial guesses for $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (68), and next period liquidation $\hat{a}^\ell_{t+1}$ if a liquidity shock hits at $t+1$.

2. For each grid point, we derive five equations for five unknowns $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (68), and $\hat{a}^\ell_{t+1}$. We then solve for these unknowns using a non-linear solver. Below are the steps:

   (a) Having the five unknowns, (65) gives R_t.

   (b) We assume $\hat{s}_t > 0$ and $\hat{\nu}_t = 0$. Then (68) gives $\hat{c}_t$.

   (c) We plug $\hat{c}_t$, $\hat{b}_t$, and $\hat{z}_t$ into (66) to check whether $\hat{s}_t > 0$. If $\hat{s}_t < 0$, then we set $\hat{s}_t = 0$ and compute $\hat{c}_t$ from (66) and $\hat{\nu}_t$ from (68). Now we have $\hat{c}_t$, $\hat{b}_t$, $\hat{s}_t$, $\hat{z}_t$, R_t, $\hat{\xi}_t$, and $\hat{\nu}_t$.

   (d) Next-period shocks $(\theta_{t+1}, \varepsilon^R_{t+1})$ are realized. For each realization,

      - If no liquidity shock, then $\hat{a}^\ell_{t+1} = 0$, and q_{t+1} and (73) are irrelevant. (72) implies $\hat{\psi}_{t+1} = 0$, (74) implies $\hat{\varphi}_{t+1} > 0$. Compute g_t from (75), $\hat{a}^*_{t+1}$ from (76), and $\hat{w}_{t+1}$ from (77).
      - If a liquidity shock hits, then (72) implies $\hat{\psi}_{t+1} > 0$, (73) implies $\hat{\varphi}_{t+1} = 0$. We use a guess for $\hat{a}^\ell_{t+1}$ and compute g_t from (75), $\hat{a}^*_{t+1}$ from (76), q_{t+1} from (73), and $\hat{w}_{t+1}$ from (77).
      - Given the next-period state variables ($\hat{w}_{t+1}$, $\hat{a}^*_{t+1}$, θ_{t+1}, ε^R_{t+1}), we compute $\hat{b}_{t+1}$, $\hat{z}_{t+1}$, and the right-hand side of (68) at $t+1$ by linearly interpolating the initial guesses. Then we compute next-period endogenous variables $\hat{c}_{t+1}$, $\hat{b}_{t+1}$, $\hat{s}_{t+1}$, $\hat{z}_{t+1}$, R_{t+1}, and $\hat{\xi}_{t+1}$ following the same steps as above. Also compute $\hat{\psi}_{t+1}$ using (78).

   (e) At this point, unused equations are (67), (69), (71), and a binding liquidity constraint (72) when $\hat{a}^\ell_{t+1} > 0$. We also compute the right-hand side of (68) by explicitly taking the expectations over shocks. Then we have five equations for five unknowns $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (68), and $\hat{a}^\ell_{t+1}$. We solve these five simultaneous equations using a non-linear solver to obtain the decision rules for the five unknowns.

3. After obtaining the decision rules for $\hat{b}_t$, $\hat{z}_t$, $\hat{\xi}_t$, the right-hand side of (68), and $\hat{a}^\ell_{t+1}$ for all grid points, we check the gap between the initial guesses and the obtained decision

rules. If they are close enough, we stop. If the gap is large, we update the guesses by the obtained decision rules and go back to step 2. We repeat this process until the decision rules converge.

The social planner's solution is numerically solved by the same steps, just replacing q_{t+1} with $q_{t+1} - \zeta q_{t+1}$ in (78).